DETERMINING THE GAS-WATER CONTACT MOVING BOUNDARY IN UNDERGROUND GAS STORAGE OPERATION

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Abstract: We considered the characteristics of key technological objects involved in gas storage. Mathematical models of groups of hydraulically related objects (system mathematical models) are constructed and described. Problems are set and examples of application of analytical and numerical methods for their solution are provided.

Keywords: gas injection process modelling (withdrawal), porous media, gas replacing, gas-water contact (GWC), hydraulic integration

1. Introduction

To construct a model of gas-water or water-gas displacement in the process of filtration in porous media and motion of two-phase mixtures in vertical wells and sloping areas of pipelines, it is necessary to take into account many dynamic parameters [1−6]. Despite a large number of studies in this field, there has been no comprehensive theory to describe these processes so far[7−10]. The calculation of filtration and motion of two-phase systems is even more complicated due to the uncertain parameters of porous media and their heterogeneity [1−4, 10]. This requires building adaptive models and applying methods that make it possible to specify the model parameters based on the measured parameters (pressure, flow rate, concentration of water vapour in gas, *etc.*).

Water has been found in almost all gas fields; it is also present in all storage facilities created in depleted fields [4, 5, 10]. Its amount varies within the range from several to 70 percent. The content of residual water in gas-bearing reservoirs is 20–30% [10]. Water is retained in pores mainly due to adsorption and capillary forces [8−10].

To take the water factor into account, it is necessary to study the following:

- the process of water displacement with gas in porous media;

- the process of gas displacement with water in porous media;
- solubility of gas in water at a given pressure and temperature level;
- diffuse infiltration of gas into water at a given gas pressure and temperature level;
- the intensity of gas liberation from the liquid phase depending on the pressure;
- the impact of gas humidity and the speed of its movement on the reservoir and packing site permeability;
- the impact of structural reservoir parameters (the porous medium parameters) on the permeability to gas, water and mixtures thereof.

An important feature of the gas inflow to the well pertains to the significant loss of pressure in the packing reservoir area. As a result of numerical experiments, it has been found that a packing area with a radius of up to 1 m in real reservoir seams at low flow rates and stationary gas filtration by the Darcy law accounts for about 50% of the whole pressure loss. The pressure loss on the well site increases with an increase in depression in the reservoir. Such factors as imperfect borehole liberation, violation of the Darcy law and unsteady inflow of gas to the well also result in the corresponding increase in the share of the total pressure loss in the packing reservoir area. At the water drive regime, the pressure loss on the well site significantly affects the water cone tightening, which restricts its maximum flow rate value. The main parameters of gas storage facilities include their load peak (the maximum total selection of gas for a given period of time) and the minimum time intervals of selections and injection. In practice, any reservoir contains water; therefore, it requires a detailed study of gas-hydro-dynamic processes that take place in porous media. Recently, for various reasons, the average gas pressure level in underground storage facilities has been decreasing. As a result, the GWC may come closer to working wells. All this calls for a more detailed study of the GWC position.

The purpose of this work is to study the speed of the GWC movement depending on the parameters of the porous medium and gas, specifically those of the gas pressure in the process of injection (withdrawal).

2. Basic filtration laws

In 1856, engineer Darcy discovered the filtration that expresses the linear relationship between the rate of filtration and the pressure gradient [4]:

$$\upsilon = -\frac{k}{\mu}\frac{dp}{dx} \tag{1}$$

In case of a joint motion of water and gas, the concept of phase permeability is introduced [3, 4] by formulas similar to (1):

$$\upsilon_g = -\frac{k_g}{\mu_g}\frac{\partial p_1}{\partial x}, \quad \upsilon_w = -\frac{k_w}{\mu_w}\frac{\partial p_2}{\partial x} \tag{2}$$

In the latter formula where p_1, p_2 stand for the pressure in the gas and water phases, respectively. Their difference is equal to the capillary pressure. The weight of water or gas should be considered when filtering them in the vertical direction. Then

$$v = -\frac{k}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right) \tag{3}$$

In the formulas (1)–(3): $p = p(x_1, x_2, x_3, t)$ is the pressure distribution in the medium, ρ is the fluid density, g is the gravity acceleration, k is the medium permeability coefficient, and μ is the dynamic viscosity of the fluid.

3. Study of the gas extraction process at a steady speed of its movement in a packing area based on succession of stationary states

The problem reduces the following differential equation to integrating:

$$q\,dt = -\Omega dp \tag{4}$$

at the limit condition:

$$q = cp_c \tag{5}$$

Based on the equation (4), we get the following:

$$q = -\Omega\frac{dp}{dt} \tag{6}$$

Here, Ω is the volume of the porous medium filled with gas. Assuming that the pressure distribution is the same as at the steady motion of gas at any moment, we have the right correlation [4]:

$$q = \frac{\pi k h p_{\text{at}}}{\mu}\frac{p_k^2 - p_c^2}{\ln\frac{r_k}{r_c}} \tag{7}$$

In the formulas (4), (7), pressures p, p_k, p_c stand for their correlation to the atmospheric pressure, respectively. Since, the given values are dimensionless. Let us indicate:

$$A = \frac{\pi k h p_{\text{at}}}{\mu \ln\frac{r_k}{r_c}} \tag{8}$$

Then, the equation (7) is written as:

$$q = A\left(p_k^2 - p_c^2\right) \tag{9}$$

Assuming the error at replacing $\frac{dp}{dt}$ with $\frac{dp_k}{dt}$ to be negligible, we get the following differential equation to determine pressure in a packing area:

$$-\Omega\frac{dp_k}{dt} = \frac{c^2}{2A}\left[-1 + \sqrt{1 + \left(\frac{2A}{c}\right)^2 p_k^2}\right] \tag{10}$$

If we mark:

$$y = \frac{\pi k h p_{\text{at}}}{c\mu \ln\frac{r_k}{r_c}} p_k, \quad a = \frac{2A}{c} \tag{11}$$

then with the initial condition of:

$$y = y_n = \frac{2A}{c} p_n \tag{12}$$

we get the following solution of the differential equation (10):

$$t=\frac{Q}{c}\left\{2\left[\frac{1}{-1+ap_k+\sqrt{1+a^2p_k^2}}-\frac{1}{-1+ap_k+\sqrt{1+a^2p_n^2}}\right]+\ln\frac{ap_k+\sqrt{1+a^2p_n^2}}{p_k+\sqrt{1+a^2p_k^2}}\right\} \tag{13}$$

Now by assigning certain values to variable t the equation (13) allows us to find the corresponding value of pressure p_k. Based on the found value of p_k by the formula:

$$p_c=\frac{c}{2A}\left[-1+\sqrt{1+\left(\frac{2A}{c}\right)^2p_k^2}\right] \tag{14}$$

we can calculate the pressure on the well contour and the well capacity.

4. Hydraulic integration of the process of methane replacement with water at the steady mass transfer

The hydraulic integration will be constructed in a cylindrical coordinate system. We will consider three zones: the first one is the zone of quadratic law of gas movement to the well; the second one represents the gas flow by the linear law; while the third zone is that of the water movement to the well.

The first zone represents the quadratic law of gas movement set as the following:

$$p_1^2-p_2^2=A_aq_a+B_aq_a^2 \tag{15}$$

In the second zone, the nitrogen movement develops under the linear Darcy law:

$$p_2^2-p^2=C_aq_a \tag{16}$$

In the third zone, the distribution of water pressure at the boundary conditions of $r=R_c$, $p=p_c$ in the inner contour and $r=R_k$, $p=p_k$ at the outer one at the steady movements is as follows:

$$p(r)=p_k+\frac{p_k-p_c}{(\ln R_k/R_c)}\ln\frac{r}{R_k} \tag{17}$$

The water pressure p_k on the outer contour is determined by hydrostatics and is permanent. Pressure p_c on the inner contour is determined from the hydraulic integration of the gas condensate fuel-well packing system. The steady speed of movement of the internal water contour to the well is determined by the following formula [10]:

$$\upsilon=-\frac{k_\nu}{\mu_\nu}\left(\frac{p_k-p_c}{r(\ln R_k/R_c)}+\rho g\right) \tag{18}$$

Since:

$$\frac{dp(r)}{dr}=\frac{p_k-p_c}{r(\ln R_k/R_c)} \tag{19}$$

we can calculate the radial velocity of the formula:

$$\upsilon=-\frac{k}{\mu}\left(\frac{p_k-p_c}{r(\ln R_k/R_c)}+\rho g\right) \tag{20}$$

Let us introduce the following designation:

$$\alpha=-\frac{k}{\mu}\frac{p_k-p_c}{r(\ln R_k/R_c)},\quad \beta=-\frac{k\rho g}{\mu} \tag{21}$$

Thus, from the formula (20) we obtain a differential equation with separated variables to determine changes in the radius of the inner water movement contour:

$$\frac{dr}{dt}=\frac{\alpha}{r}+\beta \tag{22}$$

We reduce the equation (22) to an equation with separated variables:

$$\frac{(\alpha+\beta r-\alpha)dr}{\beta(\alpha+\beta r)}=\frac{1}{\beta}dr-\frac{\alpha}{\beta^2}\frac{d(\alpha+\beta r)}{\alpha+\beta r}=dt \tag{23}$$

If the radius of the inner water contour changes from r_1 to r_2, for some time from t_1 to t_2, the solution of the equation (23) is:

$$\frac{1}{\beta}(r_2-r_1)-\frac{\alpha}{\beta^2}\ln\frac{\alpha+\beta r_2}{\alpha+\beta r_1}=t_2-t_1 \tag{24}$$

At the set values for the pressure and the time of $\Delta t=t_2-t_1$, the formula (24) makes it possible to determine the reduction of the inner water contour radius for the following amount:

$$\Delta r=\frac{\alpha+\beta r_2}{r_2}(t_2-t_1) \tag{25}$$

It should be noted that the obtained values should be consistent with the volume of the gas received from the reduced volume. If q is the permanent well capacity, then for the time t_2-t_1 the following equation should be performed

$$q(t_2-t_1)=\pi h(r_2^2-r_1^2) \tag{26}$$

5. Differential model of the process of gas replacement

The part of the reservoir filled with water is limited with contour Σ_0 (Figure 1). The water pressure on this contour is p_0. At the initial moment, methane and water are separated by contour S^0 with pressure p_0^1 at any given moment. At any given moment, methane and water are separated with contour S with pressure p_1. The pressure on contour S_2 is equal to p_2. The water pressure satisfies the following equation:

$$\Delta p=0 \tag{27}$$

The components of water velocity are determined by the following formulas [4]:

$$u=-\frac{k}{\mu_1}\frac{\partial p}{\partial x},\quad \upsilon=-\frac{k}{\mu_1}\frac{\partial p}{\partial y} \tag{28}$$

Here μ_1 stands for the absolute water viscosity, and k stands for the permeability of the reservoir filled with water.

For water, the boundary conditions are as follows:

1. $p=p_0$ on the external reservoir boundary;
2. $p=p_1$ on the moving contour;

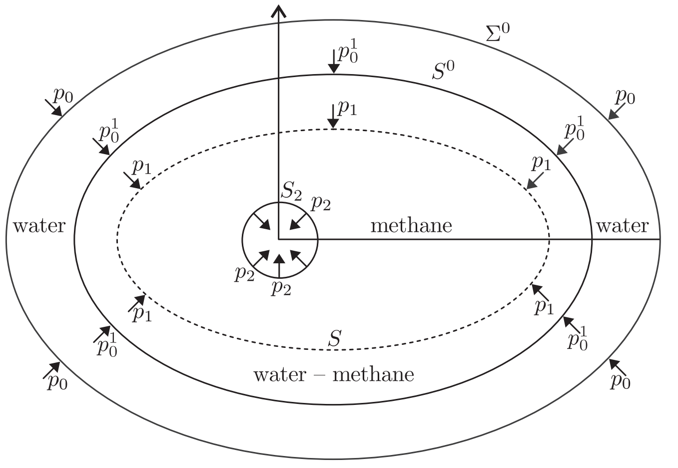

Figure 1. PSG layer area distribution

3. equation of delivery on the moving contour

$$-m\frac{\partial n_0}{\partial t}=\frac{k}{\mu_1}\frac{\partial p}{\partial n_0} \tag{29}$$

dn_0 stands for an element of the external normal to S contour. Diffusion of methane under unstable conditions is described by the following equation:

$$\Delta P=\frac{D}{p}\frac{\partial P}{\partial t} \tag{30}$$

Here $D=\frac{m\mu}{k}$, μ stand for absolute methane viscosity, velocity elements:

$$\rho u=-\frac{k}{2\beta\mu}\frac{\partial P}{\partial x},\quad \rho v=-\frac{k}{2\beta\mu}\frac{\partial P}{\partial y} \tag{31}$$

Boundary conditions for methane:

1. $P=P_1=p_1^2$ on the moving contour;
2. $p=p_2$ on the well contour;
3. equation of delivery on the moving contour

$$-m\frac{\partial n_0}{\partial t}=\frac{k}{2\mu p}\frac{\partial P}{\partial n_0} \tag{32}$$

Along with these conditions, it is necessary to specify the initial condition for $t=0$ for nitrogen and methane $p=p_0$.

6. The distribution of pressure on the well site under unsteady mass transfer

The work of the underground gas storage associated is with non-stationary processes of change of pressure in the reservoir. Clearly, to build an adaptive gas and water filtering model with moving GWC, we must be able to determine this distribution.

We consider a layer region which is limited with nested cylinders containing the well in its centre and divided into corresponding zones: zone I is filled with extracted gas; zone II emerges due to the movement of water that restrains the number of pores to the well; zone III is filled with water. Then, r_i and P_i, where $i=1,2,3$ are distances from the zone surfaces to the centre of the well and pressures in the respective zones.

Let us assume that there is water pressure P_1 on surface $r_1=a$, and that there is pressure P_2 on surface $r_2=b$, while the initial distribution is provided in the formula $f(r)$. In this case, the solution of the original problem of mathematical physics is presented in the following form: $P=P_s+P_n$, where: [10]

$$P_s=\frac{P_1\ln(b/r)+P_2\ln(r/a)}{\ln(b/a)} \tag{33}$$

determines the steady pressure distribution among the surfaces, and the second solution component is as follows:

$$\begin{aligned} P_n=&\frac{\pi^2}{2}\sum_{n=1}^{\infty}\frac{\alpha_n^2 J_0^2(a\alpha_n)}{J_0^2(a\alpha_n)-J_0^2(b\alpha_n)}e^{-\kappa\alpha_n^2\tau}U_0(r\alpha_n)\int_a^b rf(r)U_0(r\alpha_n)dr-\\ &-\pi\sum_{n=1}^{\infty}\frac{\left[P_2J_0(a\alpha_n)-P_1J_0(b\alpha_n)\right]J_0(a\alpha_n)U_0(r\alpha_n)}{J_0^2(a\alpha_n)-J_0^2(b\alpha_n)}e^{-\kappa\alpha_n^2\tau} \end{aligned} \tag{34}$$

Here $U_0(ar)=J_0(ar)Y_0(ab)+J_0(ab)Y_0(ar)$, $J_i(x)$ and $Y_i(x)$ are the first and second kind Bessel functions of actual argument of order i, respectively. Considering the equality (1) of correlations (33) and (34) we obtain the formula for determining the GWC velocity:

$$\begin{aligned} \upsilon=\Bigg\{&\frac{P_2-P_1}{\ln(b/a)}+r\frac{\pi^2}{2}\sum_{n=1}^{\infty}\frac{\alpha_n^2 J_0^2(a\alpha_n)}{J_0^2(a\alpha_n)-J_0^2(b\alpha_n)}e^{-\kappa\alpha_n^2\tau}U_1(r\alpha_n)\int_a^b xf(x)U_0(x\alpha_n)dx-\\ &-r\pi\sum_{n=1}^{\infty}\frac{\left[P_2J_0(a\alpha_n)-P_1J_0(b\alpha_n)\right]J_0(a\alpha_n)U_1(r\alpha_n)}{J_0^2(a\alpha_n)-J_0^2(b\alpha_n)}e^{-\kappa\alpha_n^2\tau}\Bigg\}\frac{\pi kh}{\beta\mu}\frac{\chi RT}{pS} \end{aligned} \tag{35}$$

where: $U_1(r\alpha_n)=-\alpha\left[J_1(\alpha_n r)Y_0(ab)+J_0(ab)Y_1(\alpha_n r)\right]$, α_n stand for roots of nonlinear equations

$$J_0(\alpha a)Y_0(\alpha b)+J_0(\alpha b)Y_0(\alpha a)=0 \tag{36}$$

The formula (35) allows us to find the value of pressure on surfaces r_1 and r_2, respectively:

$$P_1=\frac{P_2-\ln(b/a)\left[\dfrac{\upsilon}{C_2}-S_1+r\pi P_1\displaystyle\sum_{n=1}^{\infty}C_1J_0(a\alpha_n)\right]}{1-r\pi\ln(b/a)\displaystyle\sum_{n=1}^{\infty}C_1J_0(b\alpha_n)} \tag{37}$$

$$P_1 = \frac{P_1 + \ln(b/a)\left[\dfrac{\upsilon}{C_2} - S_1 - r\pi P_1 \sum\limits_{n=1}^{\infty} C_1 J_0(b\alpha_n)\right]}{1 - r\pi \ln(b/a) \sum\limits_{n=1}^{\infty} C_1 J_0(a\alpha_n)} \tag{38}$$

where

$$C_1 = \frac{J_0(a\alpha_n) U_1(r\alpha_n)}{J_0^2(a\alpha_n) - J_0^2(b\alpha_n)} \tag{39}$$

$$C_2 = \frac{\pi k h}{\beta\mu} \frac{\chi R T}{p S} \tag{40}$$

$$S_1 = r\frac{\pi^2}{2} \sum_{n=1}^{\infty} \frac{\alpha_n^2 J_0^2(a\alpha_n)}{J_0^2(a\alpha_n) - J_0^2(b\alpha_n)} e^{-\kappa\alpha_n^2 \tau} U_1(r\alpha_n) \int_a^b x f(x) U_0(x\alpha_n) dx \tag{41}$$

For gas under similar conditions, formulas for pressure distribution and velocity in a respective hollow cylinder will be the same, except for the fact that $P_i = p_i^2$ and $\chi = 1$.

7. Computational experiment

In order to test our theoretical results a computer experiment was performed with the following input parameter values: $p_0 = 50 \cdot 98066.5$ (N/m^2); $p_{gzp} = 35 \cdot 98066.5$ (N/m^2); $\Delta h = 400$ (m); $R = 8.3144621$ (J/mol·K); $T = 293$ (K); $\rho_0 = 0.68$ (kg/m^3); $\mu = 0.0008$ (m^2/s); $g = 9.8$ (m/s^2); $\rho = 998$ (kg/m^3).

Table 1. The dependence of water velocity υ on gas pressure on GWC p_{gzp}, seam pressure p_{pl} under the following parameter values: $q = 2$ (m^3/s), $R_k = 350$ (m), $R_c = 330$ (m)

p_{gzp}	45	43	41	39	37	35	33
p_{pl}	51.89	50.17	48.46	46.78	45.13	43.5	41.91
υ	$3 \cdot 10^{-6}$	$40 \cdot 10^{-6}$	$80 \cdot 10^{-6}$	$100 \cdot 10^{-6}$	$200 \cdot 10^{-6}$	$200 \cdot 10^{-6}$	$200 \cdot 10^{-6}$

Table 2. The dependence of contour radiuses R_k, R_c and water velocity υ under the following parameter values $q = 2$ (m^3/s), $p_{gzp} = 35 \cdot 98066.5$ (N/m^2), $p_{pl} = 43.5 \cdot 98066.5$ (N/m^2), $p_0 = 50 \cdot 98066.5$ (N/m^2), $R_k = 400$ (m)

$R_k - R_c$	5	0	15	20	25	30	35
υ	$600 \cdot 10^{-6}$	$300 \cdot 10^{-6}$	$300 \cdot 10^{-6}$	$200 \cdot 10^{-6}$	$200 \cdot 10^{-6}$	$200 \cdot 10^{-6}$	$100 \cdot 10^{-6}$

Table 3. The dependence of the well capacity q and water velocity υ under the following parameter values $p_{gzp} = 35 \cdot 98066.5$ (N/m^2), $R_k = 350$ (m), $R_c = 330$ (m)

q	5	4	3	2	1
p_{pl}	62.48	55.59	49.19	43.5	38.84
υ	$1 \cdot 10^{-6}$	$80 \cdot 10^{-6}$	$60 \cdot 10^{-6}$	$200 \cdot 10^{-6}$	$300 \cdot 10^{-6}$

Table 4. The dependence of pressures P_1 (at $P_2 = 55 \cdot 98066.5$ (N/m^2)) on surface r_1 and P_2 (at $P_1 = 45 \cdot 98066.5$ (N/m^2)) on surface r_2 on water velocity υ under the following parameter values $r_2 = 400$ (m), $r_1 = 350$ (m)

υ	$200 \cdot 10^{-6}$	$20 \cdot 10^{-6}$	$2 \cdot 10^{-6}$
P_1	44.41	47.21	49.32
P_2	48.01	49.88	51.23

Table 5. The dependence of pressures P_1 (at $P_2 = 55 \cdot 98066.5$ (N/m^2)) on surface r_1 and P_2 (at $P_1 = 45 \cdot 98066.5$ (N/m^2)) on surface r_2 on radiuses of contours r_1, r_2 at the following parameter values $\upsilon = 20 \cdot 10^{-6}$ (m/s), $r_1 = 350$ (m), $P_2 = 55 \cdot 98066.5$ (N/m^2)

$r_2 - r_1$	50	40	30	20	10
P_1	47.21	47.92	48.42	49.07	49.55
P_2	48.01	47.88	47.03	46.77	46.21

8. Conclusions

Our numerical results coincide with those well-known from the literature [2, 3, 10] and with the experimental data [9], which validates our theoretical results.

The results make it possible to calculate the speed of the GWC, depending on the parameters of the porous medium and the parameters of the underground gas storage. This allows us to calculate the work regime of an underground gas storage which prevents flooding of wells.

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