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Experimental Verification of Similarity Criteria for Sound Absorption of Simple Metamaterials

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Abstract The paper concerns the dimensional analysis of simple acoustic metamaterials and its experimental verification in a computer model. Due to their decreased thickness possible because of the thermoviscous losses and sound dispersion that occur in acoustic metamaterials, such structures are gaining popularity, both as sound absorbers and diffusers. This implies the need to find their equivalents to be used at scale – both for modeling interiors with metamaterials and developing more complicated structures. The paper discusses the dimensional analysis performed for a generalized unit cell of a metamaterial with a resonator. The dimensional analysis shows the need for scaling both the geometrical dimensions of the structure and the parameters of the medium – air. The dimensional analysis was derived based on the transfer matrix method and was proven correct with the finite element method model. The paper also discusses the consequences of neglecting the air criteria, which are impossible to be fulfilled. This opens the question of finding new criterial numbers allowing the correct reflection of acoustic metamaterials at scale.

Keywords: scale models, scale model tests, MES, TMM, sound absorption coefficient.

1. Introduction

Acoustic metamaterials are structures that due to their construction present unusual properties, such as negative bulk modulus or negative mass density [1, 2] which translates into desired values of acoustic parameters, such as high sound absorption coefficient, sound diffusion, or transmission loss. The concept of metamaterials was translated into acoustics from electromagnetics in the 1990s – a photonic crystal was translated into a phononic crystal and presented frequency band gaps [3]. Since then, the development of acoustic metamaterials has begun and now different types of metamaterials are being investigated, such as membrane-type acoustic metamaterials [4], metamaterials with a periodic distribution of scatterers, i.e., phononic crystals [5], metamaterials with volumetric scatterers, such as Helmholtz resonators or quarter-wavelength resonators [6, 7], or complex hybrid metamaterials [8]. This paper concerns acoustic metamaterials with volumetric resonators, working as sound absorbers or diffusers, such as the ones described in [6, 9, 10]. The advantage of metamaterial structures over classic sound absorbers and diffusers, such as porous absorbers or QR diffusers is the reduction of their thickness. Due to the sound dispersion, metamaterials work in a subwavelength frequency regime. This makes us expect that in a short time after the manufacturing technology is developed, acoustic metamaterials will become more and more popular in architectural acoustics. Therefore, there arises the need to find a way for scaling acoustic metamaterials to be used in scale models of the designed interiors.

This paper concerns sound absorbing properties of acoustic metamaterials, so the dimensional analysis is performed for the sound absorption coefficient. A classic dimensional analysis is presented, and criterial numbers are derived for the elementary unit cells. The performed analysis is then verified with the use of the mathematical model and finite element method modeling. Next, it is shown that not all the criterial numbers derived in the classical dimensional analysis are possible to be realized in a real-life scenario, so the results of neglecting the unrealistic similarity criteria are investigated.

2. Acoustic metamaterials with volumetric scatterers

Acoustic metamaterials with volumetric scatterers (resonators) vary in terms of construction. Most often a unit cell is built of a Helmholtz resonator [7] or slits or ducts loaded with resonators [9]. The effectiveness of such structures can be assessed with different computational methods; the most popular being the plane wave expansion method (PWE), multiple scattering theory (MST), transfer matrix method (TMM), and finite element method modeling (FEM) [11]. In this paper, two of them were employed: TMM and FEM. The mathematical model TMM was used for the dimensional analysis, and FEM modeling was used for the verifications of the results. The transfer matrix method works for the low-frequency regime – only for a plane wave propagating in the system. Usually, this restriction is not an issue since metamaterials are specially designed for low frequencies.

2.1. Transfer Matrix Method

Transfer Matrix Method (TMM) has been widely used in the literature to describe wave propagation in phononic crystals and acoustic metamaterials [12, 6]. It describes effectively sound dispersion and acoustic properties of the structures, such as reflection and transmission, leading to the typically used parameters, such as sound absorption coefficient, sound diffusion, scattering, and transmission loss. The method can be used to derive effective parameters of resonant structures, but also more complicated, multi-layered systems, including layers loaded with resonators (including the thermoviscous phenomena observed in ducts and slits), porous materials, microperforated plates, and membranes. Together with low computational costs, it is a perfect tool for the analysis and design of acoustic metamaterials.

The transfer matrix method provides the relationship between the initial sound pressure p and acoustic velocity v at the start ($x = 0$) and at the end ($x = L$) of the modelled system. A transfer matrix takes the general form of:

$$\begin{bmatrix} p \\ v \end{bmatrix}_{x=0} = \mathbf{T} \begin{bmatrix} p \\ v \end{bmatrix}_{x=L} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix}_{x=L}. \tag{1}$$

In the case of a multi-layered system, e.g., a duct loaded with resonators, the matrix $\mathbf{T}$ is the product of the transfer matrixes for M subsequent layers:

$$\mathbf{T} = \prod_{m=1}^{M} \mathbf{T}^{(m)}. \tag{2}$$

The form of a single transfer matrix depends on the type of the element and the connection between the elements. For a continuous fluid layer, such as a duct, slit, or a fluid layer, the transfer matrix takes the form of:

$$\mathbf{T} = \begin{bmatrix} \cos k_{\mathrm{eff}} l_m & i Z_{\mathrm{eff}} \sin k_{\mathrm{eff}} l_m \\ {}^{i}/_{Z_{\mathrm{eff}}} \sin k_{\mathrm{eff}} l_m & \cos k_{\mathrm{eff}} l_m \end{bmatrix}, \tag{3}$$

where k_{eff} is the effective wave number in the layer, l_m is the length of the layer, and Z_{eff} is the effective characteristic impedance of the medium. A point element in-series (e.g., a change of a cross-section, radiation of a waveguide in the free air) is accounted for as:

$$\mathbf{T} = \begin{bmatrix} 1 & Z_{\mathrm{eff}} \\ 0 & 1 \end{bmatrix}, \tag{4}$$

and a point element in parallel (e.g., a quarter-wavelength resonator or a Helmholtz resonator loaded in parallel):

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 1/Z_{\mathrm{eff}} & 1 \end{bmatrix}. \tag{5}$$

For a rigidly backed acoustic panel, which is usually the case for sound absorbers and diffusers, sound reflection coefficient R can be derived from the transfer matrix as:

$$R = \frac{T_{11} - Z_0 T_{21}}{T_{11} + Z_0 T_{21}}, \tag{6}$$

where $Z_0 = \rho_0 c$ is the characteristic impedance of the surrounding medium (typically air), where ρ_0 is the air density and c – the speed of sound. Sound absorption coefficient α is then defined as:

$$\alpha = 1 - |R|^2. \tag{7}$$

2.2. Effective parameters

For narrow ducts and slits, it is necessary to account for the viscothermal losses. It is done by evaluating complex and frequency-dependent density ρ_{eff} and bulk modulus κ_{eff} for each segment of the metamaterial structure. For ducts and slits of a constant cross-section Stinson [13] gave the following formulas: for a slit:

$$\rho_{\mathrm{eff}} = \rho_0 \left[1 - \frac{\tanh(rG_\rho)}{rG_\rho}\right]^{-1}, \tag{8}$$

$$\kappa_{\mathrm{eff}} = \kappa_0 \left[1 - (\gamma - 1)\frac{\tanh(rG_\kappa)}{rG_\kappa}\right]^{-1}, \tag{9}$$

where ρ_0 is the equilibrium density of the medium, $2r$ is the width of the slit, $\kappa_0 = \gamma P_0$, with $\gamma = 1.4$ being the heat capacity ratio, and P_0 static pressure; $G_\rho = \sqrt{i\omega\rho_0/\eta}$, $G_\kappa = \sqrt{i\omega\mathrm{Pr}\rho_0/\eta}$, with Prandtl number $P_r = 0.71$, and dynamic viscosity of air $\eta = 1{,}813 \cdot 10^{-5}\,\frac{\mathrm{kg}}{\mathrm{ms}}$.
For a circular duct of radius r:

$$\rho_{\mathrm{eff}} = \rho_0 \left[1 - \frac{2J_1(rG_\rho)}{rG_\rho J_0(rG_\rho)}\right], \tag{10}$$

$$\kappa_{\mathrm{eff}} = \kappa_0 \left[1 + (\gamma - 1)\frac{2J_1(rG_\kappa)}{rG_\kappa J_0(rG_\kappa)}\right], \tag{11}$$

where J_0 and J_1 are Bessel functions of the first kind and order 0 and 1, $G_\rho = \sqrt{-i\omega\rho_0/\eta}$, $G_\kappa = \sqrt{-i\omega\mathrm{Pr}\rho_0/\eta}$, the other symbols as described above.

For a rectangular duct of width $2a$ and height $2b$:

$$\rho_{\mathrm{eff}} = -\frac{\rho_0 a^2 b^2}{4{G_\rho}^2 \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\left[\alpha_k^2\beta_m^2(\alpha^2\beta^2 - {G_\rho}^2)\right]^{-1}}, \tag{12}$$

$$\kappa_{\mathrm{eff}} = \frac{\kappa_0}{\gamma + 4(\gamma - 1){G_\kappa}^2/a^2b^2 \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\left[\alpha_k^2\beta_m^2(\alpha^2\beta^2 - {G_\rho}^2)\right]^{-1}}, \tag{13}$$

where $\alpha_k = \left(k + \frac{1}{2}\right)\pi/a$, $\beta_m = \left(m + \frac{1}{2}\right)\pi/b$, remaining symbols as for a circular duct. Effective wave number k_{eff} and effective characteristic impedance Z_{eff} can be calculated as:

$$k_{\mathrm{eff}} = \omega\sqrt{\rho_{\mathrm{eff}}\kappa_{\mathrm{eff}}}, \tag{14}$$

$$Z_{\mathrm{eff}} = \frac{S_0}{S_a}\sqrt{\frac{\rho_{\mathrm{eff}}}{\kappa_{\mathrm{eff}}}}, \tag{15}$$

where $\omega = 2\pi f$ is the angular frequency, where f – considered frequency, S_0 is the area of the unit cell and S_a is the area of the duct (in the case of a slit – areas are replaced by linear dimensions). For more accurate calculations, the corrections for a change of the cross-section and radiation to the free air may be included according to [14]. These corrections depend on the ratio of hydraulic diameters of the ducts and the open area of the top layer, i.e., the geometry of the structure. Any combination of ducts and slits constituting the acoustic metamaterial can be described as the product of the building blocks characterized above.

3. Dimensional analysis

Finding a scaled equivalent of a structure means finding such a structure, that at scale 1:S presents the same acoustic properties as its full-size equivalent in a shifted frequency range (S times higher) [15]. In this paper, the sound absorption coefficient is considered. According to the Buckingham π theorem, a system can be described by a set of non-dimensional criterial numbers. If two systems: full-size one and scaled one have the same criterial numbers, they can be considered similar, and the conclusions made for one system can be transferred to the other [16, 17].

The first step of the dimensional analysis is determining the input quantities that define the output value of the considered parameter – the sound absorption coefficient. In the case of acoustic metamaterials, we can see in Sects. 2.1 and 2.2 that these input quantities depend on the geometry of a particular system. Generally, we can point to the following: frequency f, geometrical dimensions of ducts and slits building a unit cell – generally referred to as l_G, speed of sound c, dynamic viscosity η, static pressure P_0, Prandtl number Pr, heat capacity ratio γ. We can express sound absorption coefficient α as a function of these quantities:

$$\alpha = F(f, l_G, c, \rho_0, \mu, P_0, \mathrm{Pr}, \gamma) \tag{16}$$

Then, we should define the dimensional base – a set of dimensional quantities that can be used to express the remaining dimensional quantities. It is a good practice to use quantities of SI units. In these case, three quantities are chosen: frequency f [Hz = 1/s] – representing time [s], sound speed c [m/s] – representing length [m], and density ρ_0 [kg/m^3] – representing mass [kg]. Having defined the dimensional base, we can express the remaining dimensional quantities using the quantities from the base and define so-called criterial numbers:

$$\Pi_{l_G} = \frac{l_G * f}{c} \qquad \Pi_\mu = \frac{\mu * f}{\rho_0 c^2} \qquad \Pi_{p_0} = \frac{p_0}{\rho_0 c^2}$$

According to the Buckingham π theorem, sound absorption coefficient α can be expressed as the function of these criterial numbers and remaining non-dimensional input quantities:

$$\alpha = \check{F}(\Pi_{l_G}, \Pi_\mu, \Pi_{p_0}, \mathrm{Pr}, \gamma) \tag{17}$$

This means that in the case of a 1:S scale model in the air of normal density and sound speed, the geometrical dimensions of the system and dynamic viscosity of air must be decreased S-times, while the static pressure remains unchanged. It should be noted that also Prandtl number Pr and heat capacity ratio γ must remain unchanged in the model in comparison with its full-size equivalent. As per [15], the speed of sound and air density remain unchanged and the frequency is S times higher. The dimensional analysis was proven to be correct in the example of a rectangular slit of the width of 10 mm, the length of 120 mm, and the width of a unit cell of 50 mm (full-size sample, the dimensions for the considered scale factors were decreased according to the derived similarity criteria). We can see in Fig. 1 that the sound absorption curves determined with TMM and with respect to all the derived similarity criteria are identical.

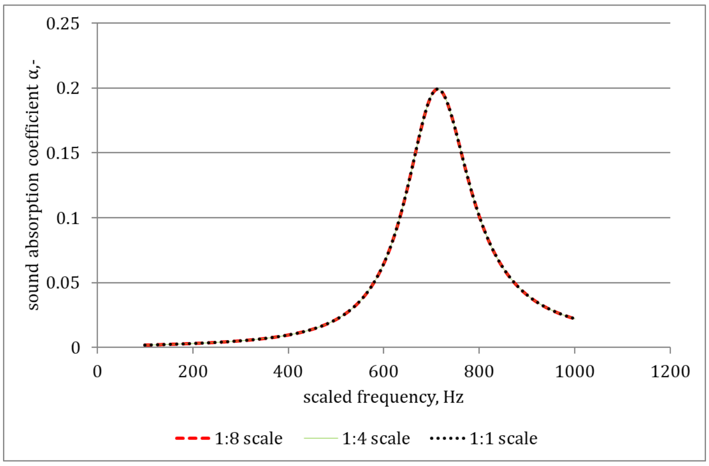

Figure 1. Sound absorption coefficient of a slit (width: 10 mm, length: 120 mm, the width of a unit cell: 58 mm), determined with TMM for a full-size sample and its equivalents for scale factors 1:4 and 1:8.

4. Neglecting similarity criteria regarding the medium

Fulfilling the requirement of the similarity criteria derived in Sect. 3 regarding the medium is particularly difficult. As was shown previously, it is impossible to change air viscosity by changing its easily-modificative parameters, such as temperature and relative humidity for any reasonable scale factors 1:S [18]. What is more, the Prandtl number which should remain unchanged depends on the dynamic viscosity of the medium, which on the other hand should be modified. Also, the speed of sound and air density should remain unchanged. This means that we would have to manipulate air parameters very specifically. The solution could be changing the composition of air or replacing the air with other fluids, chosen specifically to meet the requirements of the dimensional analysis. However, it would make the measurement procedure much more challenging and using scaled metamaterials while building room models – ineffective. What is more, changing the medium for the measurement could cause additional troubles with electro-acoustic equipment and would require the verification of the similarity criteria derived and commonly used for other types of acoustic materials, such as porous materials or perforated plates. This makes the efforts to fulfill all the requirements of the classical dimensional analysis highly illogical. Therefore, in the next step, the influence of neglecting the similarity criteria regarding the medium (air) was investigated in a FEM model.

4.1. Finite element method models

Finite element method modeling included two examples: the model of a single slit, and the model of a slit loaded with a quarter-wavelength resonator. Both models were prepared in 2D geometry, each time a unit cell was modeled with Floquet periodic conditions. Thermoviscous losses were accounted for with the use of the narrow region acoustics module, in which effective parameters are calculated. A triangular mesh was used, with a maximum element size equal to 1/20 of the length of the shortest wavelength in the narrow regions and close to the panel surface and 1/6 of the length of the shortest wavelength in the remaining parts of the geometry. The incident plane wave was modeled with the background pressure field, and in order to avoid sound reflection from the termination of the model, the plane wave radiation condition was applied, which means that no reflection was allowed from the top surface of the model (Fig. 2). Sound reflection coefficient was determined as the ratio of the reflected and incident acoustic pressure at 1/5 of the distance of the total air modeled above the panel. The walls were assumed rigid and perfectly reflecting. The calculations were performed at 1:1 scale, 1:4 scale, and 1:8 scale – only the geometrical criterial numbers were considered, and the parameters of air remained unchanged.

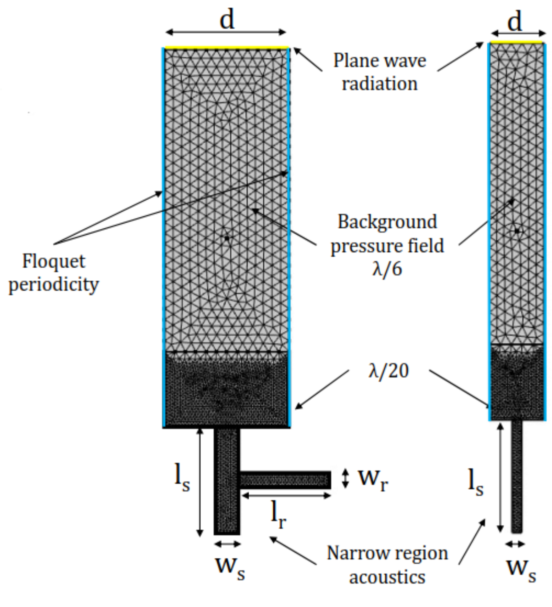

Figure 2. Meshes of the FEM models of the unit cell of a single slit and a slit loaded with a quarter-wavelength resonator.

Table 1. Parameters of the FEM models at 1:1 scale.

Model	Total width of the unit cell d [mm]	Length of the main slit l_s [mm]	Width of the main slit w_s [mm]	Length of the resonator l_r [mm]	Width of the resonator w_r [mm]
slit	58	120	10	–	–
slit with a quarter-wavelength resonator	132	112	24	96	16

5. Results and discussion

The results of the finite element method modeling are shown in Fig. 3. All the results were transposed for the full-size frequency range. We can see in Fig. 3 that neglecting similarity criteria regarding the medium (air) has no effect on the resonance frequency of the structure. On the other hand, it has a significant impact on the values of the modeled sound absorption coefficient. In both cases, the maximum sound absorption value increases with increasing scale factor. In the case of the simple slit unit cell, the maximum values of sound absorption are 0.2, 0.36, and 0.47 for scale factors 1:1, 1:4, and 1:8. This means that for the scale factor 1:8 the maximum absorption doubles. The observations are analogous in the case of the unit cell with a slit loaded with a quarter-wavelength resonator. This is caused by the increasing importance of the thermoviscous phenomena in slits of decreased dimensions.

a) b)

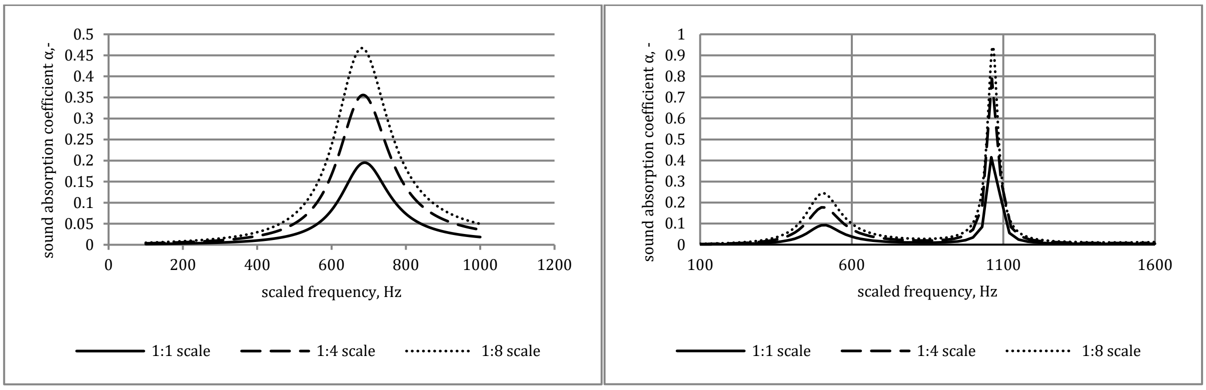

Figure 3. Sound absorption coefficient of two unit cells: a) a single slit, b) a slit loaded with a quarter-wavelength resonator of parameters given in Tab. 1, modeled with the FEM method for three scale factors 1:1, 1:4, and 1:8.

6. Conclusions

The paper discusses the possibility of scaling a generalized building block of the acoustic metamaterial with volumetric scatterers. A classical dimensional analysis of a generalized building block of a unit cell was performed in terms of sound absorption coefficient, the criterial numbers were derived. It was shown that in order to correctly scale a metamaterial, all the geometrical dimensions and dynamic viscosity of air must be scaled, and the Prandtl number and heat capacity ratio must remain unchanged. Since it is impossible to manipulate air parameters this selectively, the influence of neglecting similarity criteria regarding the surrounding medium was investigated. It was shown that disregarding these conditions does not influence the resonant frequency of the system but causes an increase in the sound absorption coefficient. Therefore, new criterial numbers must be derived in order to scale such metamaterials correctly.

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Additional information

The author declares: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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