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Theoretical predictions on the influence of moisture on movements of wooden construction elements

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Abstract. The paper presents a method for determining the creep function of wood. A key feature of this method is a constitutive relation which includes the continuous effect of wood moisture on creep. The method can be applied to improve the precision of design of wooden structures as well to assess the safety of biodegraded (biologically corroded) structural elements fabricated from wood.
Keywords: construction, wooden constructions, rheology, constitutive relation, creep function
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1. Introduction

The paper proposes a constitutive model for assessing the impact of biological corrosion and moisture variations on the rheological effects in wood. The model should provide better precision of the analysis of changes in the stiffness of new designs of wooden structures and legacy wooden structures. Legacy wooden structures feature a reduced life and strength, and require bracing (reinforcement) or upgrading.

Wood is a natural (orthotropic) composite material with a polymer matrix reinforced by continuous polymer fibres. The main components of wood include cellulose and lignin, both natural polymers. The components share several characteristics with synthetic polymers (including heterogeneity and anisotropy). This allows the description of the rheological processes evolving in wood using models applicable to synthetic polymers [11].

The papers [2, 5, 8, 11] present studies of the rheological properties of wood, stressing the impact of moisture, duration of applied loads, and biological corrosion. More simplified engineering studies of these issues assume that wood has a constant moisture content. Experimental observations proved, however, that variations in moisture content and biological corrosion factors have a significant impact on the relaxation and creep of wood.

Given this, this paper presents the rheological model proposed by the authors and which considers the complex structure of wood. The creep function of the model depends on the variable moisture content and biological corrosion factors of wood.

Section 2 provides a general formulation of the problem contemplated in this work. The rheological model considers a selection of most significant rheological properties of wood. An operational form was used of a differential equation output, using the rheological model. The creep function was determined which satisfied the thermodynamic limits [11].

For the range of moisture content variations, an approximation of creep was proposed in the form of a quadratic function, the factors of which were identified from experimental test results. See Section 3. Section 4 presents a calculation example. The subject of the rheological analysis shown here was a wooden, circular arc. The calculated values of its deflection were compared to the deflection values produced with the Eurocode 5 rules. The conclusions are shown in Section 5.

2. The rheological model of wood

The formulation of an effective rheological model of wood required an in-depth understanding of the structure of wood and the properties of individual components of the structure. Wood is an orthotropic material, comprising cellulose, lignin, solutions, and water. The mechanical properties of each of these components affect the magnitude of deflection of the medium. The model contemplated here was a system of suitably connected springs and suppressors, which were Hookean solids and Kelvin-Voight solids (Fig. 1).

The deflection properties of cellulose were defined with a standard model connected in series with a Kelvin-Voight model [9, 11, 14]. Given its structure, cellulose features properties not unlike those found in high polymers. Twisting the links of a polymer around its bondings results in delayed elastic strain, which is reversible.

The mechanisms initiating the creep of lignin are of a different nature, however. The main factors here are viscous slips between the elements within the structure of lignin. The viscous slips can be triggered by variations in moisture content. The experimental observations discussed in [8, 11] showed that the processes take a different time than they do in cellulose structures. The works referenced to above described the strain properties of lignin with a standard model.

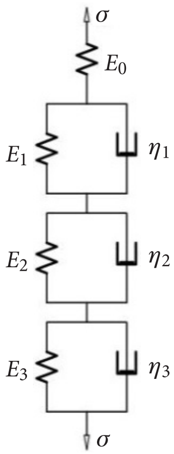

Fig. 1. Rheological model of wood.

The components of the proposed rheological model (Fig. 1) conformed to Boltzmann's linear superposition principle. As a consequence, the strains and their increments were determined with additional rules.

The proposed rheological model also satisfied the conservation laws for multicomponent media.

In the rheological model of wood, total strain had the following notation:

$$\varepsilon = \varepsilon_0 + \sum_{i=1}^{3} \varepsilon_i^{(KV)}, \tag{1}$$

with $\varepsilon_0 = \dfrac{\sigma}{E_0}$ being the linear elastic strain at the longitudinal modulus of elasticity E_0. The second component in the formula (1) included the inelastic (viscous) strains in three Kelvin-Voight components.

The stresses occurring in the links of the model were determined with the formula:

$$\sigma = \sigma_i = E_i \varepsilon_i^{(KV)} + \eta_i \dot{\varepsilon}_i^{(KV)} = \left(E_i + \eta_i \frac{d}{dt} \right) \varepsilon_i^{(KV)}, \tag{2}$$

with η_i and E_i being, respectively, the coefficient of viscosity and Young modulus for Kelvin-Voight solid *i*. $\dot{\varepsilon}_i^{(KV)}$ was the notation of the velocity of strain in the Kelvin--Voight solid *i*. With (2), the Kelvin-Voight strain was determined:

$$\varepsilon_i^{(KV)} = \frac{\sigma}{\left(E_i + \eta_i \dfrac{d}{dt} \right)}. \tag{3}$$

Having applied this convenient operational form in the equation (1), the following operational differential equation was derived for the proposed rheological model:

$$a_1\sigma + a_2\dot{\sigma} + a_3\ddot{\sigma} + a_4\dddot{\sigma} = b_1\varepsilon + b_2\dot{\varepsilon} + b_3\ddot{\varepsilon} + b_4\dddot{\varepsilon}. \tag{4}$$

The factors in this equation were determined from these formulas:

$$\begin{aligned}
&a_1 = E_3E_2E_1 + E_0E_3E_2 + E_0E_3E_1 + E_0E_2E_1,\\
&a_2 = E_3E_2\eta_1 + E_3\eta_2E_1 + \eta_3E_2E_1 + E_0E_3\eta_2 + E_0\eta_3E_2 +\\
&+E_0E_3\eta_1 + E_0\eta_3E_1 + E_0E_2\eta_1 + E_0\eta_2E_1,\\
&a_3 = E_3\eta_2\eta_1 + \eta_3E_2\eta_1 + \eta_3\eta_2E_1 + E_0\eta_3\eta_2 + E_0\eta_3\eta_1 + +E_0\eta_2\eta_1,\\
&a_4 = \eta_3\eta_2\eta_1,\\
&b_1 = E_0E_3E_2E_1,\\
&b_2 = E_0E_3E_2\eta_1 + E_0E_3\eta_2E_1 + E_0\eta_3E_2E_1,\\
&b_3 = E_0E_3\eta_2\eta_1 + E_0\eta_3E_2\eta_1 + E_0\eta_3\eta_2E_1,\\
&b_4 = E_0\eta_1\eta_2\eta_3.
\end{aligned} \tag{5}$$

The creep function was determined from the relation (4) by assuming an input function being a positive pulse:

$$\sigma(0)^+ = \sigma_0 H(t), \tag{6}$$

with σ_0 used to determine the input stress, and $H(t)$ being a Heaviside step function. The strain step function was converted to:

$$\varepsilon(t) = \sigma_0\tilde{J}(t), \tag{7}$$

where

$$\tilde{J}(t) = \left[\frac{1}{E_0} + \sum_{i=1}^{3}\frac{1}{E_i}\left(1 - e^{-\frac{E_i}{\eta_i}t}\right)\right]H(t) \tag{8}$$

is the sought creep function in time. The creep function facilitated a standard notation for the creep factor of wood:

$$k_{def} = E_0\left[\sum_{i=1}^{3}\frac{1}{E_i}\left(1 - e^{-\frac{E_i}{\eta_i}t}\right)\right] \tag{9}$$

The constants of the creep function (8) were identified by a series of tests.

3. Determination of creep factors vs. moisture content

In this work, the results were used from the testing of healthy pine wood [11] with three different moisture content values (Table 1). Biocorrosion-stricken parts were omitted from the analysis. The tests were done with creep testing machines capable of applying a constant bending moment. The test specimens had a square cross-section of 2 × 2 cm and were 160 cm long, with a support spacing of each test specimen of 40 cm. Constant air temperature and humidity were maintained throughout testing. The applied load values on the wood gave results at 30% of its immediate strength. The objective of the experiment was to measure the deflection increment in time. This value was used to identify the assumed creep function. The results of the identification are summarized in Table 1.

Table 1

Laboratory test results [11]

Item	Wood moisture content [%]	E_0 [GPa]	E_1 [GPa]	η_1 [GPa·h]	E_2 [GPa]	η_2 [GPa·h]	E_3 [GPa]	η_3 [GPa·h]
1.	4.8	13.986	258.398	66769.492	543.478	10451.505	571.429	109.827
2.	9.5	13.479	129.534	16778.974	270.270	4290.004	389.105	104.542
3.	37	11.989	56.915	3239.34	166.945	2981.159	153.139	29.164

The material constants shown in Table 1 were then applied to determine the creep factor values. Four different load application times were assumed: $t = 87600$, 4392, 168 and 12 hours. These values corresponded to standard loads which follow: constant load, long-term load, mid-term load, and short-term load [21]. To the defined measurement points, approximating square functions were extended:

$$k_{def}(w) = aw^2 + bw + c. \tag{10}$$

Table 2 summarizes the creep function factors determined as above.

Table 2

Factors of creep function $k_{def}(w)$

Load duration class	a	b	c
Constant	–0.0003674	0.0233391	0.0004377
Long-term	–0.0003674	0.0233391	0.0004377
Mid-term	–0.0003146	0.0179037	–0.0066882
Short-term	–0.0001053	0.0068255	0.0066647

The dependence of the creep function in the tested range of moisture content variations is shown in Fig. 2 versus the load duration classes.

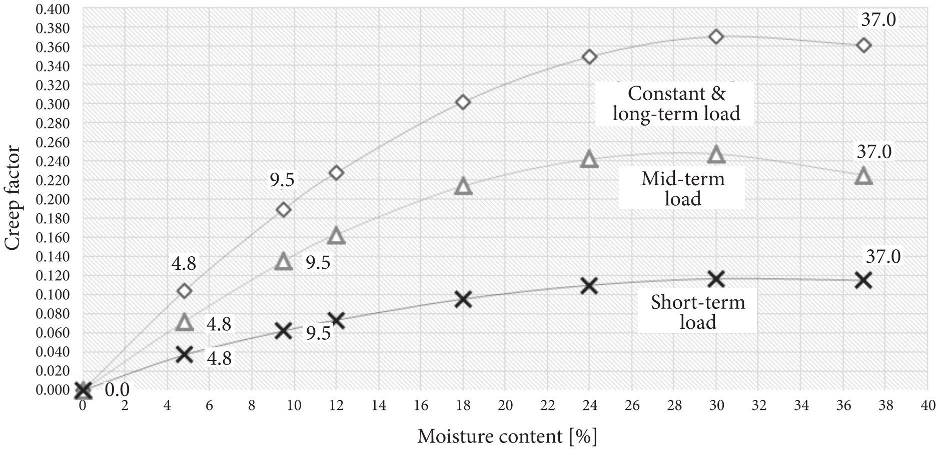

Fig. 2. Relationship between creep factor, moisture content and load duration.

The proposed creep functions were applied to determine the deflection of a wooden arch loaded for different durations with variations in moisture content. The results of these calculations were compared with the proposal conforming to the standard [21].

4. Calculation example

The subject of analysis was a wooden circular arch (Fig. 3) with a span $L = 12$ m, a height $f = 6$ m, and a square cross-section of 275 × 550 mm.

The loads used, q_α (α = 1, 2, 3, 4) had the following values: constant load $q_1 = 3.3$ kN/m, long-term load $q_2 = 2.5$ kN/m, mid-term load $q_3 = 1.9$ kN/m, short--term load $q_4 = 1.5$ kN/m.

The redundant thrust force was determined with the method of forces:

$$H_a = H_b = -\frac{4q_\alpha r}{3\pi}, \tag{11}$$

with r being the arch radius.

With the thrust force known, the vertical displacement of the arch apex was expressed with the formula:

$$w(r)_{inst,q\alpha} = \frac{\left(\pi^2 + 4\pi - 16\right)}{8\pi}\frac{q_\alpha r^4}{EJ} \approx 0,256\frac{q_\alpha r^4}{EJ}, \tag{12}$$

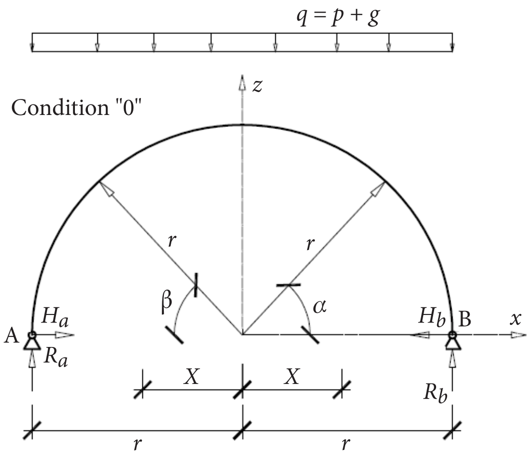

Fig. 3. Circular arch.

where *EJ* is the standard cross-sectional stiffness.

According to the proposal in Eurocode 5, the final deflection at the service limit state for the wooden circular arch was calculated following a quasi-constant combination [21]:

$$w(r)_{fin} = w(r)_{inst,q_1}\left(1+k_{def}\right) + w(r)_{inst,q_2}\left(1+\psi_{2,1}k_{def}\right) + \\ + w(r)_{inst,q_3}\left(\psi_{0,1}+\psi_{2,1}k_{def}\right) + w(r)_{inst,q_4}\left(\psi_{0,2}+\psi_{2,1}k_{def}\right), \tag{13}$$

with $\psi_{2,1}$ as the factors of the quasi-constant variable reactions, $\psi_{0,i}$ as the coincidence factors of duration variable loads, and k_{def} as a factor of the rheological properties of wood in the function of the load duration class. The standard factors k_{def} of three structural service classes had the following values, respectively [21]: k_{def} = 0.6,·0.8, and 2.0.

The following coincidence factors of duration variable loads were assumed [17]:

— long-term duration (from equipment), $\psi_{2,1} = 0,8$

— mid-term load (from snow), $\psi_{0,1} = 0,7$ and $\psi_{2,1} = 0,2$;

— short-term load (from wind), $\psi_{0,2} = 0,6$ and $\psi_{2,1} = 0$;

A summary of the calculated creep factors k_{def} is shown in Table 3. The values depended on the load duration classes and the moisture content. Note the significant differences between the creep factor values obtained according to Eurocode 5 and those from the proposed rheological method for wood.

TABLE 3

Calculated values of creep factor k_{def}

Moisture content	Class	k_{def}	
		Eurocode 5	Formula 9
4.5%	Constant	0.600	0.104
	Long-term	0.600	0.104
	Mid-term	0.600	0.072
	Short-term	0.600	0.037
9.5%	Constant	0.800	0.189
	Long-term	0.800	0.189
	Mid-term	0.800	0.135
	Short-term	0.800	0.062
37%	Constant	2.000	0.361
	Long-term	2.000	0.361
	Mid-term	2.000	0.225
	Short-term	2.000	0.115

The values shown in Table 3 were applied to calculate the final vertical deflection of the wooden circular arch, and the formula (13) was used. The calculation results are summarized in Table 4 according to the respective load duration class and moisture content values. The causes of the discrepancies should be attributed to the standard; while it exposes the factor of wood moisture content, it also considers the reaction of material defects caused by biological corrosion. The material constants of the proposed rheological model were identified by experimental testing of healthy wood [11].

TABLE 4

Calculated results of circular arch deflection

<table>
<tr><th rowspan="3">Moisture content</th><th rowspan="3">Load</th><th colspan="4">Deflection [mm]</th></tr>
<tr><th colspan="2">EC5</th><th colspan="2">Formulas 9 & 13</th></tr>
<tr><th>$u_{fin,q\alpha}$</th><th>u_{fin}</th><th>$u_{fin,q\alpha}$</th><th>u_{fin}</th></tr>
<tr><td rowspan="4">4.5%</td><td>Constant</td><td>33.204</td><td rowspan="4">53.7</td><td>22.918</td><td rowspan="4">40.3</td></tr>
<tr><td>Long-term</td><td>6.908</td><td>5.056</td></tr>
<tr><td>Mid-term</td><td>9.824</td><td>8.559</td></tr>
<tr><td>Short-term</td><td>3.734</td><td>3.734</td></tr>
</table>

continuation of table 4

9.5%	Constant	38.760	61.3	25.594	44.0
	Long-term	7.943		5.576	
	Mid-term	10.691		9.038	
	Short-term	3.875		3.875	
37%	Constant	72.628	106.5	32.943	54.7
	Long-term	14.158		7.018	
	Mid-term	15.374		10.412	
	Short-term	4.356		4.356	

5. Conclusions

The work proposes a constitutive model for wood which considers the rheological reactions of wood. The assumed input rheological model was defined according to the structure of wood. The material constants of the model were identified by testing at different moisture content values. The creep function, dependent on moisture content, biocorrosion factors, and time, was derived with an input function defined as a positive step of stress. A more detailed analysis would require more extensive testing, especially of constant and long-term loads, and biological corrosion factors.

A simple form of the creep function also enables the effective identification of material constants, as required by design conditions. It is also possible that there are other forms of the creep function available which would better describe the actual behaviour of wooden structures.

This method can be applied in the design of wooden structures, provided that an adequate level of safety is assumed.

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Prognoza teoretyczna wpływu wilgotności na przemieszczenia elementów konstrukcji drewnianych

Streszczenie. W artykule przedstawiono metodę wyznaczania funkcji pełzania dla drewna. Kluczową rolę w tej metodzie pełni relacja konstytutywna, uwzględniająca w sposób ciągły wpływ zmiany wilgotności drewna na pełzanie. Rozważana metoda może być wykorzystana w bardziej precyzyjnym projektowaniu konstrukcji drewnianych, jak również przy ocenie bezpieczeństwa elementów konstrukcyjnych skorodowanych biologicznie.

Słowa kluczowe: budownictwo, konstrukcje drewniane, reologia, relacja konstytutywna, funkcja pełzania

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