**JOURNAL OF THEORETICAL AND APPLIED MECHANICS**
**51**, 3, pp. 593-602, Warsaw 2013

# THERMAL ANALYSIS OF THE CONVECTIVE-RADIATIVE FIN WITH A STEP CHANGE IN THICKNESS AND TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

Mohsen Torabi, Hessameddin Yaghoobi
*Young Researchers Elite Club, Central Tehran Branch, Islamic Azad University, Tehran, Iran*
*e-mail: torabi_mech@yahoo.com*

Mohammad Reza Kiani
*Department of Mechanical Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran*

This paper provides heat transfer analysis in a straight fin with a step change in thickness and variable thermal conductivity, which is losing heat by simultaneous convection and radiation. The calculations are carried out by using the differential transformation method (DTM) that can be applied to various types of differential equations. The results obtained employing DTM are compared with an accurate numerical solution to verify the accuracy of the proposed method. Several graphs are provided to illustrate how the temperature distribution is affected by the (i) thickness parameter, (ii) dimensionless fin semi thickness, (iii) length ratio, (iv) thermal conductivity parameter, (vi) Biot's number, and (vii) radiation-conduction parameter. This collection of graphs provides a comprehensive picture of the thermal performance of the system under steady state conditions.

*Key words:* step straight fin, simultaneous convection and radiation, temperature dependent thermal conductivity, DTM analytical technique

## Nomenclature

| | | |
|---|---|---|
| $A_c, a$ | – | dimensional and dimensionless area of cross-section |
| Bi | – | Biot number |
| $C_1, C_2, C_3$ | – | integration constants |
| $h$ | – | coefficient of heat transfer |
| $x_1, x_2, x_3$ | – | axial coordinate for entire fin and for thin and thick section of fin, respectively |
| $\alpha$ | – | thickness parameter |
| $t, \delta$ | – | unreduced and dimensionless semi-thickness of fin, respectively |
| $L, l, \lambda$ | – | length of entire fin, length of its thin section and l-to-L ratio, respectively |
| $T$ | – | temperature |
| $T_a, T_b$ | – | temperature of ambient fluid and base of fin, respectively |
| $T_1, T_2$ | – | temperature within thin and thick section of fin, respectively |
| $k, k_b$ | – | temperature-dependent thermal conductivity and its value at fin base, respectively |
| $\kappa$ | – | slope of thermal conductivity curve |
| $N_r, \beta$ | – | conduction-radiation and thermal conductivity, respectively |
| $\varepsilon$ | – | emissivity |
| $\sigma$ | – | Stefan-Boltzmann constant |
| $\theta, \phi$ | – | dimensionless temperature within thin and thick section of fin, respectively |
| $\xi, \tau$ | – | dimensionless axial coordinate of thin and thick section of fin, respectively |

## 1. Introduction

Extended surfaces (fins) are widely used in many engineering applications which include, but are not limited to, air conditioning, refrigeration, automobile and chemical processing equipment. The primary objective of using fins is to enhance the heat transfer between the base surface and its convective, radiative or convective-radiative environment. An extensive review on this topic is presented by Kraus *et al.* (2002). The assumptions of constant thermo-physical properties and uniform heat transfer coefficient reduce the mathematical complexity of the energy equation and allow closed form analytical solutions for a number of cases as documented in Kraus *et al.* (2002). If a large temperature difference exists within a fin, the thermal conductivity varies from the base to the tip of the fin, the variation being dependent on the material of the fin. In real operating conditions, the heat transfer coefficient also varies along a fin. The variation may be a function of the spatial coordinate along the fin or the local temperature difference between the fin surface and the surrounding fluid. A brief review of published works that is of immediate relevance to the present paper is now presented. Bert (2002) applied DTM to steady-state heat transfer in a triangular-profile fin with constant properties. Kundu (2007) analytically carried out the thermal analysis and optimization of longitudinal and pin fins of uniform thickness subject to fully wet, partially wet and fully dry surface conditions. Sharqawy and Zubair (2008) carried out an analysis to study the efficiency of straight fins with different configurations when subjected to simultaneous heat and mass transfer mechanisms. Domairry and Fazeli (2009) solved the nonlinear straight fin differential equation by the homotopy analysis method (HAM) to evaluate the temperature distribution within the fin. Arslanturk (2009)developed correlation equations for the optimum design of annular fins with temperature-dependent thermal conductivity. Kulkarni and Joglekar (2009) proposed and implemented a numerical technique based on residue minimization to solve the nonlinear differential equation governing the temperature distribution in a straight convective fin having temperature-dependent thermal conductivity. Khani *et al.* (2009) used HAM to derive approximate analytical solutions for the temperature distribution and efficiency of a convective fin with simultaneous variation of the thermal conductivity and heat transfer coefficient with temperature. Fouladi *et al.* (2010) utilized the variational iteration method as an approximate analytical method to overcome some inherent limitations arising as uncontrollability to the nonzero endpoint boundary conditions and used this method to solve some examples in the field of heat transfer. Khani and Aziz (2010) used HAM to develop an analytical solution for the thermal performance of a straight fin of trapezoidal profile when both thermal conductivity and heat transfer coefficients are temperature dependent. Aziz and Torabi (2012) numerically studied convective-radiative straight fins. They considered simultaneous variation of the thermal conductivity, heat transfer coefficient, and surface emissivity with temperature for that research.

These studies consider fins with constant cross-sectional area or tapered fins. Aziz (1994) investigated optimum dimensions of convective rectangular fins with a step change in the cross-sectional area. A similar profile was also adopted for radial fins by Kundu and Das (2001). Malekzadeh *et al.* (2006) used the differential quadrature method for optimization of convective-radiative flat and step fins. Recently, Kundu (2009) analyzed an annular fin with a step change in thickness under fully and partially wet surface conditions. The optimization study demonstrated that an annular fin with a step change in thickness is a better choice for transferring rate of heat in comparison with the concentric-annular disc fin for the same fin volume and identical surface conditions. Kundu and Aziz (2010) used a numerical procedure to study the thermal performance of a step convective-radiative fin with temperature-dependent thermal conductivity and convective base heating.

A careful assessment of the foregoing literature shows that the most studies of step fins assume that the fins lose heat either by convection or radiation. The purpose of the present paper is to demonstrate the usefulness of DTM to analytically solve the problem of convective-radiative heat transfer from a step fin with temperature dependent thermal conductivity. The results to be presented will highlight the effects of the thickness parameter $\alpha$, dimensionless fin semi-thickness $\delta$, length ratio $\lambda$, thermal conductivity parameter $\beta$, Biot number Bi, radiation-conduction parameter $N_r$, on the temperature distribution. Thermal analysis of step fins is a new application for DTM which was used for other engineering applications (Yaghoobi *et al.*, 2011; Yaghoobi and Torabi, 2011, 2012).

## 2. Description of the problem

A rectangular step fin of unreduced thickness $2t$ and length $L$ is shown in Fig. 1. The fin is attached to a primary surface at fixed temperature $T_b$ and loses heat by simultaneous convection and radiation to the surrounding medium. The sink temperatures for convection and radiation are denoted by $T_a$ and $T_s$, respectively. As is customary, the fin tip is assumed to be adiabatic. It is also assumed that the convection heat transfer coefficient $h$, and the emissivity coefficient of surface $\varepsilon$, are both constant. The radiative exchange between the fin and its base is neglected. Since the fin is assumed to be thin, the temperature distribution within the fin does not depend on the vertical direction.

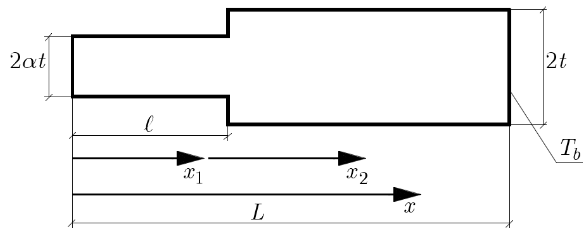


Fig. 1. Schematic of a convective-radiative fin with a step change in thickness

The energy balance equation for a differential element of the fin is given as

$$
\begin{aligned}
&2\alpha t\frac{d}{dx_1}\Big[k(T_1)\frac{dT_1}{dx_1}\Big] - 2h(T_1 - T_a) - 2\varepsilon\sigma(T_1^4 - T_s^4) = 0\\
&2t\frac{d}{dx_2}\Big[k(T_2)\frac{dT_2}{dx_2}\Big] - 2h(T_2 - T_a) - 2\varepsilon\sigma(T_2^4 - T_s^4) = 0
\end{aligned}
\tag{2.1}
$$

where $k(T)$ and $\sigma$ are the thermal conductivity and Stefan-Boltzmann constant, respectively. The thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$
k(T) = k_b[1 + \kappa(T - T_b)]
\tag{2.2}
$$

where $k_b$ is the thermal conductivity at the base temperature and $\kappa$ is the slope of the thermal conductivity-temperature curve. Invoking the continuity of temperature and heat current at the junction, the boundary conditions of the governing equations can be expressed as

$$
\begin{aligned}
&\frac{dT_1}{dx_1}\Big|_{x_1=0} = 0 \qquad T_1(\ell) = T_2(0) \qquad T_2(L-\ell) = T_b\\
&\Big[k\frac{dT_2}{dx_2}\Big]_{x_2=0} - \big[h(1-\alpha)(T_2 - T_a)\big]_{x_2=0} - \big[\sigma\varepsilon(1-\alpha)T_2^4\big]_{x_2=0} = \Big[\alpha k\frac{dT_1}{dx_1}\Big]_{x_1=\ell}
\end{aligned}
\tag{2.3}
$$

Introducing the following dimensionless parameters

$$\begin{array}{llll}
\theta = \dfrac{T_1}{T_b} & \phi = \dfrac{T_2}{T_b} & \theta_a = \dfrac{T_a}{T_b} & \theta_s = \dfrac{T_s}{T_b} \\
\zeta = \dfrac{x}{L} & \xi = \dfrac{x_1}{L} & \tau = \dfrac{x_2}{L} & \lambda = \dfrac{\ell}{L} \\
\delta = \dfrac{t}{L} & \beta = \kappa T_b & \mathrm{Bi} = \dfrac{hL}{k_b} & N_r = \dfrac{\varepsilon\sigma L T_b^3}{k_0} \\
\Psi_1 = \sqrt{\dfrac{\mathrm{Bi}}{\alpha\delta}} & \Omega_1 = \sqrt{\frac{\mathrm{Bi}}{\delta}} & \Psi_2 = \dfrac{N_r}{\alpha\delta} & \Omega_2 = \dfrac{N_r}{\delta}
\end{array} \tag{2.4}$$

and by assuming $\theta_a = \theta_s = 0$, the formulation of the fin problem reduces to the following equation

$$\begin{aligned}
&\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta\Big(\frac{d\theta}{d\xi}\Big)^2 - \Psi_1^2\theta - \Psi_2\theta^4 = 0 && 0 \leqslant \xi \leqslant \lambda \\
&\frac{d^2\phi}{d\tau^2} + \beta\phi\frac{d^2\phi}{d\tau^2} + \beta\Big(\frac{d\phi}{d\tau}\Big)^2 - \Omega_1^2\phi - \Omega_2\phi^4 = 0 && 0 \leqslant \tau \leqslant 1-\lambda
\end{aligned} \tag{2.5}$$

with the following boundary conditions

$$\begin{aligned}
&\frac{d\theta}{d\xi}\Big|_{\xi=0} = 0 \qquad \theta(\lambda) = \phi(0) \qquad \phi(1-\lambda) = 1 \\
&\Big[(1+\beta\phi)\frac{d\phi}{d\tau}\Big]_{\tau=0} - \big[(1-\alpha)\mathrm{Bi}\phi\big]_{\tau=0} - \big[(1-\alpha)N_r\phi^4\big]_{\tau=0} = \Big[\alpha(1+\beta\theta)\frac{d\theta}{d\xi}\Big]_{\xi=\lambda}
\end{aligned} \tag{2.6}$$

## 3. Fundamentals of differential transformation method (Zhu, 1986)

Let $x(t)$ be analytic in a domain $D$ and let $t = t_i$ represent any point in $D$. The function $x(t)$ is then represented by one power series whose center is located at $t_i$. The Taylor series expansion of $x(t)$ is in form of

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!}\Big[\frac{d^k x(t)}{dt^k}\Big]_{t=t_i} \qquad \forall t \in D \tag{3.1}$$

The particular case of Eq. (3.1) when $t_i = 0$ is referred to as the Maclaurin series of $x(t)$ and is expressed as

$$x(t) = \sum_{k=0}^{\infty} \frac{(t)^k}{k!}\Big[\frac{d^k x(t)}{dt^k}\Big]_{t=0} \tag{3.2}$$

As explained by Hassan (2004), the differential transformation of the function $x(t)$ is defined as follows

$$X(k) = \frac{(H)^k}{k!}\Big[\frac{d^k x(t)}{dt^k}\Big]_{t=0} \tag{3.3}$$

where $x(t)$ is the original function and $X(k)$ is the transformed function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where $H$ is a constant. The differential inverse transform of $X(k)$ is defined as follows

$$x(t) = \sum_{k=0}^{\infty} \Big(\frac{t}{H}\Big)^k X(k) \tag{3.4}$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of the function $X(k)$ for values of the argument $k$ are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. The more discrete available, the more precise restoration of the unknown function is possible. The function $x(t)$ consists of $T$-function $X(k)$, and its value is given by the sum of the $T$-function with $(t/H)^k$ as its coefficient. In real applications, at the right choice of the constant $H$, larger values of the argument $k$, the discrete of spectrum reduces rapidly. The function $x(t)$ is expressed by a finite series, and Eq. (3.4) can be written as

$$x(t)=\sum_{k=0}^{n}\Big(\frac{t}{H}\Big)^k X(k) \tag{3.5}$$

Mathematical operations performed by the differential transform method are listed in Table 1.

**Table 1.** The fundamental operations of differential transform method

| Original function | Transformed function |
|---|---|
| $x(t)=\alpha f(t)\pm\beta g(t)$ | $X(k)=\alpha F(k)\pm\beta G(k)$ |
| $x(t)=\dfrac{df(t)}{dt}$ | $X(k)=(k+1)F(k+1)$ |
| $x(t)=\dfrac{d^2f(t)}{dt^2}$ | $X(k)=(k+1)(k+2)F(k+2)$ |
| $x(t)=t^m$ | $X(k)=\delta(k-m)=\begin{cases}1 & k=m\\ 0 & k\neq m\end{cases}$ |
| $x(t)=\exp(\lambda t)$ | $X(k)=\dfrac{\lambda^k}{k!}$ |
| $x(t)=f(t)g(t)$ | $X(k)=\sum\limits_{l=0}^{k}F(l)G(k-l)$ |

### 3.1. Solution with DTM

Now we apply the differential transformation method into Eq. $(2.5)_1$. Taking the differential transform of Eq. $(2.5)_1$ with respect to $\xi$, and considering $H=1$ according to Table 1, gives

$$\begin{aligned}&(k+2)(k+1)\Theta(k+2)+\beta\Big(\sum_{l=0}^{k}\Theta(l)(k+2-l)(k+1-l)\Theta(k+2-l)\Big)\\&\quad+\beta\Big(\sum_{l=0}^{k}(l+1)\Theta(l+1)(k+1-l)\Theta(k+1-l)\Big)-\Psi_1^2\Theta(k)\\&\quad-\Psi_2\left(\sum_{m=0}^{k}\Theta(k-m)\Big(\sum_{v=0}^{m}\Theta(m-v)\Big(\sum_{w=0}^{v}\Theta(v-w)\Theta(w)\Big)\Big)\right)=0\end{aligned} \tag{3.6}$$

From the boundary condition in Eq. $(2.6)_1$, that we have it in point $\xi=0$, and exerting transformation

$$\Theta(1)=0 \tag{3.7}$$

The other boundary condition is considered as follows

$$\Theta(0)=C_1 \tag{3.8}$$

Accordingly, from a process of inverse differential transformation, in this problem we calculated $\Theta(k+2)$ from Eq. (3.6) as follows

$$\begin{aligned}
&\Theta(2) = \frac{C_1(\Psi_1^2 + \Psi_2 C_1^3)}{2(1+\beta C_1)} \qquad \Theta(3) = 0\\
&\Theta(4) = -\frac{C_1(\Psi_1^2 + \Psi_2 C_1^3)}{24(1+\beta C_1)^3}(-\Psi_1^2 + 2\Psi_1^2\beta C_1 - \Psi_2\beta C_1^4 - 4\Psi_2 C_1^3)\\
&\Theta(5) = 0\\
&\Theta(6) = \frac{C_1(\Psi_1^2 + \Psi_2 C_1^3)}{720(1+\beta C_1)^5}(\Psi_1^4 - 16\Psi_1^4\beta C_1 + 28\Psi_1^4\beta^2 C_1^2 - 2\Psi_1^2\Psi_2\beta C_1^4\\
&\qquad + 44\Psi_1^2\Psi_2\beta^2 C_1^5 + 44\Psi_1^2\Psi_2 C_1^3 + 25\Psi_2^2\beta^2 C_1^8 + 32\Psi_2^2\beta C_1^7 + 52\Psi_2^2 C_1^6\\
&\vdots
\end{aligned} \tag{3.9}$$

The above process is continued further. Substituting Eq. (3.9) into the main equation based on DTM, the solutions are obtained as

$$\begin{aligned}
\theta(\xi) &= C_1 + \frac{C_1(\Psi_1^2 + \Psi_2 C_1^3)}{2(1+\beta C_1)}\xi^2 - \frac{C_1(\Psi_1^2 + \Psi_2 C_1^3)}{24(1+\beta C_1)^3}(-\Psi_1^2 + 2\Psi_1^2\beta C_1 - \Psi_2\beta C_1^4 - 4\Psi_2 C_1^3)\xi^4\\
&+ \frac{C_1(\Psi_1^2 + \Psi_2 C_1^3)}{720(1+\beta C_1)^5}(\Psi_1^4 - 16\Psi_1^4\beta C_1 + 28\Psi_1^4\beta^2 C_1^2 - 2\Psi_1^2\Psi_2\beta C_1^4\\
&+ 44\Psi_1^2\Psi_2\beta^2 C_1^5 + 44\Psi_1^2\Psi_2 C_1^3 + 25\Psi_2^2\beta^2 C_1^8 + 32\Psi_2^2\beta C_1^7 + 52\Psi_2^2 C_1^6)\xi^6 + \cdots
\end{aligned} \tag{3.10}$$

Also, we apply the differential transformation method into Eq. $(2.5)_2$. Taking the differential transform of Eq. $(2.5)_2$ with respect to $\tau$, and considering $H = 1$ according to Table 1, gives

$$\begin{aligned}
&(k+2)(k+1)\Phi(k+2) + \beta\Big(\sum_{l=0}^{k}\Phi(l)(k+2-l)(k+1-l)\Phi(k+2-l)\Big)\\
&\quad + \beta\Big(\sum_{l=0}^{k}(l+1)\Phi(l+1)(k+1-l)\Phi(k+1-l)\Big) - \Omega_1^2\Phi(k)\\
&\quad - \Omega_2\left(\sum_{m=0}^{k}\Phi(k-m)\Big(\sum_{v=0}^{m}\Phi(m-v)\Big(\sum_{w=0}^{v}\Phi(v-w)\Phi(w)\Big)\Big)\right) = 0
\end{aligned} \tag{3.11}$$

Letting $\phi(0) = C_2$ and $(d\phi/d\tau)|_{\tau=0} = C_3$, and exerting transformation

$$\Phi(0) = C_2 \qquad \Phi(1) = C_3 \tag{3.12}$$

Using the same procedure as introduced in Eq. (3.9), the closed form of the solutions is obtained as

$$\begin{aligned}
\phi(\tau) &= C_2 + C_3\tau + \frac{\Omega_2 C_2^4 - \beta C_3^2 + \Omega_1^2 C_2}{2(1+\beta C_2)}\tau^2\\
&- \frac{C_3(-\Omega_1^2 + 2\Omega_1^2\beta C_2 - 3\beta^2 C_3^2 - \beta\Omega_2 C_2^4 - 4\Omega_2 C_2^3)}{6(1+\beta C_2)^2}\tau^3 + \cdots
\end{aligned} \tag{3.13}$$

The integration constant $C_1$ represents the temperature at the fin tip. $C_2$ and $C_3$ are the temperature and temperature gradient at the cross-section where the step change in thickness occurs, respectively. The constants can be evaluated from the boundary conditions given in Eqs. $(2.6)_{2,3,4}$ using Newton-Raphson method.

The calculations reported in this paper use $n = 7$, which was found to be sufficient to give an accurate solution. An implication of this is that Eq. (2.5) only requires the summation of a limited number of terms, and therefore the solution can be computed without excessive computational effort.

## 4. Results and discussion

The differential transformation method was applied to provide an analytical solution in terms of an infinite power series. Figures 2a and 2b show the differences among the DTM and the numerical solution (NS) for Eq. (2.5). In these figures, we assume that the fin is without a step in the thickness, i.e. $\alpha = 1$. These figures clearly show that with the DTM, a highly accurate analytical solution of the problem is achievable. It should be noted that, for all numerical results reported here, the following values of variables were used unless otherwise indicated by the graphs or tables: $\alpha = 0.5$, $\lambda = 0.5$, $\delta = 0.05$, $\beta = -0.4$, $\mathrm{Bi} = 0.01$, $N_r = 0.01$.

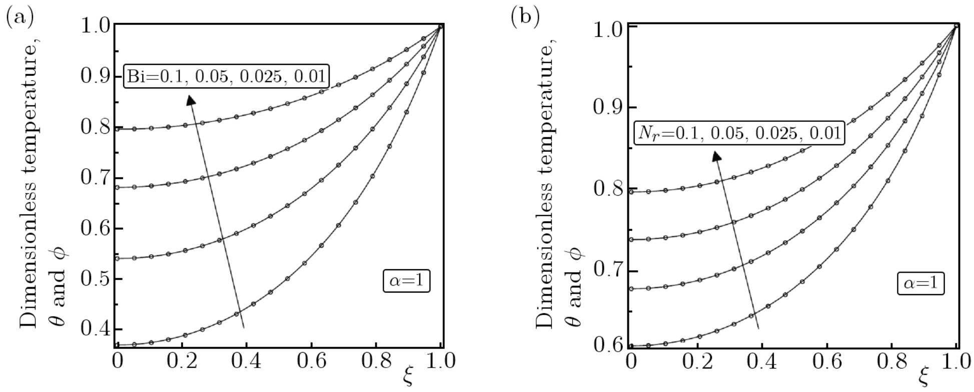


Fig. 2. Comparison of dimensionless temperature variation for various Biot number (a) and for various conduction-radiation parameters (b) obtained by DTM (solid line) and NS (circle)

Figure 3a shows the effect of the thickness parameter, i.e. parameter $\alpha$ on the temperature distribution in the step fin. The bottom curve corresponds to $\alpha = 0.2$ and the top curve corresponds to $\alpha = 0.8$. As the parameter $\alpha$ increases, the temperature distribution within the thin section of the fin increases, and the temperature distribution within the thick section of the fin decreases but, as expected it is not significant.

Figure 3b illustrates the effect of the length ratio, i.e. $\lambda$ on the temperature distribution in the fin. As $\lambda$ increases, i.e. as the thin section increases, the temperature distribution within the thin section of the fin decreases.

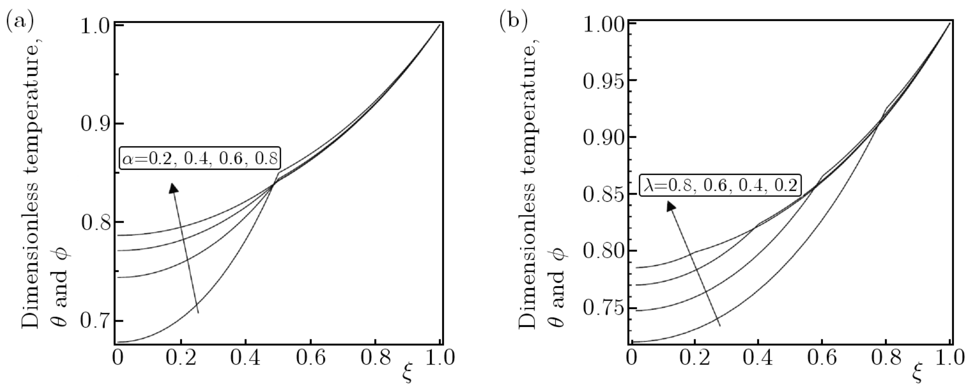


Fig. 3. Dimensionless temperature variation for various values of $\alpha$ (a) and of $\lambda$ (b) obtained by DTM

For the case of different values for the dimensionless fin semi-thickness results of the present analysis are depicted in Fig. 4a. As $\delta$ decreases, the cooling becomes more effective, promoting lower temperatures in the fin. This interesting behavior occurs for both thin and thick sections of the step fin. In Fig. 4b, we have plotted the effect of the thermal conductivity parameter on the temperature distribution within the fin. The results in the figure reveal that as the value of $\beta$ increases, the temperature distribution within the both sections increases.

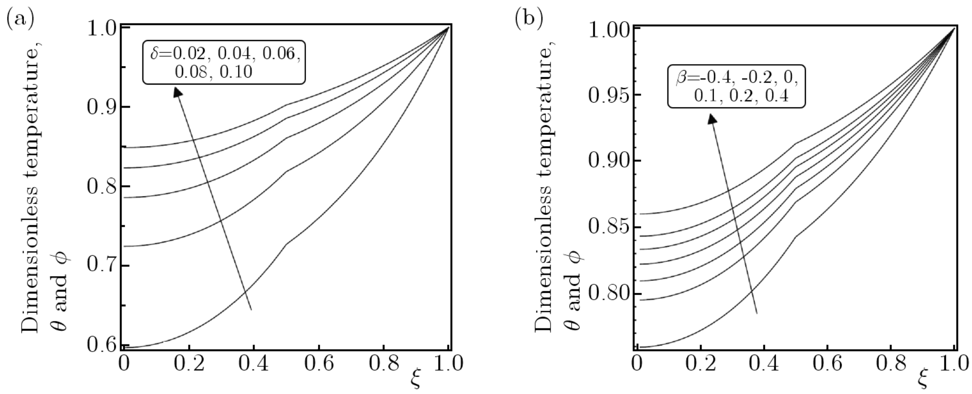


Fig. 4. Dimensionless temperature variation for various values of $\delta$ (a) and of $\beta$ (b) obtained by DTM

In Fig. 5a, the effect of Biot's number Bi on the temperature distribution in the fin is illustrated. The top curve corresponds to Bi = 0 (without convection effect) and the bottom curve corresponds to Bi = 0.1. As Biot's number increases, i.e. as convection gets stronger, the cooling becomes more effective, promoting lower temperatures in the fin. In Fig. 5b, we illustrate the effect of radiation-conduction parameter $N_r$ on the temperature distribution in the fin. As the radiative transport becomes stronger, the radiative cooling becomes more effective, which in turn causes the lowering of temperatures in the fin. Moreover, all the mentioned parameter effects on the tip temperature (TT) and junction temperature (JT) are tabulated in Table 2.

**Table 2.** The effects of various parameters on the tip (TT) and junction temperature (JT)

| Bi | $\delta$ | $\lambda$ | $\alpha$ | $\beta = -0.2$ | | | | $\beta = 0.2$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $N_c = 0.01$ | | $N_c = 0.1$ | | $N_c = 0.01$ | | $N_c = 0.1$ | |
| | | | | TT | JT | TT | JT | TT | JT | TT | JT |
| 0.01 | 0.05 | 0.4 | 0.4 | 0.7943 | 0.8528 | 0.5941 | 0.6853 | 0.8426 | 0.8895 | 0.6413 | 0.7296 |
| | | | 0.8 | 0.8222 | 0.8533 | 0.6288 | 0.6815 | 0.8657 | 0.8902 | 0.6757 | 0.7261 |
| | | 0.8 | 0.4 | 0.7286 | 0.9404 | 0.5347 | 0.8624 | 0.7851 | 0.9558 | 0.5795 | 0.8877 |
| | | | 0.8 | 0.8091 | 0.9356 | 0.6149 | 0.8450 | 0.8547 | 0.9528 | 0.6615 | 0.8732 |
| | 0.1 | 0.4 | 0.4 | 0.8745 | 0.9105 | 0.6913 | 0.7652 | 0.9083 | 0.9354 | 0.7382 | 0.8059 |
| | | | 0.8 | 0.8945 | 0.9134 | 0.7242 | 0.7664 | 0.9238 | 0.9378 | 0.7694 | 0.8074 |
| | | 0.8 | 0.4 | 0.8293 | 0.9635 | 0.6361 | 0.8999 | 0.8716 | 0.9738 | 0.6830 | 0.9207 |
| | | | 0.8 | 0.8863 | 0.9625 | 0.7120 | 0.8909 | 0.9174 | 0.9734 | 0.7576 | 0.9137 |
| 0.1 | 0.05 | 0.4 | 0.4 | 0.3360 | 0.4942 | 0.3124 | 0.4640 | 0.4101 | 0.5703 | 0.3740 | 0.5300 |
| | | | 0.8 | 0.3968 | 0.4877 | 0.3689 | 0.4571 | 0.4752 | 0.5650 | 0.4342 | 0.5235 |
| | | 0.8 | 0.4 | 0.2352 | 0.7898 | 0.2228 | 0.7644 | 0.2927 | 0.8330 | 0.2719 | 0.8071 |
| | | | 0.8 | 0.3707 | 0.7605 | 0.3460 | 0.7338 | 0.4466 | 0.8098 | 0.4093 | 0.7811 |
| | 0.1 | 0.4 | 0.4 | 0.5196 | 0.6437 | 0.4748 | 0.5992 | 0.6028 | 0.7152 | 0.5445 | 0.6633 |
| | | | 0.8 | 0.5798 | 0.6486 | 0.5307 | 0.6016 | 0.6600 | 0.7207 | 0.5994 | 0.6660 |
| | | 0.8 | 0.4 | 0.4117 | 0.8537 | 0.3814 | 0.8277 | 0.4912 | 0.8880 | 0.4471 | 0.8628 |
| | | | 0.8 | 0.5562 | 0.8417 | 0.5099 | 0.8123 | 0.6374 | 0.8799 | 0.5787 | 0.8506 |

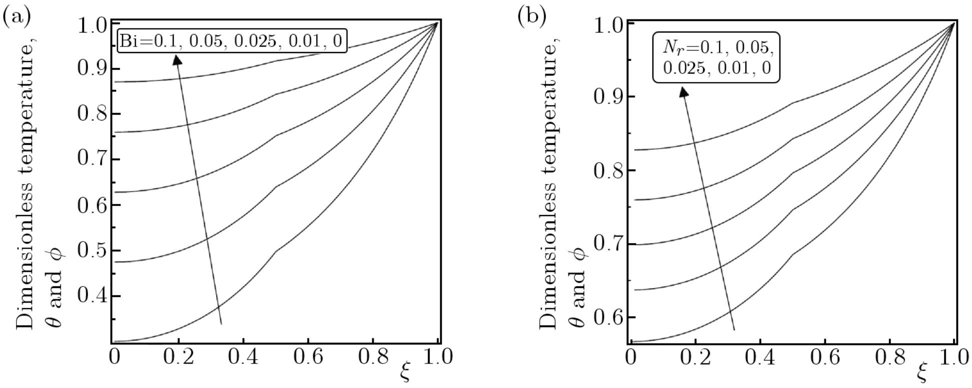


Fig. 5. Dimensionless temperature variation for various values of Bi (a) anf of $N_r$ obtained by DTM

## 5. Conclusions

The present work is concentrated on the performance analysis of convective-radiative step fins with temperature-dependent thermal conductivity. Since the fins have a step change in thickness, the fin problem has been divided into two parts as thin and thick sections. Resulting two nonlinear heat transfer equations with nonlinear boundary conditions have been solved by the differential transformation method (DTM). As both the radiation and convection effects increase, they lower the fin temperature. Similarly, as the thermal conductivity of the fin increases, i.e. the parameter $\beta$ increases, it promotes slower cooling accompanied by higher local fin temperatures. As a prominent result, it was found that the DTM solution can achieve extremely accurate results. This paper shows us the validity and great potential of the DTM for nonlinear problems in science and engineering.

## References

1. Arslanturk C., 2009, Correlation equations for optimum design of annular fins with temperature dependent thermal conductivity, *Heat and Mass Transfer*, **45**, 4, 519-525

2. Aziz A., 1994, Optimum design of a rectangular fin with a step change in cross-sectional area, *International Journal of Heat and Mass Transfer*, **21**, 389-401

3. Aziz A., Torabi M., 2012, Convective-radiative fins with simultaneous variation of thermal conductivity, heat transfer coefficient, and surface emissivity with temperature, *Heat Transfer-Asian Research*, **41**, 2, 99-113

4. Bert C.W., 2002, Application of differential transform method to heat conduction in tapered fins, *ASME Journal of Heat Transfer*, **124**, 208-209

5. Domairry G., Fazeli M., 2009, Homotopy analysis method to determine the fin efficiency of convective straight fins with temperature-dependent thermal conductivity, *Communications in Nonlinear Science and Numerical Simulation*, **14**, 489-499

6. Fouladi F., Hosseinzadeh E., Barari A., Domairry G., 2010, Highly nonlinear temperature-dependent fin analysis by variational iteration method, *Heat Transfer Research*, **41**, 155-165

7. Hassan A.H., 2004, Differential transformation technique for solving higher-order initial value problems, *Applied Mathematics and Computation*, **154**, 299-311

8. Khani F., Ahmadzadeh Raji M., Hamedi Nejad H., 2009, Analytical solutions and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient, *Communications in Nonlinear Science and Numerical Simulation*, **14**, 3327-3338

9. Khani F., Aziz A., Thermal analysis of a longitudinal trapezoidal fin with temperature-dependent thermal conductivity and heat transfer coefficient, *Communications in Nonlinear Science and Numerical Simulation*, **15**, 590-601

10. Kraus A.D., Aziz A., Welty J.R., 2002, *Extended Surface Heat Transfer*, John Wiley, New York

11. Kulkarni D.B., Joglekar M.M., 2009, Residue minimization technique to analyze the efficiency of convective straight fins having temperature-dependent thermal conductivity, *Applied Mathematics and Computation*, **215**, 2184-2191

12. Kundu B., 2007, Performance and optimum design analysis of longitudinal and pin fins with simultaneous heat and mass transfer: unified and comparative investigations, *Applied Thermal Engineering*, **27**, 976-987

13. Kundu B., 2009, Analysis of thermal performance and optimization of concentric circular fins under dehumidifying conditions, *International Journal of Heat and Mass Transfer*, **52**, 2646-2659

14. Kundu B., Aziz A., 2010, Performance of a convectively heated rectangular fin with a step change in cross-sectional area and losing heat by simultaneous convection and radiation (step fins under radiation environment), *Journal of Heat Transfer – Trans. ASME*, **132**, 104502-1

15. Kundu B., Das P.K., 2001, Performance analysis and optimization of annular fin with a step change in thickness, *ASME Journal of Heat Transfer*, **123**, 601-604

16. Malekzadeh P., Rahideh H., Karami G., 2006, Optimization of convective-radiative fins by using differential quadrature method, *Energy Conversion and Management*, **47**, 1505-1514

17. Sharqawy M.H., Zubair S.M., 2008, Efficiency and optimization of straight fins with combined heat and mass transfer – an analytical solution, *Applied Thermal Engineering*, **28**, 2279-2288

18. Torabi M., Yaghoobi H., 2013, Accurate solution for acceleration motion of a vertically falling spherical particle in incompressible Newtonian media, *The Canadian Journal of Chemical Engineering*, **91**, 376-381

19. Yaghoobi H., Khoshnevisrad P., Fereidoon A., 2011, Application of the differential transformation method to a modified van der pol oscillator, *Nonlinear Science Letter A*, **2**, 171-180

20. Yaghoobi H., Torabi M., 2011, The application of differential transformation method to nonlinear equations arising in heat transfer, *International Communications in Heat and Mass Transfer*, **38**, 815-820

21. Yaghoobi H., Torabi M., 2012, Analytical solution for settling of non-spherical particles in incompressible Newtonian media, *Powder Technology*, **221**, 453-463

22. Zhou J.K., 1986, *Differential Transform and its Applications for Electrical Circuits*, Wuhan, Huarjung University Press


*Manuscript received March 6, 2012; accepted for print October 11, 2012*