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RELATIVISTIC EFFECTS IN THE ROTATION OF JUPITER'S INNER SATELLITES

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ABSTRACT. The most significant relativistic effects (the geodetic precession and the geodetic nutation, which consist of the effect of the geodetic rotation) in the rotation of Jupiter's inner satellites were investigated in this research. The calculations of the most essential secular and periodic terms of the geodetic rotation were carried out by the method for studying any bodies of the solar system with long-time ephemeris. As a result, for these Jupiter's satellites, these terms of their geodetic rotation were first determined in the rotational elements with respect to the International Celestial Reference Frame (ICRF) equator and the equinox of the J2000.0 and in the Euler angles relative to their proper coordinate systems. The study shows that in the solar system there are objects with significant geodetic rotation, due primarily to their proximity to the central body, and not to its mass.

Keywords: relativistic effects, geodetic (relativistic) rotation, inner Jupiter satellites, solar system bodies

1. INTRODUCTION

The most significant relativistic effects in the rotation of celestial bodies are the effects of the geodetic precession and the geodetic nutation, together making up the geodetic rotation. The geodetic precession effect, first considered by Willem de Sitter in 1916 (De Sitter, 1916), is a secular change in the direction of the axis of rotation of a celestial body as a result of a parallel transfer of the angular momentum vector of the body along its orbit in curved space-time. The geodetic nutation effect, first introduced by Toshio Fukushima in 1991 (Fukushima, 1991), is a periodic change in the direction of the axis of its rotation arising for the same reason.

In 2015, Francesco Biscani and Sante Carloni (Biscani and Carloni, 2015), considering a simplified model of the rotation of the Jupiter's satellites: Io (J1) and Metis (J16). They assumed that the satellites are homogeneous spheres, and reference planes were chosen perpendicular to the planet's rotation axis. As the result, is obtained the theoretical value of their geodetic precession. The two satellites are considered separately, and thus assumed not to interact with each other.

In our previous investigations (Melnikov et al., 2020), the dynamics of rotation of small satellites of the planets of the solar system were studied. Satellites with well-established rotation parameters (Archinal et al., 2018) (Jupiter: Amalthea (J5); Saturn: Iapetus (S8), Phoebe (S9), Prometheus (S16) and Pandora (S17); Uranus: Miranda (U5)) were considered.

It was found that the value of the geodetic precession of Amalthea, which is one from the closest satellites of Jupiter, is 50 times greater than the value of the geodetic precession of Mercury. Recall that Mercury has the largest value of geodetic precession among the planets of the solar system (Pashkevich, 2016), as the planet closest to the Sun (the most massive body of the solar system). Thus, a more detailed study of the relativistic effects in the rotation of the nearest satellites of Jupiter of the Amalthea group becomes relevant and interesting.

The purpose of this study is to determine the most significant secular and periodic terms of the geodetic rotation of the inner satellites of Jupiter (nearest Jupiter's satellites of the Amalthea group): Thebe (J14), Amalthea (J5), Adrastea (J15), and Metis (J16). Calculations will be made using the method for studying the geodetic rotation of any bodies of the solar system (Pashkevich, 2016) with long-time ephemeris.

2. MATHEMATICAL MODEL OF THE PROBLEM

The effects of the geodetic rotation of the inner satellites of Jupiter are studied with respect to their proper coordinate systems (Archinal et al., 2018). The positions, velocities, and orbital elements for Thebe, Amalthea, Adrastea, and Metis are taken from the Horizons On-Line Ephemeris System (Giorgini et al., 1996) at all periods of the ephemeris existence (Table 1). The positions and velocities for the Sun, the Moon, Pluto, and the major planets are calculated using the fundamental ephemeris JPL DE431/LE431 (Folkner et al., 2014).

Table 1. The parameters of the studies of geodetic rotation

Satellite	**The time span (years)**	**Spacing**
Metis	400 (from AD1799 19 December to AD2200 13 January)	42 minutes
Adrastea	400 (from AD1799 19 December to AD2200 13 January)	42 minutes
Amalthea	1000 (from AD1600 07 February to AD2599 06 December)	60 minutes
Thebe	400 (from AD1799 19 December to AD2200 13 January)	90 minutes

The geodetic rotation of the inner satellites of Jupiter is calculated in the rotational elements (α_0, δ_0, W) (Archinal et al., 2018) relative to the International Celestial Reference Frame (ICRF) equator (Ma et al., 1998) and the equinox of the J2000.0 and in the Euler angles (ψ, θ, φ) (Pashkevich and Vershkov, 2019) with respect to their proper coordinate systems (Archinal et al., 2018). The most essential relativistic component of the rotational motion of a body *i* around the proper center of mass is defined by the angular velocity vector of this body geodetic rotation (Brumberg, 1972; Eroshkin and Pashkevich, 2007, 2009):

$$\bar{\sigma}_i = \frac{1}{c^2} \sum_{j \neq i} \frac{G m_j}{\left|\bar{R}_i - \bar{R}_j\right|^3} \left(\bar{R}_i - \bar{R}_j\right) \times \left(\frac{3}{2}\dot{\bar{R}}_i - 2\dot{\bar{R}}_j\right). \quad (1)$$

Here c is the velocity of light; G is the gravitational constant; m_j is the mass of a body j; $\bar{R}_i, \dot{\bar{R}}_i, \bar{R}_j, \dot{\bar{R}}_j$ are the vectors of the barycentric position and velocity of the bodies i and j. The symbol $\times$ means a vector product; the subscripts i for the investigated body and j for perturbing bodies are taken from the set bodies: Thebe, Amalthea, Adrastea, Metis, the Moon, the major planets, Pluto, and the Sun. Further in the formulas the body index *i* will be omitted.

The geodetic rotation velocities values are determined over various time spans with different time spacing (Table 1) for each investigated inner satellites of Jupiter.

The expressions of the geodetic rotation velocities values for a body are determined in the Euler angles (Pashkevich and Vershkov, 2019) as follows:

$$\left.\begin{aligned} \Delta\dot{\psi} &= -\frac{\sigma_1 \sin\varphi + \sigma_2 \cos\varphi}{\sin\theta} \\ \Delta\dot{\theta} &= -\sigma_1 \cos\varphi + \sigma_2 \sin\varphi \\ \Delta\dot{\varphi} &= \sigma_3 - \Delta\dot{\psi}\cos\theta \end{aligned}\right\}. \tag{2}$$

Here ψ, θ, and φ are the Euler angles (ψ is the longitude of the descending node of epoch J2000 of the body equator, θ is the inclination of the body equator to the fixed ecliptic J2000, and φ is the proper rotation angle of the body between the descending node of epoch J2000 and the principal axis of the minimum moment of inertia [point B, Figure 1]); the dot denotes differentiation with respect to time; $\Delta\dot{\psi} = \dot{\psi}_r - \dot{\psi}$, $\Delta\dot{\theta} = \dot{\theta}_r - \dot{\theta}$, and $\Delta\dot{\varphi} = \dot{\varphi}_r - \dot{\varphi}$ are the differences between the relativistic and Newtonian components of the velocities geodetic rotation of the solar system body; σ_1, σ_2, and σ_3 are reduced (Pashkevich, 2016) components of the angular velocity vector (1) of the geodetic rotation of the inner satellites of Jupiter from the geocentric reference frame epoch J2000 to the body-centric reference frames, given by Archinal et al. (2018).

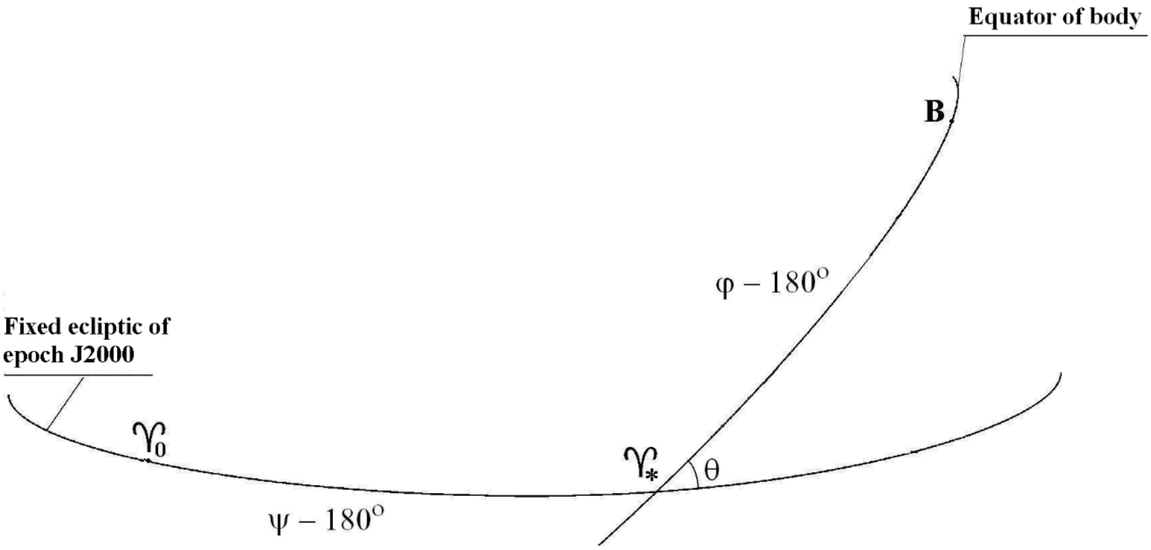

Figure 1. Euler angles used to define the direction of the angular velocity vector of the geodetic rotation for a body of the solar system

Figure 2 shows that the configuration of the location of the rotational elements (α_0, δ_0, W) of the solar system bodies is similar to the configuration for the Euler angles (ψ, θ, φ) of these bodies (see Figure 1). Here α_0 is the right ascension of the north pole of rotation of the body; δ_0 is the declination of the north pole of rotation of the body; W = QB is the angular distance of the meridian zero of the body measured along the equator of the body from a fixed Earth's equator J2000.0 epoch. From expressions (2) were obtained the expressions for the geodetic rotation velocity values of the inner satellites of Jupiter in the rotational elements

$(\alpha_0,\ \delta_0,\ \mathrm{W})$ by replacing the Euler angles with the corresponding rotational elements of the satellites $(\psi \to 270^\circ + \alpha_0,\ \theta \to 90^\circ - \delta_0,\ \varphi \to 180^\circ + \mathrm{W})$:

$$\left.\begin{aligned}
\Delta\dot{\alpha}_0 &= \frac{\sigma_1 \sin \mathrm{W} + \sigma_2 \cos \mathrm{W}}{\cos \delta_0} \\
\Delta\dot{\delta}_0 &= -\sigma_1 \cos \mathrm{W} + \sigma_2 \sin \mathrm{W} \\
\Delta\dot{\mathrm{W}} &= \sigma_3 - \Delta\dot{\alpha} \sin \delta_0
\end{aligned}\right\}, \tag{3}$$

where $\Delta\dot{\alpha}_0 = \dot{\alpha}_{0r} - \dot{\alpha}_0$, $\Delta\dot{\delta}_0 = \dot{\delta}_{0r} - \dot{\delta}_0$, and $\Delta\dot{\mathrm{W}} = \dot{\mathrm{W}}_r - \dot{\mathrm{W}}$ are the differences of the relativistic and Newtonian angles of rotation of the investigated body, respectively; dot means time differentiation.

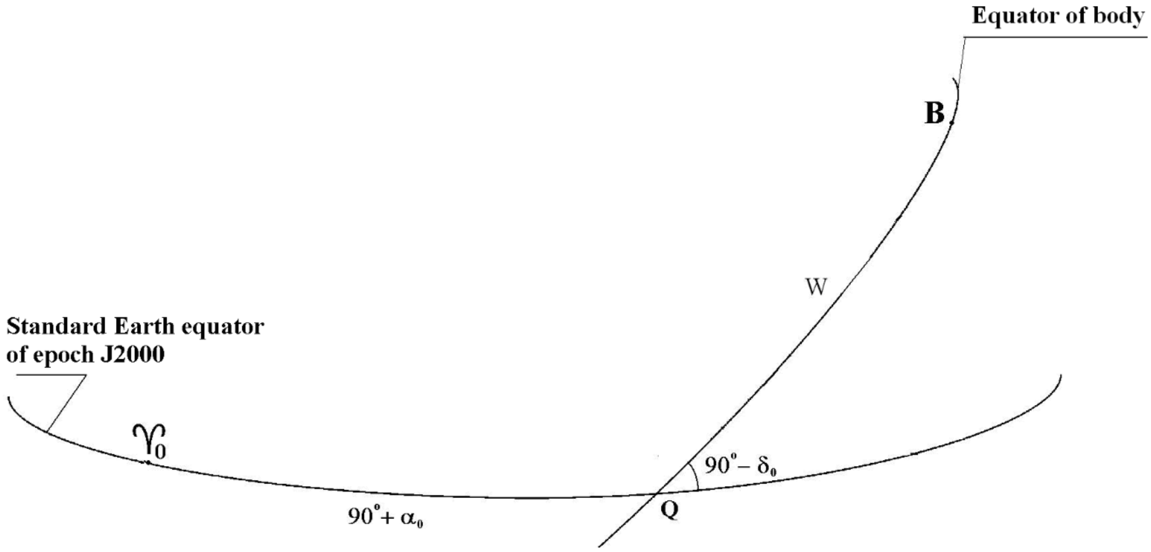

Figure 2. The rotational elements $(\alpha_0,\ \delta_0,\ \mathrm{W})$ used to define the direction of the angular velocity vector of the geodetic rotation for a body of the solar system

The most significant components of the geodetic rotation velocity of the studied body were found by the least-squares method and spectral analysis (Pashkevich, 2016). As a result, the values of the coefficients of the main secular and periodic terms of the body geodetic rotation velocities were calculated. The expressions for the secular and periodic terms of the geodetic rotation velocity of the body are presented in the following form:

$$\Delta\dot{x} = \sum_{n=1}^{N} \Delta\dot{x}_n t^{n-1} + \sum_{l} \sum_{k=0}^{M} (\Delta\dot{x}_{Clk} \cos(\nu_{l0} + \nu_{l1} t) + \Delta\dot{x}_{Slk} \sin(\nu_{l0} + \nu_{l1} t)) t^k, \tag{4}$$

where $\Delta\dot{x}_n$ are the coefficients of the secular terms; $\Delta\dot{x}_{Slk}$, $\Delta\dot{x}_{Clk}$ are the coefficients of the periodic terms; $\dot{x} = \dot{\psi}, \dot{\theta}, \dot{\varphi}, \dot{\alpha}_0, \dot{\delta}_0, \dot{\mathrm{W}}$; ν_{l0}, ν_{l1} are phases and frequencies of the body under study, which are combinations of the corresponding Delaunay arguments (Smart, 1953) and the mean longitudes of the perturbing bodies; the summation index l is the number of added periodic terms and its value changes for each body under study; t is the time in the Julian days; N and M are approximation parameters.

Figure 3 shows the calculated values of the velocities of change of the geodetic rotation of the inner satellites of Jupiter in the Euler angles. The yellow line in the graphs shows a secular trend.

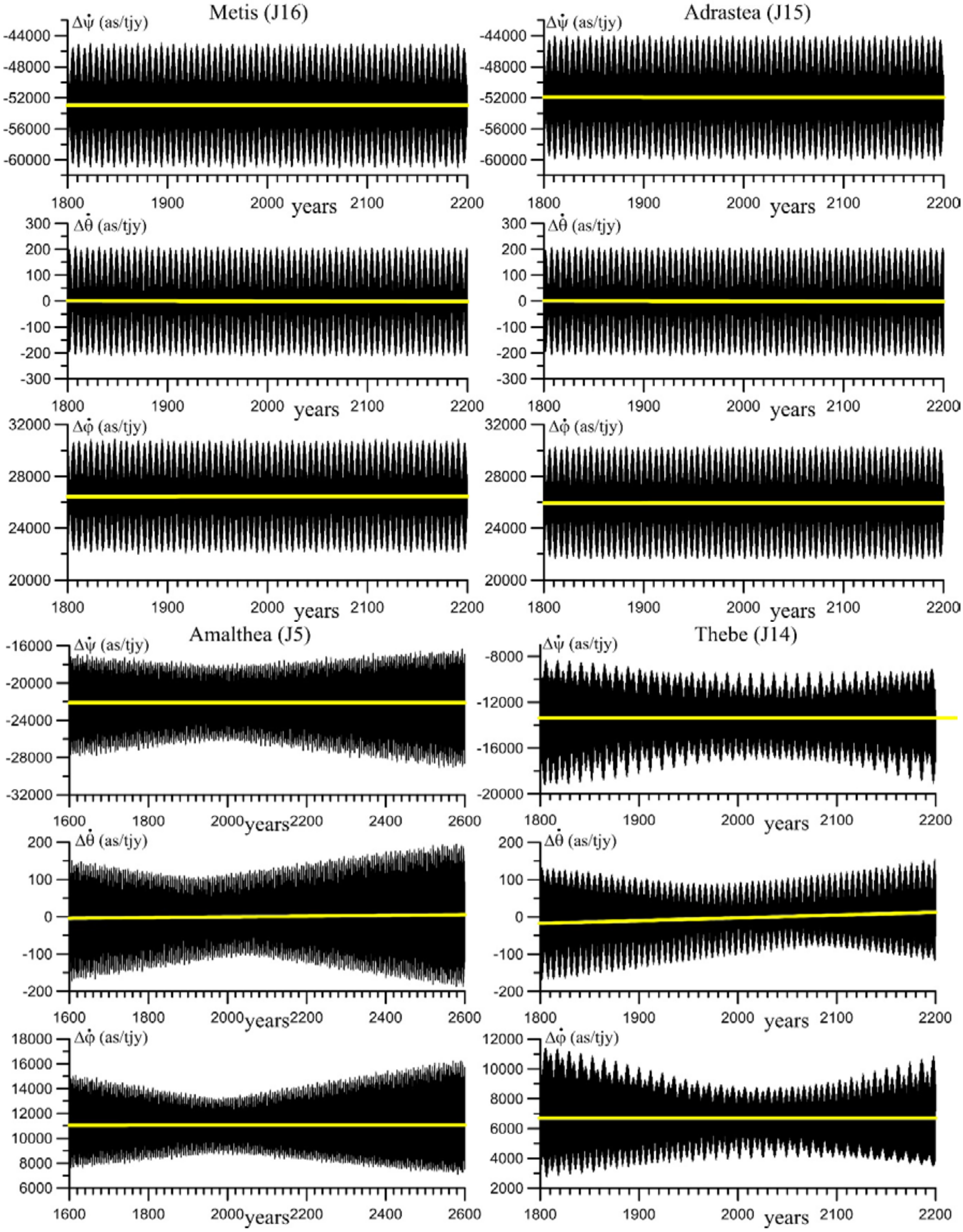

Figure 3. The values of the velocities of the change in geodetic rotations for the inner satellites of Jupiter in the Euler angles

After analytical integration of equation (4) $\Delta x = \int \Delta\dot{x}\, \mathrm{dt}$, the expressions of the secular and periodic terms of the body geodetic rotation can be obtained:

$$\Delta x=\sum_{n=1}^{N}\Delta x_n t^n+\sum_{l}\sum_{k=0}^{M}(\Delta x_{Clk}\cos(\nu_{l0}+\nu_{l1}t)+\Delta x_{Slk}\sin(\nu_{l0}+\nu_{l1}t))t^k, \qquad (5)$$

where $x=\psi, \theta, \varphi, \alpha_0, \delta_0, W$, the coefficients $\Delta x_n=\frac{\Delta\dot{x}_n}{n}$, and the coefficients for sine Δx_{Slk} and cosine Δx_{Clk} terms are calculated using the "Cascade" method (Pashkevich, 2016).

As a result of the least-squares calculations, the values of the approximation parameters for providing the best approximation of the parameters of geodetic rotation were obtained: N = 2 and M = 1.

3. RESULTS

In this research, the values of the secular (Tables 2 and 4) and periodic (Tables 3 and 4) terms of the geodetic rotation for the inner Jupiter's satellites were first determined in the rotational elements (α_0, δ_0, W) with respect to the ICRF equator (Tables 4–7) and the equinox of the J2000.0 and in the Euler angles (ψ, θ, φ) relative to their proper coordinate systems (Tables 2 and 3). For ease of use, in Tables 2 and 3, expression (5) is presented in the form:

$$\Delta x=\Delta x_1+\Delta x_2, \qquad (6)$$

where $\Delta x_1=\sum_{n=1}^{N}\Delta x_n t^n$, $\Delta x_2=\sum_{j}\sum_{k=0}^{M}(\Delta x_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)+\Delta x_{Sjk}\sin(\nu_{j0}+\nu_{j1}t))t^k$, $x=\psi, \theta, \varphi$.

Table 2. The secular terms of geodetic rotation for the inner Jupiter's satellites in the Euler angles

	Metis (J16) ***a* = 128 000 km**	**Adrastea (J15)** ***a* = 129 000 km**	**Amalthea (J5)** ***a* = 181 400 km**	**Thebe (J14)** ***a* = 221 900 km**
	$\Delta\psi_1$ ('')	$\Delta\psi_1$ ('')	$\Delta\psi_1$ ('')	$\Delta\psi_1$ ('')
t	– 52 957.2516	– 51 932.8456	– 22 118.2274	– 13 372.5500
t^2	– 20.0929	– 19.7509	– 0.7460	– 2.8287
	$\Delta\theta_1$ ('')	$\Delta\theta_1$ ('')	$\Delta\theta_1$ ('')	$\Delta\theta_1$ ('')
t	– 0.4232	– 0.4151	– 0.0923	– 2.4703
t^2	– 3.9838	– 3.9067	4.7351	37.7619
	$\Delta\varphi_1$ ('')	$\Delta\varphi_1$ ('')	$\Delta\varphi_1$ ('')	$\Delta\varphi_1$ ('')
t	26 460.9380	25 949.0709	11 055.1784	6 693.8317
t^2	19.8858	19.5347	0.5755	2.8902

In Tables 2 and 3: t is Dynamical Barycentric Time (TDB), which is measured in thousand Julian years (tjy) (of 365 250 days) from the epoch J2000.0, and a is the semi-major axis of the satellite orbit.

As can be seen from Table 2, the calculated value of the geodetic precession of Metis $\Delta\psi_1$ = -1°.4710348 per century, which is in good agreement with the theoretical value of this value -1°.473 per century obtained in (Biscani and Carloni, 2015) for a simplified model of satellite rotation.

Table 3. The periodic terms of geodetic rotation for the inner Jupiter's satellites in the Euler angles

Body	Angle	Period	Argument	Coefficient of sin(Argument) (μas)		Coefficient of cos(Argument) (μas)	
Metis (J16) e = 0.0012	$\Delta\psi_2$	7.0752 h	$\lambda_{516} - \lambda_5$	– 491.02	+66.84t	406.71	+77.88t
		7.0742 h	$\lambda_{516} + \lambda_5$	36.97	+70.96t	326.50	–3.60t
	$\Delta\theta_2$	7.0752 h	$\lambda_{516} - \lambda_5$	–12.11	–0.68t	– 4.19	+1.84t
		7.0742 h	$\lambda_{516} + \lambda_5$	12.68	–0.25t	– 1.43	–2.68t
	$\Delta\varphi_2$	7.0752 h	$\lambda_{516} - \lambda_5$	191.16	–11.21t	– 46.80	–30.55t
		7.0742 h	$\lambda_{516} + \lambda_5$	–36.99	–70.18t	–326.96	+3.90t
Adrastea (J15) e = 0.0018	$\Delta\psi_2$	7.1587 h	$\lambda_{515} - \lambda_5$	–619.25	+144.03t	–96.70	–880.88t
		7.1578 h	$\lambda_{515} + \lambda_5$	–216.77	–332.46t	240.02	–294.02t
	$\Delta\theta_2$	7.1587 h	$\lambda_{515} - \lambda_5$	– 4.80	+16.84t	–11.64	–7.06t
		7.1578 h	$\lambda_{515} + \lambda_5$	9.32	–11.54t	8.41	+2.88t
	$\Delta\varphi_2$	7.1587 h	$\lambda_{515} - \lambda_5$	159.02	–163.79t	110.20	+221.18t
		7.1578 h	$\lambda_{515} + \lambda_5$	217.02	+333.29t	–240.32	+295.24t
Amalthea (J5) e = 0.0032	$\Delta\psi_2$	143.7475 d	Ω_{L55}	14 788.07	–20 766.41t	–8 065.69	–240 278.07t
		71.8737 d	$2\Omega_{L55}$	–705.83	+8 528.53t	1 110.30	+17 772.11t
		11.9577 h	$\lambda_{55} - \lambda_5$	428.18	–518.50t	221.47	+672.87t
		11.9549 h	$\lambda_{55} + \lambda_5$	215.16	+138.37t	–118.79	+403.48t
	$\Delta\theta_2$	143.7475 d	Ω_{L55}	290.56	+9 211.87t	585.78	–792.88t
		71.8737 d	$2\Omega_{L55}$	–16.61	–340.18t	–27.57	+162.20t
		11.9577 h	$\lambda_{55} - \lambda_5$	0.57	–17.16t	9.58	–2.20t
		11.9549 h	$\lambda_{55} + \lambda_5$	– 4.61	+15.81t	–8.33	–5.39t
	$\Delta\varphi_2$	143.7475 d	Ω_{L55}	–14 505.42	+20 809.18t	7 964.79	+240 812.87t
		71.8737 d	$2\Omega_{L55}$	705.95	–8 528.58t	–1 110.22	–17 772.10t
		11.9577 h	$\lambda_{55} - \lambda_5$	–90.20	+232.96t	–118.14	– 114.91t
		11.9549 h	$\lambda_{55} + \lambda_5$	–215.48	–138.61t	118.75	– 404.03t
Thebe (J14) e = 0.0176	$\Delta\psi_2$	291.3118 d	Ω_{L514}	–20 924.96	–523 207.02t	41 738.74	–1 108 462.06t
		145.6559 d	$2\Omega_{L514}$	871.19	+180 592.02t	–9 582.70	+175 870.07t
		16.1914 h	$\lambda_{514} - \lambda_5$	– 463.34	+113.40t	137.40	+384.96t
		16.1863 h	$\lambda_{514} + \lambda_5$	–76.85	+288.85t	205.30	+55.46t
	$\Delta\theta_2$	291.3118 d	Ω_{L514}	–1 560.60	+39 956.22t	–871.89	–19 067.79t
		145.6559 d	$2\Omega_{L514}$	170.03	–3 080.78t	11.00	+3 191.90t
		16.1914 h	$\lambda_{514} - \lambda_5$	–7.24	–3.94t	–6.81	+4.93t
		16.1863 h	$\lambda_{514} + \lambda_5$	8.24	+2.12t	2.76	–9.20t
	$\Delta\varphi_2$	291.3118 d	Ω_{L514}	21 951.15	+524 366.28t	– 42 198.58	+1 110 937.40t
		145.6559 d	$2\Omega_{L514}$	–871.47	–180 591.77t	9 582.78	–175 869.98t
		16.1914 h	$\lambda_{514} - \lambda_5$	147.35	+19.52t	24.12	–124.60t
		16.1863 h	$\lambda_{514} + \lambda_5$	77.02	–288.66t	–205.62	–55.10t

In Table 3: the superscripts h and d are the hours and days, respectively; Ω_{L55} and Ω_{L514} are the longitudes of ascending nodes (orbits of the Jupiter's satellites) on the Laplace plane for Amalthea and Thebes, respectively; λ_5 is the mean longitude of Jupiter; λ_{55}, λ_{514}, λ_{515}, and λ_{516} are the mean jovicentric longitudes of Amalthea, Thebes, Adrastea, and Metis, respectively;

and *e* is the eccentricity of the satellite orbit. The mean longitude of Jupiter was taken from the paper of Brumberg and Bretagnon (2000). The mean longitudes and longitudes of the ascending nodes of the Jupiter's satellites were taken from the paper of Archinal et al. (2018).

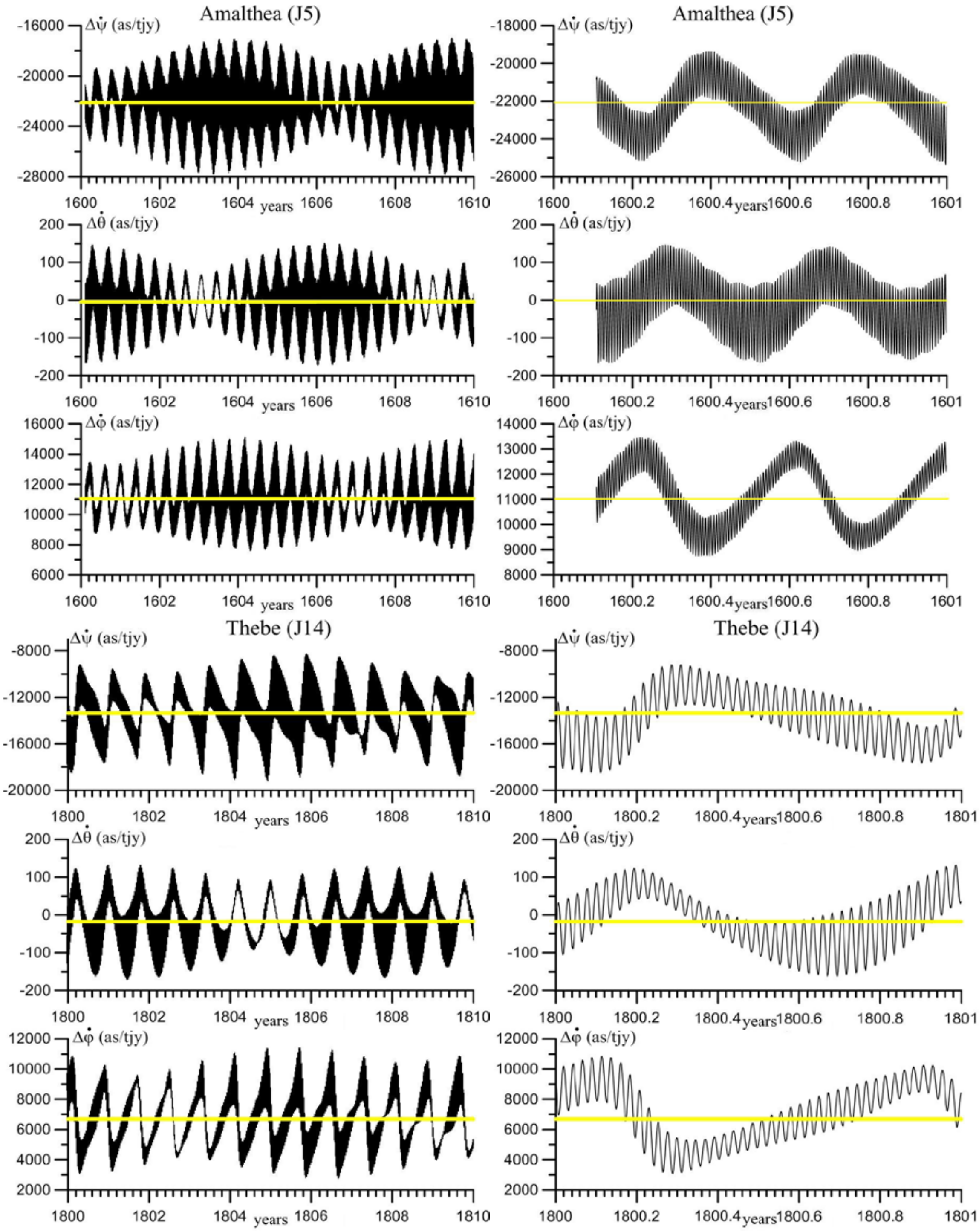

Figure 4. The values of the velocities of the change in geodetic rotations for Amalthea and Thebe in the Euler angles (fragments on 10 and 1 year)

Some asymmetry of the graphical representation of the values of the velocities of the change in geodetic rotations of Thebe for the Euler angles in comparison, for example, with Amalthea (presented in Figure 4) is explained by the relatively large eccentricity of its orbit as compared to the other satellite orbits (Table 3). The value of the eccentricity for Thebe orbit is lager than for Amalthea orbit. The sharp peaks of the curve correspond to Thebe's transits via perihelia. Thus, the values of the periodic terms of the geodetic rotation of any body of the solar system depend not only on their distance from the central body (Pashkevich and Vershkov, 2019), but also on its orbit eccentricity value (Table 3).

Tables 4–7 present the rotational elements (α_0, δ_0, W) of the inner satellites of Jupiter (Archinal et al., 2018) and the most significant secular and periodic terms of their geodetic rotation ($\Delta\alpha_0$, $\Delta\delta_0$, ΔW) calculated in this study.

Table 4. The rotational elements of Metis (J16) and their secular and periodic terms of geodetic rotation

α_0	$268.05 - 0.009\,T$
$\Delta\alpha_0$	$0.1241\,T - 0.00007\,T^2$ $-8.590\times10^{-10}\cos(J15) + 1.424\times10^{-9}\sin(J15) - 2.068\times10^{-11}T\cos(J15) - 1.413\times10^{-11}T\sin(J15)$ $-7.330\times10^{-10}\cos(J16) - 3.720\times10^{-10}\sin(J16) + 8.023\times10^{-12}T\cos(J16) - 1.750\times10^{-11}T\sin(J16)$
δ_0	$64.49 + 0.003\,T$
$\Delta\delta_0$	$-0.0199\,T - 0.00004\,T^2$ $+2.620\times10^{-10}\cos(J15) + 1.306\times10^{-10}\sin(J15) - 1.233\times10^{-12}T\cos(J15) + 3.193\times10^{-12}T\sin(J15)$ $+1.600\times10^{-10}\cos(J16) - 3.161\times10^{-10}\sin(J16) + 7.374\times10^{-12}T\cos(J16) + 3.613\times10^{-12}T\sin(J16)$
W	$33.29 + 1206.9986602\,d$
ΔW	$-0.0000232\,d + 4\times10^{-14}d^2$ $+1.076\times10^{-8}\cos(J15) - 9.604\times10^{-9}\sin(J15) + 1.500\times10^{-10}T\cos(J15) + 1.671\times10^{-10}T\sin(J15)$ $+6.420\times10^{-10}\cos(J16) + 3.344\times10^{-10}\sin(J16) - 6.380\times10^{-12}T\cos(J16) + 1.782\times10^{-11}T\sin(J16)$

Table 5. The rotational elements of Adrastea (J15) and their secular and periodic terms of geodetic rotation

α_0	$268.05 - 0.009\,T$
$\Delta\alpha_0$	$0.1217\,T - 0.00006\,T^2$ $+4.885\times10^{-10}\cos(J13) + 1.559\times10^{-9}\sin(J13) + 2.235\times10^{-10}T\cos(J13) - 7.292\times10^{-11}T\sin(J13)$ $-7.523\times10^{-10}\cos(J14) + 2.984\times10^{-10}\sin(J14) + 3.962\times10^{-11}T\cos(J14) + 1.021\times10^{-10}T\sin(J14)$
δ_0	$64.49 + 0.003\,T$
$\Delta\delta_0$	$-0.0195\,T - 0.00004\,T^2$ $+2.665\times10^{-10}\cos(J13) - 1.079\times10^{-10}\sin(J13) - 1.513\times10^{-11}T\cos(J13) - 3.958\times10^{-11}T\sin(J13)$ $-1.288\times10^{-10}\cos(J14) - 3.242\times10^{-10}\sin(J14) - 4.402\times10^{-11}T\cos(J14) + 1.734\times10^{-11}T\sin(J14)$
W	$33.29 + 1206.9986602\,d$
ΔW	$-0.0000227\,d + 4\times10^{-14}d^2$ $-6.374\times10^{-11}\cos(J13) - 1.418\times10^{-8}\sin(J13) - 2.032\times10^{-9}T\cos(J13) + 1.056\times10^{-11}T\sin(J13)$ $+6.654\times10^{-10}\cos(J14) - 2.579\times10^{-10}\sin(J14) - 3.175\times10^{-11}T\cos(J14) - 8.922\times10^{-11}T\sin(J14)$

Table 6. The rotational elements of Amalthea (J5)
and their secular and periodic terms of geodetic rotation

α_0	$268.05 - 0.009\,T - 0.84\sin(J1) + 0.01\sin(2\,J1)$
$\Delta\alpha_0$	$0.0518\,T - 0.00003\,T^2$ $-1.091\times10^{-8}\cos(J1) + 4.759\times10^{-7}\sin(J1) + 5.759\times10^{-8}\,T\cos(J1) - 1.618\times10^{-8}\,T\sin(J1)$ $+1.424\times10^{-10}\cos(2J1) - 2.774\times10^{-9}\sin(2J1) - 3.866\times10^{-10}\,T\cos(2J1) + 1.040\times10^{-10}\,T\sin(2J1)$ $-7.333\times10^{-10}\cos(J9) - 1.016\times10^{-9}\sin(J9) - 1.531\times10^{-10}\,T\cos(J9) + 1.611\times10^{-10}\,T\sin(J9)$ $+4.740\times10^{-10}\cos(J10) - 4.033\times10^{-10}\sin(J10) - 8.403\times10^{-11}\,T\cos(J10) - 6.913\times10^{-11}\,T\sin(J10)$
δ_0	$64.49 + 0.003\,T - 0.36\cos(J1)$
$\Delta\delta_0$	$-0.0083\,T - 0.00002\,T^2$ $+2.057\times10^{-7}\cos(J1) - 1.972\times10^{-9}\sin(J1) - 6.964\times10^{-9}\,T\cos(J1) - 2.489\times10^{-8}\,T\sin(J1)$ $-9.629\times10^{-10}\cos(2J1) - 1.405\times10^{-11}\sin(2J1) + 2.196\times10^{-11}\,T\cos(2J1) + 8.530\times10^{-11}\,T\sin(2J1)$ $-1.656\times10^{-10}\cos(J9) + 1.449\times10^{-10}\sin(J9) + 3.135\times10^{-11}\,T\cos(J9) + 2.542\times10^{-11}\,T\sin(J9)$ $+1.739\times10^{-10}\cos(J10) + 2.039\times10^{-10}\sin(J10) + 2.980\times10^{-11}\,T\cos(J10) - 3.628\times10^{-11}\,T\sin(J10)$
W	$231.67 + 722.6314560\,d + 0.76\sin(J1) - 0.01\sin(2\,J1)$
ΔW	$-0.0000097\,d + 2\times10^{-14}\,d^2$ $+3.907\times10^{-9}\cos(J1) - 4.044\times10^{-7}\sin(J1) - 5.001\times10^{-8}\,T\cos(J1) + 1.474\times10^{-8}\,T\sin(J1)$ $-1.267\times10^{-10}\cos(2J1) + 2.853\times10^{-9}\sin(2J1) + 3.878\times10^{-10}\,T\cos(2J1) - 1.050\times10^{-10}\,T\sin(2J1)$ $+3.527\times10^{-9}\cos(J9) + 1.030\times10^{-8}\sin(J9) + 1.687\times10^{-9}\,T\cos(J9) - 9.374\times10^{-10}\,T\sin(J9)$ $-4.263\times10^{-10}\cos(J10) + 3.506\times10^{-10}\sin(J10) + 7.343\times10^{-11}\,T\cos(J10) + 6.139\times10^{-11}\,T\sin(J10)$

Table 7. The rotational elements of Thebe (J14)
and their secular and periodic terms of geodetic rotation

α_0	$268.05 - 0.009\,T - 2.11\sin(J2) + 0.04\sin(2J2)$
$\Delta\alpha_0$	$0.0312\,T - 0.00002\,T^2$ $-1.284\times10^{-7}\cos(J2) + 1.691\times10^{-6}\sin(J2) + 3.038\times10^{-7}\,T\cos(J2) + 2.561\times10^{-8}\,T\sin(J2)$ $+2.417\times10^{-9}\cos(2J2) - 2.937\times10^{-8}\sin(2J2) - 5.145\times10^{-9}\,T\cos(2J2) - 4.619\times10^{-10}\,T\sin(2J2)$ $-1.736\times10^{-10}\cos(J11) + 1.244\times10^{-9}\sin(J11) - 1.025\times10^{-10}\,T\cos(J11) - 1.443\times10^{-11}\,T\sin(J11)$ $-6.169\times10^{-10}\cos(J12) - 3.647\times10^{-11}\sin(J12) + 4.938\times10^{-12}\,T\cos(J12) - 5.795\times10^{-11}\,T\sin(J12)$
δ_0	$64.49 + 0.003\,T - 0.91\cos(J2) + 0.01\cos(2J2)$
$\Delta\delta_0$	$-0.0050\,T - 0.00002\,T^2$ $+7.282\times10^{-7}\cos(J2) + 3.502\times10^{-8}\sin(J2) + 1.115\times10^{-8}\,T\cos(J2) - 1.311\times10^{-7}\,T\sin(J2)$ $-6.510\times10^{-9}\cos(2J2) - 4.351\times10^{-10}\sin(2J2) - 1.171\times10^{-10}\,T\cos(2J2) + 1.131\times10^{-9}\,T\sin(2J2)$ $+2.223\times10^{-10}\cos(J11) + 1.410\times10^{-11}\sin(J11) - 7.947\times10^{-13}\,T\cos(J11) + 1.754\times10^{-11}\,T\sin(J11)$ $+1.612\times10^{-11}\cos(J12) - 2.664\times10^{-10}\sin(J12) + 2.429\times10^{-11}\,T\cos(J12) + 1.468\times10^{-12}\,T\sin(J12)$
W	$8.56 + 533.7004100\,d + 1.91\sin(J2) - 0.04\sin(2J2)$
ΔW	$-0.0000058\,d + 1\times10^{-14}\,d^2$ $+9.219\times10^{-8}\cos(J2) - 1.443\times10^{-6}\sin(J2) - 2.651\times10^{-7}\,T\cos(J2) - 1.892\times10^{-8}\,T\sin(J2)$ $-2.387\times10^{-9}\cos(2J2) + 2.933\times10^{-8}\sin(2J2) + 5.164\times10^{-9}\,T\cos(2J2) + 4.613\times10^{-10}\,T\sin(2J2)$ $+4.639\times10^{-9}\cos(J11) - 9.887\times10^{-9}\sin(J11) + 8.151\times10^{-10}\,T\cos(J11) + 3.821\times10^{-10}\,T\sin(J11)$ $+5.425\times10^{-10}\cos(J12) + 3.867\times10^{-11}\sin(J12) - 3.672\times10^{-12}\,T\cos(J12) + 5.247\times10^{-11}\,T\sin(J12)$

T is Dynamical Barycentric time (TDB) measured in Julian centuries (cjy) (of 36525 days) from the epoch J2000.0; *d* is TDB measured in Julian days (jd) from the epoch J2000.0; all angle values (α_0, $\Delta\alpha_0$, δ_0, $\Delta\delta_0$, W, ΔW) measured in degrees; J1 = $\Omega_{L55} = 73^o.32 + 91472^o.9T$, J2 = $\Omega_{L514} = 24^o.62 + 45137^o.2T$, J9 = $\lambda_{55} - \lambda_5$, J10 = $\lambda_{55} + \lambda_5$, J11 = $\lambda_{514} - \lambda_5$, J12 = $\lambda_{514} + \lambda_5$, J13 = $\lambda_{515} - \lambda_5$, J14 = $\lambda_{515} + \lambda_5$, J15 = $\lambda_{516} - \lambda_5$, J16 = $\lambda_{516} + \lambda_5$, $\lambda_5 = 34^o.35 + 3034^o.9T$, $\lambda_{55} = 722^o.6314560d$, $\lambda_{514} = 533^o.7004100d$, $\lambda_{515} = 1206^o.9986602d$, $\lambda_{516} = 206^o.9986602d$.

As can be seen from Table 2 and Tables 4–7, the values of the geodetic precession of the satellites increase as their distance to the central body (Jupiter) decreases, making a significant contribution to the values of right ascents and declinations of the satellites under consideration (see Tables 4–7). So, for example, for Metis (the closest satellite to Jupiter at the moment), the value of the geodetic precession in right ascension $\Delta\alpha_0$ is 13 times greater in absolute value than the resultant value of its right ascension α_0, and the value of the geodetic precession in declination $\Delta\delta_0$ is seven times greater in absolute value than the resulting value of its declination δ_0. For Thebe (the farthest satellite from the Jupiter's satellites under consideration), these values ($\Delta\alpha_0$, $\Delta\delta_0$) are three and two times greater in absolute value than α_0 and δ_0, respectively. From this circumstance, it follows that in the solar system, there are objects with significant geodetic rotation, due primarily to their proximity to the central body, and not to its mass. In particular, the value of the geodetic precession of the inner satellites of Jupiter is comparable to their precession in the Newtonian approximation (see Tables 4–7).

4. CONCLUSIONS

In this study, for the first time, the most significant secular and periodic terms of the geodetic rotation of the inner satellites of Jupiter (Thebe (J14), Amalthea (J5), Adrastea (J15), and Metis (J16)) were determined in the rotational elements relative to the ICRF equator and the equinox of the J2000.0 and in the Euler angles with respect to their proper coordinate systems. The secular terms of geodetic rotation of Jovian satellites mainly depend on their distance from the Jupiter. The values of the periodic terms of the geodetic rotation of any body of the solar system depend not only on their distance from the central body, but also on its orbit eccentricity value.

The present study showed that the values of the geodetic rotation can be significant not only for objects that revolve around super-massive central bodies (neutron stars), but also for bodies with a short distance to the central body, for example, close satellites of giant planets.

The obtained analytical values for the geodesic rotation of the inner moons of Jupiter can be used to numerically study their rotation in the relativistic approximation.

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