

THE INFLUENCE OF HELMHOLTZ AND GOLAY COIL HOMOGENITY ON RECTANGULAR PLATE BENDING

PIOTR KĘDZIA¹ AND ZBIGNIEW KOSMA²

*¹Faculty of Mechanical Engineering
Poznań University of Technology
Piotrowo 3 11/12, 60–965, Poznan, Poland*

*²Radom Academy of Economics
Mazowieckiego 7a, 26–600, Radom, Poland*

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Abstract: The subject of the paper is a rectangular plate. The structure of the plate is symmetrical. The plate is made of a cellular structure of a core and two external faces. There is ferrofluid in each cell of the porous core. The core is made of polyethylene foam and two faces are polyurethane sheets. It is assumed that cells in the core are regular and dense and the faces are thin, therefore, a plate filled in with ferrofluid can be treated as homogeneous. The plate is placed in the magnetic field. The magnetic field is generated by two systems of coils – the Helmholtz coil (HC) and the Golay coil (GC). The former generates an almost fixed magnetic field in the considered volume (where the plate is placed), and the latter generates a gradient magnetic field. The changing size of each subsystem of magnetic field coils, the distribution and strength of the magnetic field change as well. The magnetic field induces loads in the plate, both perpendicular and in plane. The plate bending function is approximated by the bicubic spline function presented by normalized B-spline functions. The influence of changes in the homogeneity volume and the magnetic field strength (by changing the coil size) on the plate bending is analyzed in the paper. The results of the analysis are presented in tables and figures.

Keywords: composite plate, magnetic field, ferrofluid, magnetic coils

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1. Introduction

The issue of plate bending is presented in many papers which especially include smart materials having a wide range of applications. They can be found in such areas as industry, everyday objects like toys, sports equipment. Plates filled with ferrofluid, which are classified as smart materials, can be treated as smart materials, as well. A comprehensive review of the issues of strength,

stability and dynamics of plates and coatings is presented in the monograph edited by Wozniak [1]. Ventsel and Krauthammer [2] describe in detail the theory, analysis and applications of the thin plate and shells. Composite plates made of polyethylene with a cellular structure, where each cell is filled with ferrofluid and placed in an external magnetic field are described by Kędzia and Kosma [3]. The authors show the deflection of plates with ferrofluid under a homogenous magnetic field generated by electromagnetic coils. As the area of a homogeneous magnetic field is small, coils are much bigger than the considered plate. A similar problem was presented by Kędzia and Magnucki [4], but in this case ferrofluid was distributed only on the plate edges and the stability of such structure was studied. An external magnetic field acts on the ferrofluid put in the pockets on the edges, where compression load is induced. The dynamic equilibrium paths were determined.

It is not only smart plates that are placed in magnetic fields. Noncontact transport of steel plates using electromagnetic forces has been proposed by Kurihara et al. [5]. The authors proposed an addition of electromagnetic actuators to control the horizontal motion of a levitated steel plate. Hssanyan and Piliposyan [6] obtained two-dimensional equations of magnetoelasticity for a magnetoelastic ferromagnetic plate of a finite size. The assumption that magnetic susceptibility is very large compared with unity allows obtaining an analytical representation for the components of an excited magnetic field in the case of a very thin plate. Ambartsumian [7] et al. considered some problems of magnetoelastic oscillations of thin electrically conducting plates and shells situated in a stationary magnetic field. The authors formulated a hypothesis relative to the character of the variation of the electromagnetic field and the nature of elastic displacements along the shell thickness which allows reducing the three-dimensional equations of magnetoelasticity to two-dimensional ones. Zhupanska and Sierakowski [8] described the macroscopic coupling mechanism and derived a coupled nonlinear system of 2D equations of motion and Maxwell's equation that account for the mechanical and electrical anisotropies of composite orthotropic plates. Zhupanska and Sierakowski [9] studied the mechanical response of transversely isotropic graphite/epoxy composite plates in the presence of an electromagnetic field. Hasanyan and Harutyunyan [10] presented the magnetoelastic stability of a ferromagnetic plate-strip in a homogeneous transverse magnetic field and the stress-strain state of a ferromagnetic plane with a moving crack in a transverse magnetic field. The electro-magneto-elastic analysis including the medium and its environment was studied by Kuang [11]. Zhou and Zheng [12] proposed a variational principle to establish the governing equations of magnetoelastic interaction for soft ferromagnetic thin plate structures under complex magnetic fields. The authors obtained the governing equations of the magnetic field and mechanical deformation together with an expression of the equivalent magnetic force exerted on the ferromagnetic. Nguyen and Tong [13] proposed a different way to present smart plates. The authors described an iterative technique for the multi-criteria

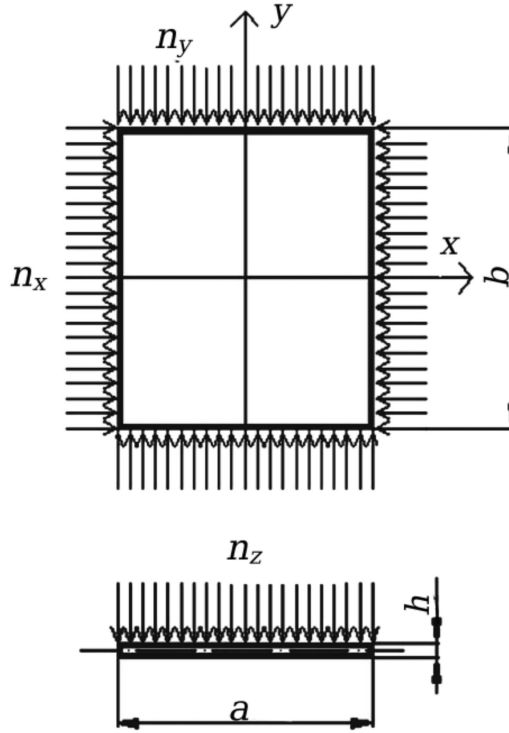


Figure 1. Scheme of rectangular plate and induced transverse and in-plane load

design problem in structural shape control applications while Sun and Tong [14] presented an investigation into the design optimization of actuator patterns for the static shape control of composite plates with piezoelectric actuator patches.

The subject of the study is a simply supported rectangular plate of length L and width b . The plate is composed of a cellular core and two external sheets of total thickness h , and average Young's modulus E and Poisson's ratio. The core is made of polyethylene foam filled in with ferrofluid. In addition, two faces are polyurethane sheets. It is assumed that each cell is separated from each other. Hence, the ferrofluid flow caused by the magnetic field along any direction is not possible. In other words, the cellular structure of the core avoids the ferrofluid flow in the plate. Nevertheless, homogenization is assumed in a further part of the paper. The plate is under a magnetic field which induces in the structure both transverse and in-plane loads of intensity n_z and $n_x(n_y)$ respectively Fig. 1. The in-plane load occurs not only on the edges of the plate, but also in the inner cells.

The load is induced in the plate by the ferrofluid placed in the magnetic field. The magnetic field is generated by the Helmholtz coil and the Golay (saddle) coil (Fig. 2). Such coils with their modification are often used in tomographs (Czechowski *et al.* [15]).

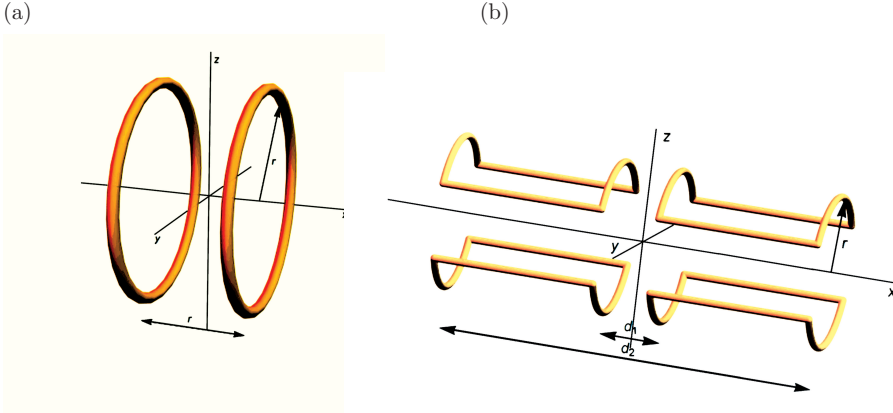


Figure 2. Scheme of a) Helmholtz coil, b) Golay coil

The Helmholtz coil consists of two circular current loops of wire of the same radius r , with the current I flowing in the same sense in each loop. Let us assume that the loops encircle the x axis and the distance between these loops is equal to r as well. In Taylor series expansion of Helmholtz coil magnetic field, the term with the second power of x vanishes (all terms with the odd power of x vanish because of the symmetry). It shows that the magnetic field expansions include the constant term only and then terms with the even power of x starting 4. It is the reason why the generated magnetic field is homogeneous.

The Golay coil consists of four saddles with the same radius of arcs r , the angle of arcs 120° . The inner arcs are separated with distance $d_1 = 0.78r$ and the outer ones with distance $d_2 = 0.78r$. Let us assume that x is an axis for all the saddles and current flows in each part of the Golay coil. In the upper inner arcs the current flows in the same sense and in the lower inner arcs current I flows in the opposite sense with respect to the upper ones. Such geometry allows obtaining relatively linear changes of the magnetic field, because of the vanishing term with the third power of x in the Taylor expansion (the term with the first power left, and then terms with odd powers from 5). In other words, the Golay coil generates a gradient magnetic field opposite to the Helmholtz coil which produces an almost constant magnetic field.

The aim of the paper is to designate how the magnetic field homogeneity influences the bending of a plate filled with ferrofluid. The homogeneity volume is changed by changing the sizes of Helmholtz and Golay coils. Ferrofluids do not conduct currents, hence, it is assumed that the magnetic field acts only on the ferrofluid and induces the Kelvin force [16]. The Kelvin force is the equivalent of the Lorentz force in the absence of a current flow through the ferromagnetic fluid. The Kelvin force influences the plate as a mechanical force.

2. Magnetic field and plate model

The magnetic field calculations are based on the Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{d\vec{l} \times \Delta\vec{r}}{(\Delta r)^3}, \quad (1)$$

where \vec{B} is the magnetic field, μ_0 is vacuum permeability and \vec{l} and $\Delta\vec{r}$ are geometrical dimensions (Fig. 3)

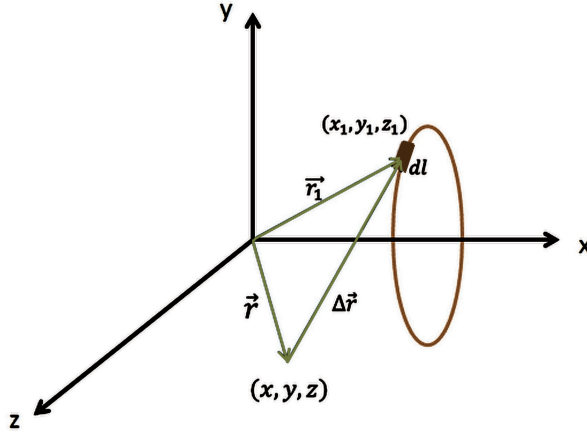


Figure 3. Scheme of magnetic field calculations for circular coils

The homogeneity of a magnetic field is defined as

$$\eta = k \frac{B - B_0}{B_0}, \quad (2)$$

where $k \in \{10^2, 10^6\}$ designate the rank of homogeneity in percent or ppm (parts per million), respectively, \vec{B} is the magnetic field at any point of the considered area and \vec{B}_0 is the magnetic field in the center of the considered area. The homogeneity formula for the gradient magnetic field Equation (2) is analogical, but the magnetic field is replaced with the magnetic field gradient. In further considerations the magnetic field \vec{H} will be used instead of the magnetic field \vec{B} .

According to the formulae Equation (1) and Equation (2) the contour lines of homogeneity are calculated and presented in Fig. 4 and Fig. 5 for Helmholtz and Golay coils, respectively.

The geometry of the Golay coil is more complicated than in the case of the Helmholtz coil, hence, the homogeneity of the gradient magnetic field of the GC is worse than the homogeneity of the magnetic field generated by the HC. In addition, the Helmholtz coil generates one component of the magnetic field vector (axial) much greater than the other components. In the Golay coil case, the magnetic field vector has two dominant components — it results from the Maxwell laws, where a concomitant gradient occurs.

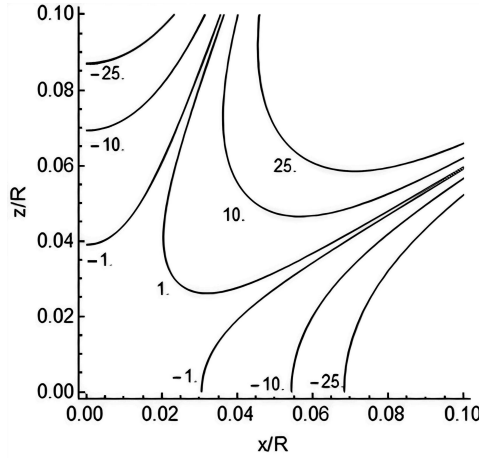


Figure 4. Homogeneity of magnetic field of Helmholtz coil in %.

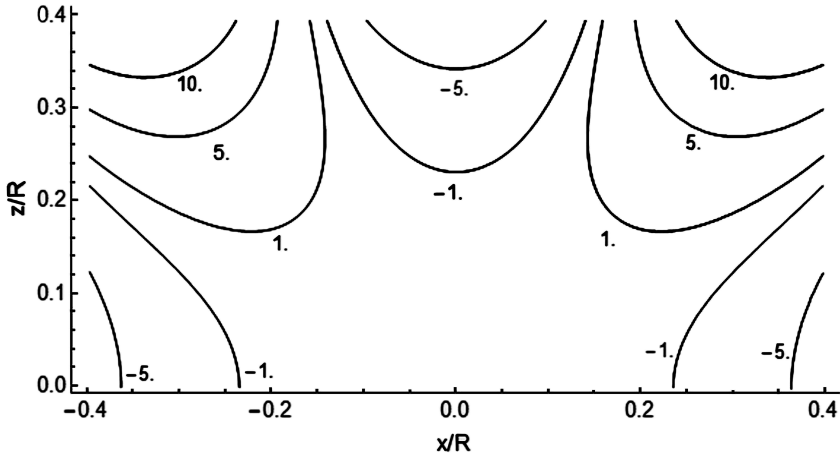


Figure 5. Homogeneity of axial magnetic field of Golay coil in %.

The magnetic field generated by the coils induces the Kelvin force [16] in the ferrofluid

$$\vec{n} = \frac{1}{2} \mu_0 \chi (1 - \chi) \nabla H^2, \quad \vec{n} = [n_x, n_y, n_z], \quad (3)$$

where χ is magnetic susceptibility.

The rectangular plate model is based on the classical Kirchhoff-Love hypothesis. The bending $w(x, y)$ of the plate shown in Fig. 1 is described by the Equation (4)

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left[n_z(x, y) + n_x(x, y) \frac{\partial^2 w}{\partial x^2} + n_y(x, y) \frac{\partial^2 w}{\partial y^2} \right], \quad (4)$$

where $D = Eh^3/12(1 - \nu^2)$ is the flexural rigidity of the plate with the boundary conditions chosen from the following cases:

- clamped edge $w = 0$, $\frac{\partial w}{\partial n} = 0$,
- simply supported edge $w = 0$, $\frac{\partial^2 w}{\partial n^2} = 0$,
- free edge $\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial s^2} = 0$, $\frac{\partial^3 w}{\partial n^3} + (2 - \nu) \frac{\partial^3 w}{\partial n \partial s^2} = 0$,

where $n, s \in [x, y]$. It is assumed that deflection w of the plate is small with respect to the plate dimensions a and b , hence, changes of the load in z direction are not taken into consideration.

3. Numerical algorithm

The solution of the Equation (4) with the boundary conditions under the magnetic field load calculated for the coil system was obtained with the collocation method described by Kosma [17, 18]. The plate bending function is approximated by the bicubic spline function presented by normalized B -spline functions $B_i(x)$ and $\widetilde{B}_j(y)$ in the form

$$w(x, y) \approx \sum_{i=-1}^{N+1} \sum_{j=-1}^{M+1} q_{i,j} B_i(x) \widetilde{B}_j(y), \quad (5)$$

where $q_{i,j}$ are unknown coefficients. In the algorithm regular mesh with points

$$x_i = i \cdot h, \quad i = -1, 0, 1, \dots, N, N+1 \quad (6)$$

$$y_j = j \cdot h, \quad i = -1, 0, 1, \dots, M, M+1 \quad (7)$$

$$h = \frac{a}{N} = \frac{b}{M} \quad (8)$$

is defined. Approximations of partial derivatives which appear in the plate bending Equation (4) and in the boundary conditions are calculated on the basis of the property that the bicubic spline function is the polynomial spline function with respect to each of the independent variables separately. The corresponding functions of deflection and its derivatives are presented in Figures 6.

Coefficients $q_{i,j}$, in each internal node of the mesh can be obtained from the algebraic linear system of equations

$$\sum_{r=-2}^2 \sum_{s=-2}^2 a_{i+r, j+s} q_{i+r, j+s} = b_{i,j}, \quad (9)$$

for $i = 1, 2, \dots, N-1$ and $j = 1, 2, \dots, M-1$, where

$$a_{i,j} = 24\alpha_1 + 324\alpha_2 + 24\alpha_3 - 72\alpha_4 - 72\alpha_5, \quad (10)$$

$$a_{i-1,j} = a_{i+1,j} = -16\alpha_1 - 144\alpha_2 + 6\alpha_3 + 32\alpha_4 - 18\alpha_5, \quad (11)$$

$$a_{i,j-1} = a_{i,j+1} = 6\alpha_1 - 144\alpha_2 - 16\alpha_3 - 18\alpha_4 + 32\alpha_5, \quad (12)$$

$$a_{i-1,j-1} = a_{i+1,j+1} = a_{i+1,j-1} = a_{i-1,j+1} = -4\alpha_1 + 64\alpha_2 - 4\alpha_3 + 8\alpha_4 + 8\alpha_5, \quad (13)$$

$$a_{i-2,j} = a_{i+2,j} = 4\alpha_1 - 18\alpha_2 + 4\alpha_4, \quad (14)$$

$$a_{i,j-2} = a_{i,j+2} = -18\alpha_1 + 4\alpha_2 + 4\alpha_5, \quad (15)$$

$$a_{i-2,j-1} = a_{i+2,j-1} = a_{i-2,j+1} = a_{i+2,j+1} = \alpha_1 + 8\alpha_2 + \alpha_4, \quad (16)$$

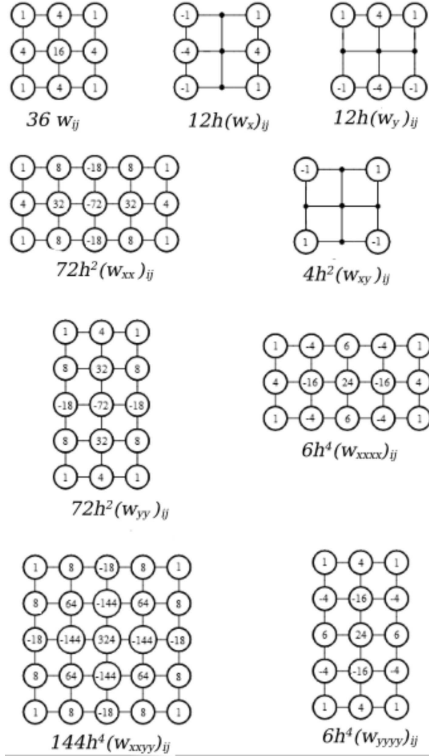


Figure 6. Schemes for the deflection function and its derivatives (Kosma [17, 18])

$$a_{i-1,j-2} = a_{i+1,j-2} = a_{i-1,j+2} = a_{i+1,j+2} = 8\alpha_2 + \alpha_3 + \alpha_5, \quad (17)$$

$$a_{i-2,j-2} = a_{i+2,j-2} = a_{i-2,j+2} = a_{i+2,j+2} = \alpha_2, \quad (18)$$

$$b_{i,j} = \frac{q_{i,j}}{D}, \quad (19)$$

$$\alpha_1 = \frac{1}{6h^4}, \quad \alpha_2 = \frac{1}{72h^4}, \quad \alpha_1 = \alpha_3, \quad \alpha_4 = \frac{-n_x}{D} \frac{1}{72h^4}, \quad \alpha_5 = \frac{-n_y}{D} \frac{1}{72h^4}, \quad (20)$$

The boundary conditions complement the system of Equation (9) to a full system, from which deflection w can be determined. The efficiency of the proposed algorithms is proved by comparing the obtained results with numerical simulations performed by FEM using Autodesk Simulation v.14 for the example plate in previous studies of Kędzia and Kosma [3] since it is impossible to compare the method with experiments. It has been confirmed that the bicubic spline approximation is good enough with respect to the estimation of admissible calculation errors.

4. Numerical examples

The numerical solutions for the problem Equation (4) with simply supported edges were conducted for two various examples. The first example describes the influence of the homogeneity of a magnetic field on the plate bending in the case

when changes in the size of the coils do not modify the strength of the magnetic field in the center of the coil system. The difference in the strength of the magnetic field caused by higher or lower values of the coil radius is compensated by a current value flowing through the wires of coils. The second numerical experiment presents the influence of the homogeneity of the magnetic field on the plate bending under an induced load when resizing coils taking into account changes in the strength of the magnetic field.

The following denotation for coils was introduced in order to better understand the influence of homogeneity on the plate bending: Hxx — Helmholtz coil, Gxx — Golay coil, where xx is the diameter of coil loops or arcs in mm.

The following data for the plate and ferrofluid was used for the calculations: $a = b = 0.3\text{m}$, $h = 0.005\text{m}$, $E = 1.07 \cdot 10^7\text{MPa}$, $\nu = 0.41$, $\chi = 0.06$. The starting diameters for Helmholtz and Golay coils are 30 mm with the magnetic field $B = 1\text{T}$ in the center and the strength of the gradient magnetic field of rank 0.1T/cm (values of magnetic field obtained in modern MRI tomographs).

4.1. Fixed magnetic field in the coil system center

The maximum deflection of the plate under a magnetic field is $w_{max} = 0.000329498\text{ mm}$ and it is obtained for the largest considered coils (H60, G45). All deflections are normalized with respect to w_{max}

Table 1. Normalized plate deflections for both fixed magnetic and gradient magnetic fields

		Helmholtz coil						
		H10	H15	H20	H30	H40	H50	H60
Golay coil	G10	0.436855	0.298243	0.233706	0.210895	0.207672	0.206863	0.206585
	G15	0.567131	0.754036	0.787522	0.796567	0.797901	0.798256	0.798382
	G20	0.57763	0.842084	0.918616	0.945606	0.94952	0.950516	0.95086
	G30	0.581152	0.866943	0.95534	0.987391	0.992036	0.993214	0.99362
	G40	0.58187	0.870896	0.960921	0.993642	0.998384	0.999585	1

In general, the deflection for the fixed magnetic field strength is smaller for lower coil radius values (Fig. 7). Different phenomena occur for the smallest Golay coil, where, if the Helmholtz coil radius increases, then the plate deflection decreases. It results from a high rank unhomogeneity of the Golay coil gradient magnetic field, which disrupts deflection. For a fixed Helmholtz coil radius, if the Golay coil radius increases, the plate deflection increases, as well. The higher the value of the radius of both coils indicates a large volume of the homogeneous magnetic field and the gradient magnetic field, hence, the deflection does not vary for them.

4.2. Variable magnetic field in the coil system center

The maximum deflection of the plate under a magnetic field is $w_{max} = 0.00064775\text{ mm}$ and it is obtained for the smallest considered coils (H10, G10).

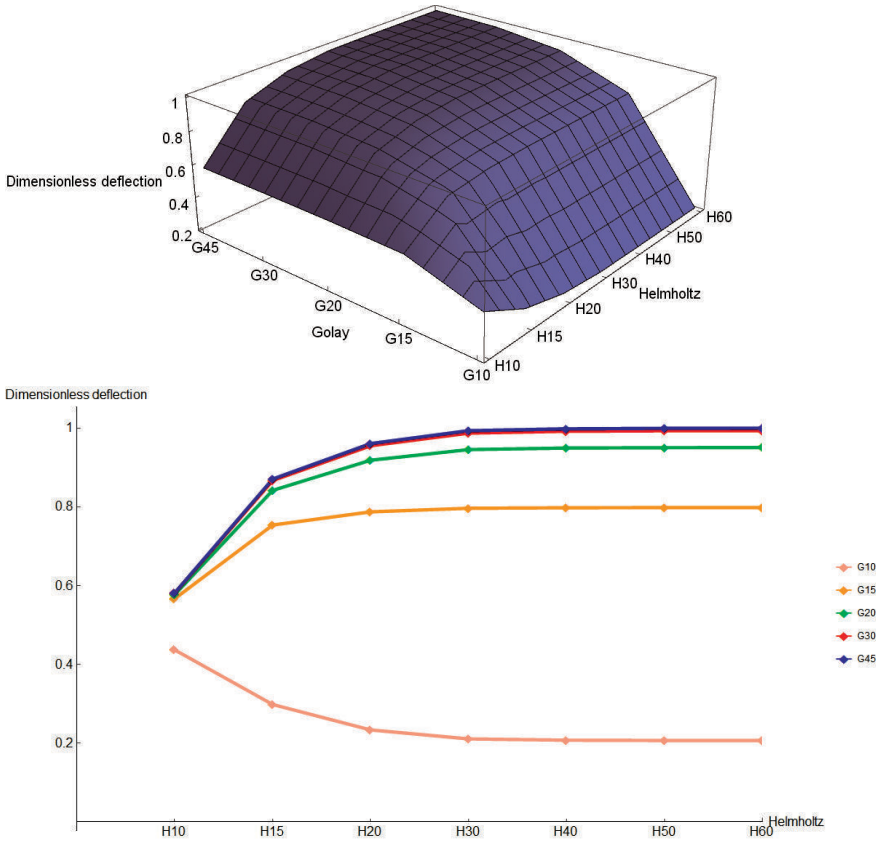


Figure 7. Plate deflection for different coil configurations (fixed and gradient magnetic fields in plate center)

Table 2. Normalized plate deflections for variable magnetic and gradient magnetic fields

		Helmholtz coil						
		H10	H15	H20	H30	H40	H50	H60
Golay coil	G10	1	0.455142	0.267474	0.160901	0.118829	0.0946916	0.0788028
	G15	0.576981	0.511444	0.400629	0.270159	0.202959	0.162439	0.135387
	G20	0.330558	0.32128	0.262869	0.180399	0.135859	0.108801	0.0907003
	G30	0.147809	0.147005	0.1215	0.083719	0.063084	0.050527	0.042123
	G45	0.065772	0.065632	0.054314	0.037442	0.028215	0.022599	0.01884

The deflection for the variable magnetic field strength is larger for lower coil radius values. It results from the fact that the strength of the magnetic field and the gradient magnetic field increases when the coil radius decreases. For Helmholtz coils, if we change the coil radius to $1/3r$, the magnetic field increases 3 times. For Golay coils, the same change of radius increases the gradient magnetic field strength 9 times (second power of 3).

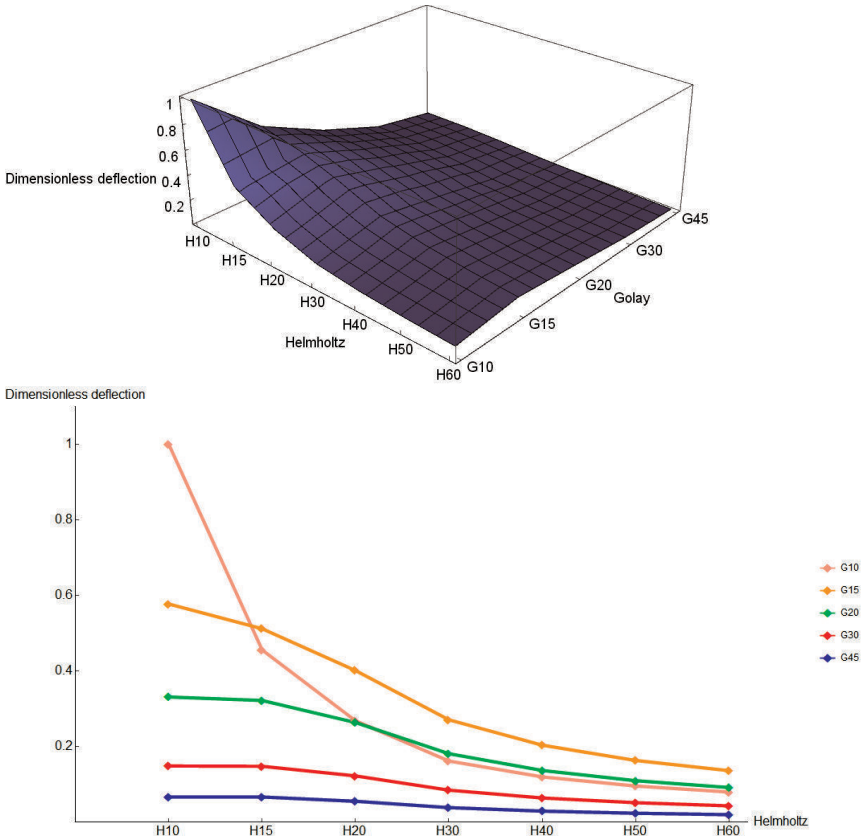


Figure 8. Plate deflection for different coil configurations (fixed and gradient magnetic fields in plate center)

5. Conclusions

Taking into account the obtained results of the plate deflection under a magnetic field generated by Helmholtz and Golay coil systems we conclude that:

For a fixed strength of a magnetic field generated by a HC and a fixed strength of a gradient magnetic field generated by the GC:

- the homogeneity influences the plate deflection. It is emphasized especially for the lowest Golay coil radius value with the largest volume of the inhomogeneous gradient magnetic field.
- a better homogeneity of magnetic and gradient magnetic fields does not influence the deflection which reaches the some value depending on the strength of magnetic fields.

For a variable strength of a magnetic field generated by a HC and a variable strength of a gradient magnetic field generated by a GC:

- the plate deflection is influenced not only by homogeneity but also by the strength of the used magnetic fields. It is emphasized especially for the lowest coil radius value (HC and GC).
- a better homogeneity of magnetic and gradient magnetic fields results in a weaker magnetic field and a weaker gradient magnetic field, hence, the deflection goes to zero.

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