Jan Długosz University in Częstochowa
Scientific Issues, Mathematics XVI, Częstochowa 2011

TESTING OF GEOMETRICAL IMAGINATION

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Abstract. The article deals with the geometric imagination in relation to intelligence tests. During an exploratory investigation of geometric imagination of pupils aged 15-18 years, a non-standardized test was created and evaluated, testing the partial and combinative abilities of students of this age group. The test consists of 40 tasks, and its evaluation process also contains a comparison of the results based on gender and mathematics mark.

1. Prologue

For movement in our world we should have an aptitude, which allows us to orient in the space, to be aware of location of our body and its parts in the space, to perceive the interrelation in the space. Varied names are used for this aptitude, e.g. Visual Thinking, Spatial Ability, Visualization, etc.

Gardner [2] talks about the spatial intelligence. He says: "Prime is aptitude, which secures the accurate perception the visual world. It allows to transform and to modify original percepts and it makes notions from own visual experience without further outward stimulus."

We define Geometrical Spatial Imagination as a "set of abilities, which relate reproduction and anticipation, static and dynamic ideas about shapes, about attributes and about relations between geometrical figures in space" [4].

Restructuring school mathematics often caused a considerable diversion from traditional parts of geometry. More time was given to more modern, more attractive parts of mathematics which are more practical. The limitation of geometry was justified by lack of time and inapplicability of traditional geometry. These remarks can be considered as tangible. The total contribution of geometry is important in a balanced education system. It should not be omitted.

2. Test of triangles

The geometrical spatial imagination is tested e.g. by standard Test of squares, which is a part of Amthauer I-S-T tests of universal intelligence and it comes out from Rybakov figures. We created a didactical test based on the similar principle. We divide an irregular plane figure into two parts only with one cut. Then we put together these two parts to create an equilateral triangle. The test, its administration and results are the components of Jana Slezáková's dissertation [7]. This dissertation was suggested at the Faculty of Science of Palacký University in Olomouc. The test was created and used in the ESF project called "The spotting of talents for the competitiveness and work with them", the area of assistance "The tantamount opportunities for children and pupils, including the pupils with a special educational needs", the registration number CZ.1.07/1.2.08/02.0017.

The test was created so as:

- it was interesting for pupils and it increases the interest in geometry,
- the teachers can easily apply it in teaching,
- it is used for the age category 15 – 18 years,
- it is focused on the geometrical spatial imagination.

The author created a coordinate grid of equilateral triangles and looked up various irregular figures, which can be divided into two parts with only one cut and put together into the equilateral triangle (only in our fantasy). The author created 40 plane figures in the first stage. These figures were tested by a small number of students and then the test was adapted. Two groups about 40 problems arised. The first group of problems – The geometrical spatial imagination (TP1) is for the age category up to 15 years, the second group of problems – The geometrical spatial imagination (TP2) is for the age category from 15 years. In both cases it is an unstandard test of geometrical spatial imagination, which is easily usable for a mathematics teacher.

The task of research was to find out whether the mark in maths and the result in the test are related, whether there exists a closeness of boys results and girls results. It also should order the problems by difficulty.

The test was carried out in June in school year 2009/2010, and 1690 pupils of a grammar school took part in this test. 548 of them (234 boys and 314 girls) were up to 15 years old (the second class, the fourth form) and 1142 of them (421 boys and 721 girls) were older than 15 years (the fifth form, the sixth form, the first class, the second class). It was realized at the faculty grammar school, which is binded by contract with Faculty of Science, Palacký

University in Olomouc. We tried to find out the quality of our surveying and we compared the validity and reliability with values of standard IQ test – Test of squares.

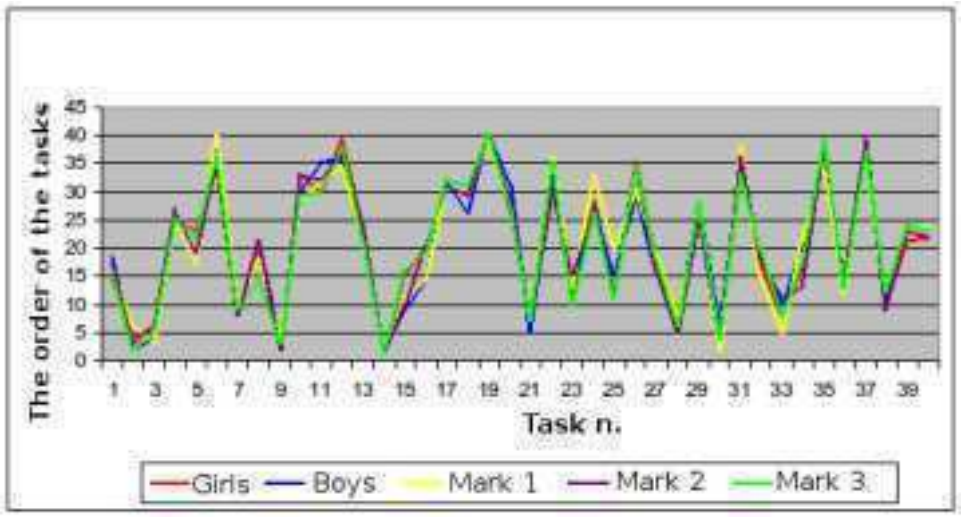

Figure 1: The order of the tasks according to correct answers in the test TP2.

3. Results

Here we present only the results of the test of the imagination – TP2 for the age group over 15 years.

Table 1 shows the relationship between success in the test TP2 and the mark in mathematics (represents the average score for each group of pupils according to marks in mathematics, including their average point difference in %).

The table shows that pupils, which have the mark 1, are clearly better than pupils with the marks 2 or 3. An interesting result is that pupils with the mark 1 were better than pupils with the marks 2 or 3 in each task. This result was not confirmed in the test of lower grammar school pupils. Next testing [7] demonstrated the correlation between success in the TP2 test and the mark in mathematics.

We also attempted to illustrate the sequence of tasks in the TP2 test with their evaluation by the number of correct answers. Now we can see how tasks were difficult for each group and how they would be sorted. Table 2 shows the average score for all the 1142 pupils in various tasks in the test TP2.

Figure 1 shows how tasks are sorted on the basis of test results of each group (particularly for girls, boys, pupils with the marks 1, 2, and 3).

Our results shows how to sort tasks according to increasing difficulty. The task numbers are as follows: 14, 9, 2, 30, 3, 28, 21, 7, 33, 38, 23, 15, 25, 36, 1, 34, 27, 32, 16, 8, 5, 40, 39, 13, 4, 29, 20, 24, 18, 17, 10, 11, 22, 26, 31, 6, 12, 35, 37, 19.

Figure 2 shows the dependence between a gender and the test results.

Task	Overall results	Mark 1	Mark 2	Mark 3	Difference between 1 and 3	Difference between 2 and 3	Difference between 1 and 2
1	81.0	89.2	80.8	76.6	12.6	4.2	8.4
2	91.6	93.7	93.2	89.7	4.0	3.5	0.5
3	89.8	94.9	90.4	87.2	7.7	3.2	4.5
4	70.8	82.9	71.2	67.2	15.7	4.0	11.7
5	76.0	88.6	79.1	68.4	20.2	10.7	9.5
6	55.4	60.1	59.3	52.3	7.8	7.0	0.8
7	86.2	91.8	88.1	82.7	9.1	5.4	3.7
8	78.2	88.6	78.8	76.9	11.7	1.9	9.8
9	92.4	94.9	94.6	89.4	5.5	5.2	0.3
10	62.3	78.5	63.3	58.1	20.4	5.2	15.2
11	62.2	75.9	64.7	58.1	17.8	6.6	11.2
12	54.5	72.2	55.4	48.0	24.2	7.4	16.8
13	74.3	86.7	75.1	69.6	17.1	5.5	11.6
14	93.3	96.8	95.5	91.5	5.3	4.0	1.3
15	82.9	91.8	86.4	76.6	15.2	9.8	5.4
16	78.3	89.9	79.1	72.6	17.3	6.5	10.8
17	62.6	76.6	65.3	55.3	21.3	10.0	11.3
18	66.8	77.2	68.4	56.8	20.4	11.6	8.8
19	43.8	60.8	43.8	39.5	21.3	4.3	17.0
20	68.0	81.0	70.3	59.6	21.4	10.7	10.7
21	88.4	93.0	89.8	86.0	7.0	3.8	3.2
22	61.3	69.0	66.7	54.4	14.6	12.3	2.3
23	83.8	91.1	83.6	81.2	9.9	2.4	7.5
24	68.0	75.9	71.5	63.2	12.7	8.3	4.4
25	82.8	88.0	85.9	81.2	6.8	4.7	2.1
26	60.9	77.2	61.6	55.0	22.2	6.6	15.6
27	79.7	88.6	82.5	73.3	15.3	9.2	6.1
28	89.8	93.0	91.2	86.9	6.1	4.3	1.8
29	68.3	82.3	72.9	59.0	23.3	13.9	9.4
30	91.0	95.6	92.9	89.1	6.5	3.8	2.7
31	57.4	67.1	57.6	55.3	11.8	2.3	9.5
32	78.8	89.9	80.5	72.0	17.9	8.5	9.4
33	86.1	94.3	86.4	83.3	11.0	3.1	7.9
34	80.0	87.3	84.2	75.1	12.2	9.1	3.1
35	53.4	69.6	54.8	47.7	21.9	7.1	14.8
36	82.7	91.8	84.2	77.8	14.0	6.4	7.6
37	52.1	69.0	50.0	48.3	20.7	1.7	19.0
38	84.4	93.0	87.3	78.4	14.6	8.9	5.7
39	74.6	86.7	76.0	67.8	18.9	8.2	10.7
40	75.2	86.1	78.2	68.4	17.7	9.8	7.9
					14.5	6.5	8.0

Table 1: The relationship between success in the test TP2 and the mark in mathematics.

TP2 (1142)	Solved:	Correct		Wrong		Did not solve	
	n	n	%	n	%	n	%
u1	1119	925	81.0	194	17.0	23	2.0
u2	1068	1046	91.6	22	1.9	74	6.5
u3	1089	1025	89.8	64	5.6	53	4.6
u4	980	809	70.8	171	15.0	162	14.2
u5	1088	868	76.0	220	19.3	54	4.7
u6	1091	633	55.4	458	40.1	51	4.5
u7	1040	984	86.2	56	4.9	102	8.9
u8	1009	893	78.2	116	10.2	133	11.6
u9	1092	1055	92.4	37	3.2	50	4.4
u10	900	711	62.3	189	16.5	242	21.2
u11	828	710	62.2	118	10.3	314	27.5
u12	754	622	54.5	132	11.6	388	34.0
u13	975	848	74.3	127	11.1	167	14.6
u14	1104	1065	93.3	39	3.4	38	3.3
u15	1037	947	82.9	90	7.9	105	9.2
u16	1069	894	78.3	175	15.3	73	6.4
u17	849	715	62.6	134	11.7	293	25.7
u18	945	763	66.8	182	15.9	197	17.3
u19	696	500	43.8	196	17.2	446	39.1
u20	997	776	68.0	221	19.4	145	12.7
u21	1073	1010	88.4	63	5.5	69	6.0
u22	799	700	61.3	99	8.7	343	30.0
u23	1023	957	83.8	66	5.8	119	10.4
u24	881	776	68.0	105	9.2	261	22.9
u25	1051	946	82.8	105	9.2	91	8.0
u26	912	695	60.9	217	19.0	230	20.1
u27	991	910	79.7	81	7.1	151	13.2
u28	1058	1025	89.8	33	2.9	84	7.4
u29	910	780	68.3	130	11.4	232	20.3
u30	1065	1039	91.0	26	2.3	77	6.7
u31	953	655	57.4	298	26.1	189	16.5
u32	963	900	78.8	63	5.5	179	15.7
u33	1027	983	86.1	44	3.9	115	10.1
u34	977	914	80.0	63	5.5	165	14.4
u35	821	610	53.4	211	18.5	321	28.1
u36	1013	944	82.7	69	6.0	129	11.3
u37	713	595	52.1	118	10.3	429	37.6
u38	1032	964	84.4	68	6.0	110	9.6
u39	905	852	74.6	53	4.6	237	20.8
u40	953	859	75.2	94	8.2	189	16.5

Table 2: The average score for all pupils in various tasks.

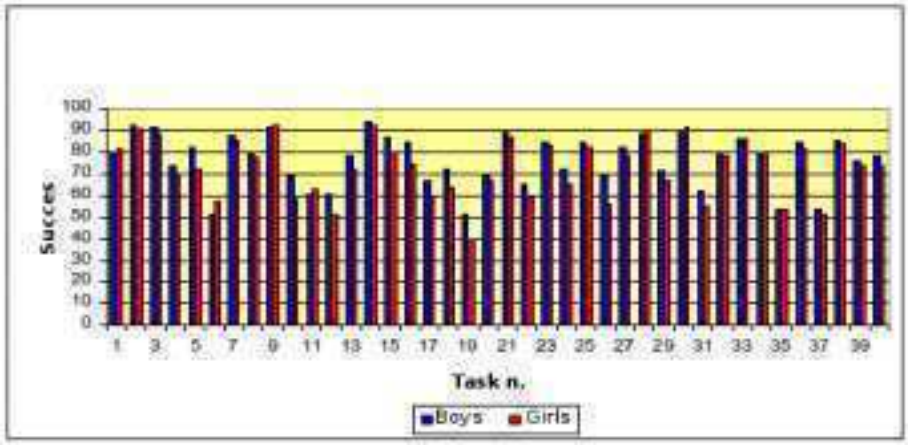

Figure 2: The dependence between a gender and the test results.

The investigation (average points from test of boys and girls) of grammar school pupils shows a big difference between the results of boys and girls. Girls were better than boys only in seven tasks (1, 6, 9, 11, 28, 30 and 34). The largest difference of overall average points was for task 28, it was 11.5% for girls. Also next investigation [7] shows that there is a correlation between success in the test solution TP2 and a gender of pupils.

4. Conclusion

Another goal was to determine the quality of our measurements and to compare the values of validity and reliability with a standardized IQ test – squares. The values of reliability, validity of measurement are in the following tables.

Statistical procedure of SPSS program which determines the value of the Cronbach alpha and the coefficient for the split-half method was used for reliability. Validity was verified by using the correlation between the mark in mathematics and test results.

Spearman's correlation coefficient was used for finding the right relations between tests. It allows to determinate quantitatively how close is the connection between variables which were used for creating orders.

These tables show how high is the grade of reliability for the test TP2. This value is higher than reliability IQ test - Test of squares. (When reliability is higher (close to +1), then precision is higher too). Reliabity is $r = 0,837$ for test TP2 and reliability for IQ–test of squares is $r = 0,812$ [9].

The mark in mathematics was chosen as a criterion to assess statistic validity. Predictive validity was used as well as in the case of square test.

Reliability Statistics

Cronbach's Alpha	Part 1	Value	.831
		N of Items	20(a)
	Part 2	Value	.80
		N of Items	20(b)
	Total N of Items		40
Correlation Between Forms			.727
Spearman–Brown Coefficient	Equal Lenght		.842
	Unequal Lenght		.842
Guttman Split-Half Coefficient			.837

a The items are: u1, u2, u3, u4, u5, u6, u7, u8, u9, u10, u11, u12, u13, u14, u15, u16, u17, u18, u19, u20.

b The items are: u21, u22, u23, u24, u25, u26, u27, u28, u29, u30, u31, u32, u33, u34, u35, u36, u37, u38, u39, u40.

Case Processing Summary

		N	%
Cases	Valid	324	28.4
	Excluded(a)	818	71.6
	Total	1142	100.0

a Listwise deletion based on all variables in the procedure.

Reliability Statistics

Cronbach's Alpha	N of Items
.902	40

Table 3: Values of reliability for split-half method in test TP2.

Correlation(research1d) Correlation are on significance level $p < .05$ Summarize the condition: TP="TP2" and research="JS"			
Variable	Mark	Points	Correct (%)
Mark	1.0000	−.2162	−.1981
	$N = 973$	$N = 973$	$N = 973$
	$p =$ —	$p = .000$	$p = .000$
Points	−.2162	1.0000	.7823
	$N = 973$	$N = 1142$	$N = 1142$
	$p = .000$	$p =$ —	$p = .000$
Correct (%)	−.1981	.7823	1.0000
	$N = 973$	$N = 1142$	$N = 1142$
	$p = .000$	$p = .000$	$p =$ —

Table 4: Values of predictive validity for test TP2.

Based on the results and from the tables, we can state that our measurements on the significance level of 0,05 can be considered valid.

It can be said that the test TP2 is suitable for verifying the level of geometric imagination of pupils of grammar schools.

Acknowledgements. The paper was created within the project ESF OP CZ.1.07/1.2.08/02.0017 "The spotting of talents for the competitiveness and work with them."

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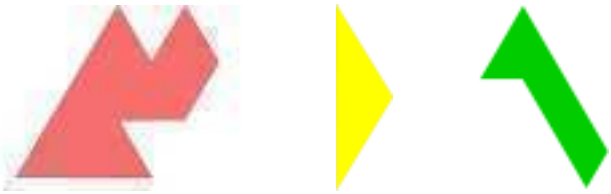
Appendix. Test TP2. Divide the polygon using only one section so that the transfer of one part to another (only in the imagination) creates an equilateral triangle.

1. 2. 3.



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