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Multi-objective genetic algorithms for the reliability analysis and optimization of electrical transmission networks

Keywords

multi-objective genetic algorithms, critical infrastructures, reliability efficiency, group closeness centrality measure

Abstract

The results of two applications of multi-objective genetic algorithms to the analysis and optimization of electrical transmission networks are reported to show the potential of these combinational optimization schemes in the treatment of highly interconnected, complex systems. In a first case study, an analysis of the topological structure of an electrical power transmission system of literature is carried out to identify the most important groups of elements of different sizes in the network. The importance is quantified in terms of group closeness centrality. In the second case study, an optimization method is developed for identifying strategies of expansion of an electrical transmission network by addition of new lines of connection. The objective is that of improving the transmission reliability, while maintaining the investment cost limited.

1. Introduction

Our society is heavily dependent on many complex systems, technological, natural and even social that are hierarchies of networks of components (also called nodes, vertices or elements) interacting through links (also called edges, arcs or connections). Identifying and quantifying the vulnerabilities of such systems is crucial for designing the adequate protections, mitigation and emergency actions against their failures [3], [10], [13].

The apparent ubiquity of networks leads to a fascinating set of problems common to biological, ecological, technological and social complex systems, regarding how the underlying network topology influences the system behavior and its characteristics of stability and robustness to faults and attacks.

Given the complexity of these highly distributed and interconnected infrastructures, performing a systematic analysis of their vulnerability and robustness to failure becomes difficult if one resorts only to traditional probabilistic safety assessment (PSA) methods, so that new complementary approaches of network analysis are emerging for characterizing the network resistance to failure and identifying its most vulnerable elements [2], [5], [7], [8], [9], [15].

A relevant outcome of the analysis of the network structure is the identification of the most important groups of elements of different sizes in the network. Various measures of importance can be defined, which capture different structural aspects of the interconnection paths among network nodes.

In this paper, two applications of multi-objective genetic algorithms (MOGAs) are reported with regards to the analysis and optimization of electrical transmission networks. In the first case study, the electrical transmission network system of the IEEE (Institute of Electrical and Electronic Engineers) 14 BUS (a portion of the American Electric Power System) is considered [12] and a MOGA is used to

solve an optimization problem in which the decision variables identify groups of components and the objectives are to maximize the importance of the groups while minimizing their dimension. This formulation guides the MOGA search towards the identification of the most important single components, couples of components, triplets, and so forth in the topological structure of the network.

In the second case study, a MOGA is developed for identifying strategies of expansion of an electrical network in terms of new lines of connection to add for improving the reliability of its transmission service, while maintaining limited the investment cost. The typical large size of electrical networks offers a combinatorial number of potential solutions of new connections, so that classical optimization techniques become inapplicable. For this reason, we resorted to a MOGA driven by the objectives of maximizing the network global reliability efficiency [16] and minimizing the cost of the added connections. To realistically restrict the search space to small numbers of new connections the so-called guided multi-objective genetic algorithm (G-MOGA) has been applied. In this approach, the search is based on the guided domination principle which allows to change the shape of the dominance region specifying maximal and minimal trade-offs between the different objectives so as to efficiently guide the MOGA towards Pareto-optimal solutions within these boundaries [17]. The performance of this search approach is tested on a case study based on the IEEE (Institute of Electrical and Electronic Engineers) RTS (Reliability Test System) 96 [1].

The paper is organized as follows. Section 2 presents the group closeness centrality measure which can be used to quantify the importance of groups of nodes. The concept of network global reliability efficiency is also presented. A short introduction to the basic concepts behind the optimization procedure by Multi-Objective Genetic Algorithms is given in Section 3. In Section 4 and Section 5, the case studies regarding the IEEE 14 BUS network system and IEEE RTS 96 are presented and solved by MOGA. Conclusions on the outcomes of the analysis are eventually drawn in Section 6.

2. Topological group closeness centrality and global reliability efficiency

Mathematically, the topological structure of a network can be represented as a graph $G(N, K)$ with N nodes connected by K edges. The connections are defined in an $N \times N$ adjacency matrix $\{a_{ij}\}$ whose entries are 1 if there is an edge joining node i to node j and 0 otherwise.

The group closeness centrality [4], $C^C(g)$, is based on the idea that a node can quickly interact with all other nodes if it is easy accessible (close to) all others. If d_{ij} is the topological shortest path length between nodes i and j (i.e., the minimum number of arcs on a path connecting them), the closeness of a group g of N_g nodes is the sum of the distances from the members of the group to all vertices outside the group:

$$C^C(g) = \frac{N - N_g}{\sum_{i \in g, j \in G} d_{ij}} \tag{1}$$

This measure is normalized by dividing the distance score into the number of non-group members, with the result that larger numbers indicate greater centrality.

When the group consists of a single node, the group closeness centrality is the same as the individual node closeness centrality [5], [11], [14].

To capture the failure behavior of the network, the reliability of its connecting edges is included in the framework of analysis by means of the formalism of weighted networks, the weight w_{ij} associated to the edge between the pair of nodes i and j being its reliability:

$$p_{ij} = e^{-\lambda_{ij} \cdot T} \tag{2}$$

where λ_{ij} is the failure rate of edge ij linking nodes i and j and T is a reference time ($T = 1$ year, in this work).

On the basis of the adjacency and reliability matrices $\{a_{ij}\}$ and $\{p_{ij}\}$, the matrix of the most reliable path lengths $\{rd_{ij}\}$ can be computed [16].

The group reliability closeness centrality can then be computed as in eq. (1), with rd_{ij} replacing d_{ij}.

The global reliability efficiency $RE[G]$ of the graph G can also be defined as [16]:

$$RE[G] = \frac{1}{N(N-1)} \sum_{i,j \in G, i \neq j} \left(1 / rd_{ij} \right) \tag{3}$$

3. Multi-objective genetic algorithms

Genetic Algorithms (GAs) are optimization methods aiming at finding the global optimum of a set of real objective functions, $F \equiv \{f(\cdot)\}$, of one or more

decision variables, $U \equiv \{u\}$, possibly subject to various linear or non linear constraints. The terminology adopted in GAs contains many terms borrowed from biology, suitably redefined to fit the algorithmic context. Thus, GAs operate on a set of (artificial) chromosomes, which are strings of numbers, generally sequences of binary digits (bits) 0 and 1, coding the values of the decision variables. The values of the objective functions in correspondence of the values of the decision variables of a chromosome, give the fitness of that chromosome. The GA search is performed by constructing a sequence of populations of chromosomes, the individuals of each population being the children of those of the previous population and the parents of those of the successive population. The initial population is generated by randomly sampling the bits of all the strings; at each step in the search sequence, the new population is obtained by probabilistically manipulating the strings of the old population with fitness-improving rules which mimic genetic evolution. The search sequence continues until a pre-established optimality termination criterion is reached.

Typically, several possibly conflicting objective functions $f_i(\cdot)$, $i = 1, 2, \ldots, n_f$, must be evaluated in correspondence of each decision variable vector U in the search space. In this case, the GA search proceeds by comparing the solutions in terms of the concepts of *Pareto optimality and dominance* [6]: with reference to a maximization problem proposed solution U_a is said to dominate U_b if

$$\forall i \in \left\{ 1, 2, \ldots, n_f \right\}, f_i(U_a) \geq f_i(U_b)$$

and

(4)

$$\exists j \in \left\{ 1, 2, \ldots, n_f \right\}, f_j(U_a) > f_j(U_b).$$

The decision variable vectors which are not dominated by any other of a given set are called *nondominated* with respect to this set; the decision variable vectors that are nondominated within the entire search space are said to be *Pareto optimal* and constitute the so called *Pareto optimal front*, which is the object of the optimization.

4. Case study 1

4.1 IEEE 14 BUS electrical transmission network

The topological structure of the electrical transmission network system of the IEEE (Institute of Electrical and Electronic Engineers) 14 BUS (a portion of the American Electric Power System) is

considered for the analysis of the importance of groups of components, measured in terms of reliability closeness centrality. The system considered represents a portion of the American Electric Power System and consists of 14 bus locations connected by 20 lines and transformers. The topology of the system can be represented by the graph $G(14,20)$ of *Figure 1*.
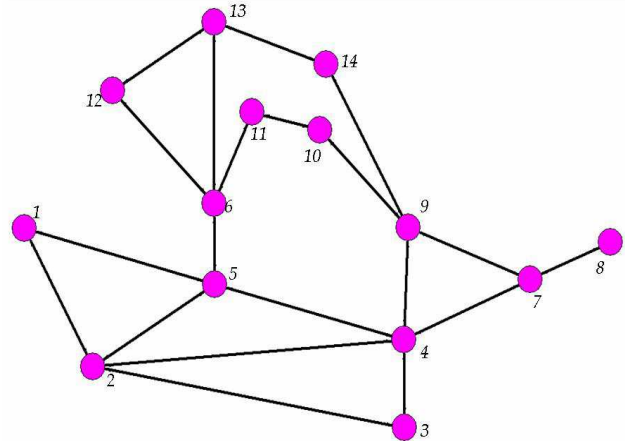

Figure 1. Graph representation of the IEEE 14 BUS transmission network

4.2 Identification of most central groups of nodes by MOGA

A MOGA has been implemented to identify the most reliability-central groups of nodes of different sizes in the network of *Figure 1*, considering the group reliability closeness centrality measure and the dimension of the group as objective functions. *Table 1* summarizes the details of the implementation of the MOGA operators described in Section 3 along with a number of parameters that control the operation of the genetic algorithm such as the population size (i.e., the size of the evolving set of candidate solutions) and the number of generations (i.e., the duration of the evolution process). The optimization runs were performed starting from an initial random population.

Figure 2 shows the results obtained on the importance of the group in terms of reliability closeness centrality. In the Figure, the values of the objective functions in correspondence of all the nondominated groups of nodes contained in the MOGA archive at convergence are shown to identify the two-dimensional Pareto-optimal surface (circles).

Table 1. MOGA parameters and rules

Number of chromosomes	200
Number of generations	200
Selection	Fit-Fit
Replacement	Children-parents
Mutation probability	0.001
Crossover probability	1
Number of generations without elitist selection	15
Fraction of parents chosen with elitist selection	0.25

The results are compared for validation with those obtained by exhaustive computation of all groups of nodes (i.e., the computation of the group reliability closeness centrality measure for all the possible combinations of n out of N nodes; due to the fact that the number of groups obtained is 2^N, its implementation is feasible here thanks to the small size of the network but would require impractical computational resources for large networks).
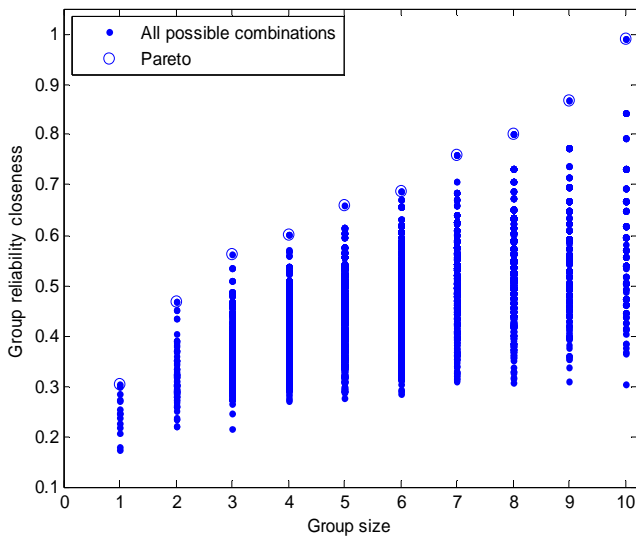

Figure 2. Results of the multi-objective search of the most central groups of nodes in terms of reliability closeness centrality

Actually, different groups of equal size can have the same centrality measure value: *Table 2* reports all the nondominated solutions contained in the archive, identified by the MOGA. For the groups composed of one to three and five nodes, there is only one combination that maximizes the group reliability closeness centrality measure; on the other hand, there is more than one solution for the groups with sizes four and six to ten; this allows choosing the solution most suitable for a given purpose, e.g. system upgrade by components renewal, replacement, or

redundancy allocation in order to use effectively the budget available.

In the present case, the smallest group with maximal reliability closeness is of size 10 and there are 2 of these. The group {1, 2, 3, 5, 7, 10, 11, 12, 13, 14} is particularly interesting because it does not contain the highly central node {4} and contains the node {1} that have the smallest individual reliability closeness centrality measure, as it can be seen in *Table 3*.

Table 2. Pareto optimal results of the multi-objective search for reliability closeness centrality groups

Group reliability closeness centrality	Group Size	Components
0.303	1	4
0.47	2	(4, 6), (6, 9)
0.562	3	(2, 6, 9)
0.602	4	(1, 2, 6, 9), (1, 3, 6, 9), (2, 3, 6, 9)
0.659	5	(1, 2, 3, 6, 9)
0.688	6	(1, 2, 3, 6, 7, 9), (1, 2, 3, 6, 8, 9)
0.761	7	(1, 2, 3, 5, 7, 10, 13), (1, 2, 3, 5, 7, 11, 13), (1, 2, 3, 6, 7, 10, 13), (1, 2, 3, 6, 7, 10, 14), (1, 2, 3, 6, 7, 11, 13), (1, 2, 3, 6, 7, 11, 14)
0.802	8	(1, 2, 3, 5, 7, 10, 11, 13), (1, 2, 3, 5, 7, 10, 12, 13), (1, 2, 3, 5, 7, 11, 12, 14), (1, 2, 3, 6, 7, 10, 11, 13), (1, 2, 3, 6, 7, 10, 12, 13), (1, 2, 3, 6, 7, 11, 12, 14) …
0.868	9	(1, 2, 3, 5, 7, 10, 11, 12, 13), (1, 2, 3, 5, 7, 10, 11, 13, 14), (1, 2, 3, 6, 7, 11, 12, 13, 14) …
0.99	10	(1, 2, 3, 5, 7, 10, 11, 12, 13, 14), (1, 2, 3, 6, 7, 10, 11, 12, 13, 14)

Table 3. Individual reliability closeness centrality

Node	Reliability closeness centrality
4	0.3031
9	0.2998
5	0.2835
7	0.2742
6	0.2716
14	0.253
10	0.2448
13	0.2448
11	0.2371
2	0.2272
8	0.2184
12	0.2081
3	0.1793
1	0.1723

To gain an engineering feeling of the impact on the network transmission performance resulting from the failure of the nodes of a group of a given reliability closeness centrality value, the relative global reliability efficiency variation of the network is reported in *Figure 3*. The relative variation of the global reliability efficiency due to the removal of a group of nodes is computed as the difference between the values of the global reliability efficiency (eq. (3)) of the network with all the nodes of the group (and all the incident arcs) removed and of the original network with no nodes removed, normalized to the latter.
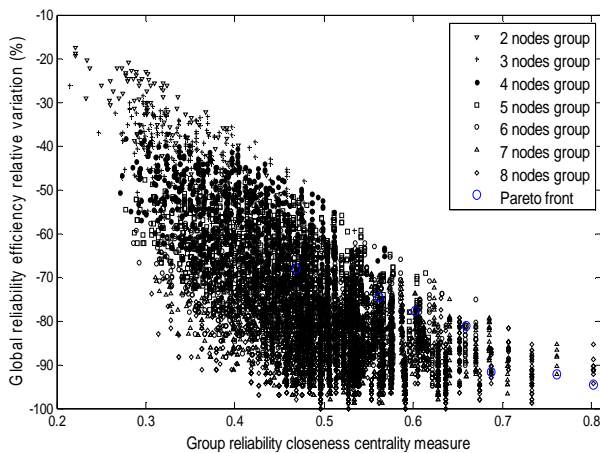

Figure 3. Relative variation of the global reliability efficiency as a function of group closeness centrality measure

The results show that there is a strong relation between the size of the groups removed and the efficiency of the network and that the most reliability-close central groups (i.e., those in the Pareto front) are indeed the most important also from the point of view of the negative impact on the network global reliability efficiency, when they fail.

5. Case study 2

5.1 IEEE RTS 96 electrical transmission network

The transmission network system IEEE RTS 96 is considered (*Figure 4*) [1]. The network consists of 24 bus locations (numbered in bold in the Figure) connected by 34 lines and transformers. The transmission lines operate at two different voltage levels, 138 kV and 230 kV. The 230 kV system is the top part of *Figure 4*, with 230/138 kV tie stations at Buses 11, 12 and 24.
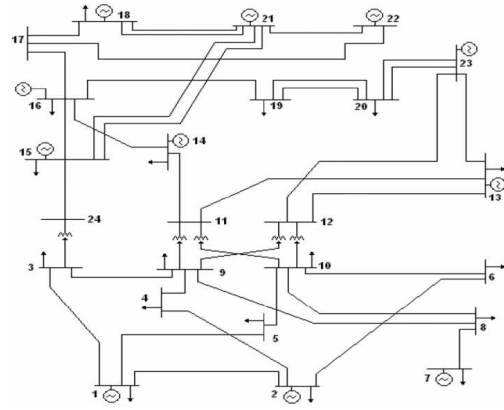

Figure 4. IEEE RTS 96 transmission network.

Figure 5 gives the representation of the graph $G(24,34)$ of the transmission network; the corresponding 24×24 adjacency matrix $\{a_{ij}\}$ has entry equal to 1 if there is a line or transformer between bus locations i and j and 0 otherwise.
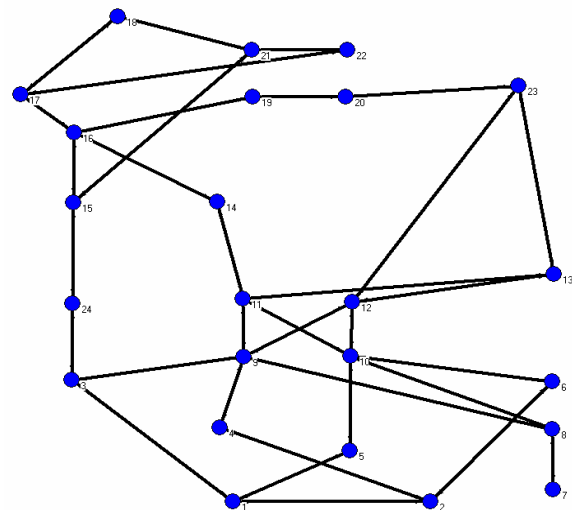

Figure 5. The IEEE RTS 96 graph representation

5.2 Optimal network expansion by MOGA

A MOGA has been constructed for identifying the best improvements in the connection of the network, aimed at increasing its global reliability efficiency in transmission at acceptable costs. The improvements are obtained by addition of new lines between nodes with no direct connection in the original network. Given the lack of geographical information on the nodes locations, for simplicity and with no loss of generality, three typologies of lines have been arbitrarily chosen as the minimum, the mean and the maximum values of the failure rates of the transmission lines taken from [1]:

$\lambda_1 = 0.2267$ outages/yr

$\lambda_2 = 0.3740$ outages/yr

$\lambda_3 = 0.5400$ outages/yr

The addition of a new line requires an investment cost assumed inversely proportional to the failure rate. The network cost can be then defined as:

$$C[G] = \sum_{i,j \in N, i \neq j} \left(1 / \lambda_{ij}\right) \qquad (5)$$

The reliability cost of the original IEEE RTS 96 is $C[G] = 332.0120$ in arbitrary monetary units and the reliability efficiency is $RE[G] = 0.2992$, which is a relatively high value representative of a globally reliable network.

The network global reliability efficiency and cost are adopted as objective functions in the MOGA optimization of the network improved structure.

From the algorithmic point of view, a proposal of improvement amounts to changing from 0 to 1 the values of the elements in the adjacency matrix corresponding to the added connections. The only physical restriction for adding direct new connections is that the connected nodes must be at the same voltage level (138 or 230 kV), otherwise the addition of a transformer would also be needed. From the genetic algorithm point of view, the generation of proposals of network improvements can be achieved by manipulating a population of chromosomes, each one with a number of bits equal to 214 which is double the number of zeros (i.e., the number of missing direct connections ij) in the upper triangular half of the symmetric adjacency matrix $\{a_{ij}\}$. The bits are dedicated to each missing direct connection ij so as to code the three different available types of lines with failure rates λ_1, λ_2 and λ_3: in other words, the bit-string (00) is used to code the absence of connection, (01) connection line with a λ_1-type line, (10) connection with a λ_2-type line and (11) connection with a λ_3-type line. The initial population of 200 individuals is created by uniformly sampling the binary bit values.

During the genetic search, each time a new chromosome is created, the corresponding matrices $\{a_{ij}\}$ and $\{p_{ij}\}$ are constructed to compute the values of the two objective functions, network global reliability efficiency and cost of the associated improved network.

Figure 6 shows the Pareto dominance front (squares) obtained by the MOGA at convergence after 10^3 generations; the circle represents the original network with $RE[G] = 0.2992$ and

$C[G] = 332.0120$, while the star represents the network fully connected by the most reliable transmission lines $\lambda_1 = 0.2267$ occ/yr, for which $RE[G] = 0.57$ and $C[G] = 804.1072$.
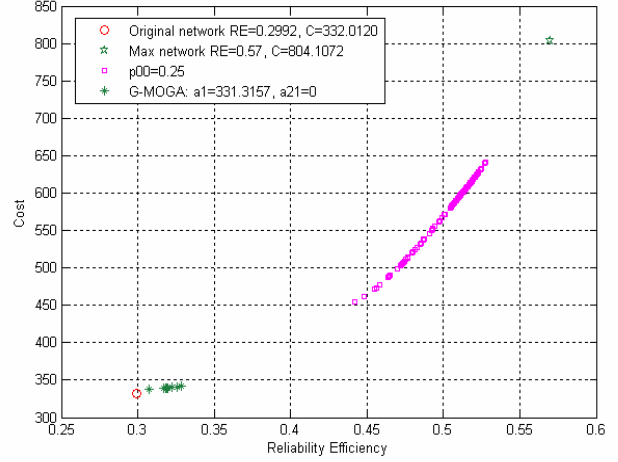

Figure 6. Pareto front reached by the MOGA

The optimality search is biased from the beginning (from the initial population) towards highly connected network solutions, because the string (00) has a probability of 0.25 whereas the probability of adding a connection of any one of the three available types (i.e., the probability of the strings 01, 10, 11) is 0.75; this drives the population evolution to highly connected networks in the Pareto front (squares in *Figure 6*), all with values $RE[G] \geq 0.4417$, $C[G] \geq 454.4738$ and numbers of added connections exceeding 60.

In practical applications only a limited number of lines can be added, due to the large investment costs and other physical constraints. To drive the genetic search towards low cost solutions (i.e., low number of added lines) maximal and minimal trade-offs to the two objectives of the optimization (network global reliability efficiency and cost) can be defined within a Guided Multi-Objective Genetic Algorithm (G-MOGA) scheme, [17]. The preferential optimization has been performed by using G-MOGA, with the same population size, evolution procedures and parameters of the previous search. In this approach, the search is guided by defining the maximal and minimal trade-offs that allow to identify a precise section of the Pareto front. The values of the trade-off parameters have been set by trial-and-error to $a_{12}=331.3157$ and $a_{21}=0$; the search converges to a small number of solutions in a Pareto front which is more concentrated on low cost

networks, characterized by a limited number of added connections (asterisks in *Figure 6*).

Table 4 lists the five solutions of lowest cost identified by the G-MOGA search: the added connections improve the network global reliability efficiency and they do so with relatively small costs.

Table 4. The five solutions on the Pareto front obtained by the G-MOGA

G-MOGA	
Reliability Efficiency	Cost
0.3072	337.6
0.3168	339.4
0.3186	339.4
0.3187	339.4
0.3193	339.4

6. Conclusions

This paper has illustrated the use of MOGA in the topological analysis of complex, interconnected network systems. The electrical transmission network system of the IEEE (Institute of Electrical and Electronic Engineers) 14 BUS (a portion of the American Electric Power System) has been taken as case study for the analysis by MOGA of the importance of groups of components, measured in terms of their centrality in the structure of interconnection paths. The results obtained using the group reliability closeness centrality measure as importance indicator have shown that the groups classified as most central indeed contain the nodes of individual highest centrality but may also include nodes with a relatively low centrality. The groups most important from the point of view of group reliability closeness have been shown to be also the most vulnerable to direct attacks, by a crude procedure of removal of nodes and arcs one at a time.

A MOGA for improving an electrical transmission network (IEEE RTS 96) has been implemented with the objective of identifying the lines to be added for maximizing the network transmission reliability efficiency, while maintaining the investment costs limited. A preferential procedure of optimization has been performed by using G-MOGA for individuating realistic network expansion solutions made of few new transmission lines.

From the point of view of the physical analysis of the electrical transmission system which the networks represent, some limitations affect the results obtained. These limitations are all related to the fact that the analysis performed focuses only on the topological features of the network, thus neglecting its physical characteristics; this is not realistic for electrical transmission networks in which: i) the "electrical" length of a path differs from the topological length, depending on the difficulty (resistance) of transmission; ii) the electrical power is not necessarily routed through the shortest paths (as assumed to compute the network reliability closeness centrality and efficiency values): rather, the transmission of power is determined by physical rules, e.g. Kirchoff's laws, nodal voltages etc.; iii) in the identification of the paths connecting nodes in the network, the direction of power flow should be taken into account in order to avoid considering paths which are not meaningful from the physical point of view; iv) the nodes on the real electrical network represent different components, i.e. generation, transmission and distribution buses. These physical aspects specific of electrical transmission networks must be included in the analysis for obtaining realistic insights on the robustness and vulnerability of these systems to faults and attacks. Work is currently undergoing in establishing effective ways of bringing these physical characteristics into the topological analysis.

Acknowledgments

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