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# A general model of industrial systems operation processes related to their environment and infrastructure

**Keywords**

system operation process, semi-markov process, system in varying operation

**Abstract**

The operation process of the complex industrial system is considered and its operation states are introduced. The semi-markov process is used to construct a general probabilistic model of the considered complex industrial system operation process. To build this model the vector of probabilities of the system operation process initials operation states, the matrix of probabilities of the system operation process transitions between the operation states, the matrix of conditional distribution functions and the matrix of conditional density functions of the system operation process conditional sojourn times in the operation states are defined. To describe the system operation process conditional sojourn times in the particular operation states the uniform distribution, the triangle distribution, the double trapezium distribution, the exponential distribution, the Weibull distribution and the normal distribution are suggested. Under these all assumptions from the constructed general model the main characteristics of the system operation process are find. The mean values of the system operation process conditional sojourn times in particular operation states having these distributions are calculated. Moreover, the unconditional distribution functions of the system operation process unconditional sojourn times in the particular operation states, the mean values of the system operation process unconditional sojourn times in the particular operation states, the limit values of the transient probabilities of the system operation process at the particular operation states and the mean values of the system operation process total sojourn times in the particular operation states are determined.

## 1. Introduction

Most real transportation systems are very complex and it is difficult to analyze their reliability and availability. Large numbers of components and subsystems and their operating complexity cause that the evaluation and optimization of their reliability and availability is complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics is often very difficult to fix and to analyze. Usually the system environment and infrastructure have either an explicit or an implicit strong influence on the system operation process. As a rule some of the initiating environment events and infrastructure conditions define a set of different operation states of the industrial system. A convenient tool for solving this problem is semi-markov modeling of the systems operation processes proposed in this work.

## 2. Modeling system operation process

We assume that the system during its operation process is taking $v, v \in N$, different operation states. Further, we define the system operation process $Z(t)$, $t \in < 0,+\infty >$, with discrete operation states from the set of states $Z = \{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is semi-markov [1] with the conditional sojourn times $\theta_{bl}$ at the operation states $z_b$ when its next operation state is $z_l$, $b, l = 1,2,...,v$, $b \neq l$. Under these assumptions, the system operation process may be described by:

- the vector of probabilities of the system operation process $Z(t)$ initial operation states

$$[p_b(0)]_{1\times v} = [p_1(0), p_2(0), ..., p_v(0)], \quad (1)$$

where

$$p_b(0) = P(Z(0) = z_b) \text{ for } b = 1,2,...,v,$$

- the matrix of probabilities of the system operation process $Z(t)$ transitions between the operation states

$$[p_{bl}]_{v\times v} = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1v} \\ p_{21} & p_{22} & \dots & p_{2v} \\ \dots \\ p_{v1} & p_{v2} & \dots & p_{vv} \end{bmatrix}, \quad (2)$$

where

$$p_{bb} = 0 \text{ for } b = 1,2,...,v,$$

- the matrix of conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ in the operation states

$$[H_{bl}(t)]_{v\times v} = \begin{bmatrix} H_{11}(t) & H_{12}(t) & \dots & H_{1v}(t) \\ H_{21}(t) & H_{22}(t) & \dots & H_{2v}(t) \\ \dots \\ H_{v1}(t) & H_{v2}(t) & \dots & H_{vv}(t) \end{bmatrix}, \quad (3)$$

where

$$H_{bl}(t) = P(\theta_{bl} < t) \text{ for } b,l = 1,2,...,v, \ b \neq l,$$

and

$$H_{bb}(t) = 0 \text{ for } b = 1,2,...,v.$$

Further, we denote the matrix of corresponding conditional density functions of the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ by

$$[h_{bl}(t)]_{v\times v} = \begin{bmatrix} h_{11}(t) & h_{12}(t) & \dots & h_{1v}(t) \\ h_{21}(t) & h_{22}(t) & \dots & h_{2v}(t) \\ \dots \\ h_{v1}(t) & h_{v2}(t) & \dots & h_{vv}(t) \end{bmatrix}, \quad (4)$$

where

$$h_{bl}(t) = \frac{d}{dt}[H_{bl}(t)] \text{ for } b,l = 1,2,...,v, \ b \neq l.$$

We assume that the typical distributions to describe the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$, $b,l = 1,2,...,v, \ b \neq l,$ in the particular operation states are:

- the uniform distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{1}{y_{bl} - x_{bl}}, & x_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \quad (5)$$

where $0 \le x_{bl} < y_{bl} < +\infty$,

- the triangle distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \quad (6)$$

where $0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty$,

- the double trapezium distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ - q_{bl}] \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ w_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ - w_{bl}] \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \quad (7)$$

where $0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty$, $0 \le q_{bl} < +\infty$,
$0 \le w_{bl} < +\infty$, $0 \le q_{bl}(z_{bl} - x_{bl}) + w_{bl}(y_{bl} - z_{bl}) \le 2$,

- the exponential distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < 0 \\ \alpha_{bl} \exp[-\alpha_{bl} t], & t \geq 0, \end{cases} \quad (8)$$

where $0 \leq \alpha_{bl} < +\infty$,

- the Weibull distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < 0 \\ \alpha_{bl} \beta_{bl} t^{\beta_{bl}-1} \exp[-\alpha_{bl} t^{\beta_{bl}}], & t \geq 0, \end{cases} \quad (9)$$

where $0 \leq \alpha_{bl} < +\infty$, $0 \leq \beta_{bl} < +\infty$,

- the normal distribution with a density function

$$h_{bl}(t) =$$

$$\frac{1}{\sigma_{bl}\sqrt{2\pi}} \exp[-\frac{(t - m_{bl})^2}{2\sigma_{bl}^2}], \ t \in (-\infty, \infty), \quad (10)$$

where $-\infty < m_{bl} < +\infty$, $0 \leq \sigma_{bl} < +\infty$.

As the mean values of the conditional sojourn times $\theta_{bl}$ are given by [2]

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int^\infty t h_{bl}(t) dt, \quad (11)$$
$$b, l = 1,2,\ldots,v, \ b \neq l,$$

then for the above distinguished distributions (5)-(10), the mean values of the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$, $b, l = 1,2,\ldots,v, b \neq l$, in particular operation sattes are respectively given by [2], [5]:
- for the uniform distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl}}{2}, \quad (12)$$

- for the triangle distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3}, \quad (13)$$

- for the double trapezium distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3} + \frac{w_{bl} y_{bl}^2 - q_{bl} x_{bl}^2}{2} + \frac{w_{bl} + q_{bl}}{6}$$
$$[(x_{bl} z_{bl} - y_{bl} z_{bl}) + \frac{x_{bl} y_{bl} (x_{bl} + y_{bl})}{y_{bl} - x_{bl}}]$$
$$- \frac{x_{bl}^3 q_{bl} + y_{bl}^3 w_{bl}}{3(y_{bl} - x_{bl})}, \quad (14)$$

- for the exponential distribution

$$M_{bl} = E[\theta_{bl}] = \frac{1}{\alpha_{bl}}, \quad (15)$$

- for the Weibull distribution

$$M_{bl} = E[\theta_{bl}] = \alpha_{bl}^{-\frac{1}{\beta_{bl}}} \Gamma(1 + \frac{1}{\beta_{bl}}), \quad (16)$$

where

$$\Gamma(u) = \int_0^{+\infty} t^{u-1} e^{-t} dt, \ u > 0,$$

is the gamma function,

- for the normal distribution

$$M_{bl} = E[\theta_{bl}] = m_{bl}. \quad (17)$$

From the formula for total probability [1] it follows that the unconditional distribution functions of the sojourn times $\theta_b$, $b = 1,2,\ldots,v$, of the system operation process $Z(t)$ at the operation states $z_b$, $b = 1,2,\ldots,v$, are given by [1]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ b = 1,2,\ldots,v. \quad (18)$$

Hence, the mean values $E[\theta_b]$ of the unconditional sojourn times $\theta_b$, $b = 1,2,\ldots,v$, are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ b = 1,2,\ldots,v, \quad (19)$$

where $M_{bl}$ are defined by the formula (11) in a case of any distribution of sojourn times $\theta_{bl}$ and by the formulae (12)-(17) in the cases of particular distinguished respectively by (5)-(10) distributions of these sojourn times.
The limit values of the transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b), \ t \in <0,+\infty), \ b = 1,2,\ldots,v,$$

are given by [1]

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \quad b = 1,2,...,v, \tag{20}$$

where $M_b$, $b = 1,2,...,v$, are given by (19), while the probabilities $\pi_b$ of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \tag{21}$$

Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its sojourn times $\hat{\theta}_b$ in the particular operation states $z_b$, $b = 1,2,...,v$. It is well known [1] that the system operation process total sojourn times $\hat{\theta}_b$ in the particular operation states $z_b$, for sufficiently large operation time $\theta$, have approximately normal distribution with the expected value given by

$$E[\hat{\theta}_b] = p_b \theta, \quad b = 1,2,...,v, \tag{22}$$

where $p_b$ are given by (20).

## 3. Conclusions

The constructed general probabilistic model of the complex system operation process is the basis for the further tasks of the Poland-Singapore Joint Research Project. Its unknown parameters will be estimated in Tasks of WP 6. The model will be used in WP2 for particular modeling the operation processes of port and shipyard transportation systems in Task 2.2 and for modeling the operation process of ships at the restricted and open water areas in Task 2.3. The model will also be used to construct in Task 4.1 of WP4 the integrated general reliability, availability and safety probabilistic models of complex industrial systems related to their operation processes and its particular case for port and shipyard transportation systems in Task 4.2 and for ships operating at the restricted and open water areas in Task 4.3.

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