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Brief note

MICROPOLAR FLOW OVER A BLACK ISOTHERMAL PLATE IN THE PRESENCE THERMAL RADIATION

A. RAPTIS
Department of Mathematics, University of Ioannina, Ioannina, 45110, GREECE
E-mail: araptis@uoi.gr

This study numerically investigates the effects of thermal radiation on the flow over a black isothermal plate for an optically thin gray micropolar fluid. The flowing medium absorbs and emits radiation, but scattering is not included. The computational results are discussed graphically for several selected flow parameters.

Keywords: Micropolar fluid, thermal radiation, isothermal plate.

1. Introduction

Micropolar fluids are fluids with a microstructure. They belong to a class of fluids with nonsymmetric stress tensor that which are called polar fluids, and include, as a special case, the well-established Navier-Stokes model of classical fluids that we shall call ordinary fluids. Physically, micropolar fluids may represent fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium, where the deformation of fluid particles is ignored. The model of micropolar fluids was introduced in by Eringen [1]. Extensive reviews of the theory and application can be found in the articles by Ariman *et al.* [2-4] and the book by Lukaszewicz [5]. The potential importance of a micropolar boundary layer flow in industrial applications has motivated a number of previous studies, of which those of Chiam [6], Hassanien and Gorla [7, 8] and Hassanien [9] are of special interest.

On the other hand, at high temperature the effects of the thermal radiation in space technology are significant. Ishak [10] investigated the thermal boundary layer flow of a micropolar fluid over a stretching sheet a with thermal radiation effect. Rashidi and Pour [11] studied the flow a micropolar fluid through a porous medium in the presence of thermal radiation. Bhattacharyya et al. [12] described the effects of thermal radiation on a micropolar fluid flow and heat transfer over a porous shrinking sheet. Srinivas *et al.* [13] examined the unsteady flow of a micropolar fluid over a porous stretching sheet with thermal radiation and chemical reaction. Singh and Kumar [14] described the effects of thermal radiation on a mixed convection flow of a micropolar fluid from an unsteady stretching surface with viscous dissipation and heat generation/absorption.

Naveed *et al*. [15] studied a magnetohydrodynamic flow of a micropolar fluid due to a curved stretching sheet with thermal radiation effect. Das and Sarkar [16] explored the effect of melting on a magnetohydrodynamic micropolar fluid flow toward a shrinking sheet in the presence of thermal radiation. Arifuzzaman *et al.* [17] described the magnetohydrodynamic micropolar fluid flow in the presence of nanoparticles through porous plate. Atif *et al.* [18] studied the magnetohydrodynamic micropolar Carreau nanofluid flow in the presence of thermal radiation and induced magnetic field. Also, Atif *et al.* [19] analyzed the magnetohydrodynamic stratified bioconvective flow of a micropolar nanofluid due to gyrotactic microorganisms in the presence of thermal radiation. Reddy and Ferdows [20] investigated the species and thermal radiation effects on a micropolar hydromagnetic dusty fluid flow across a paraboloid revolution. In all these studies, the micropolar fluid was assumed to be optically thick and used the Rosseland approximation for the thermal radiation. To the author knowledge a micropolar flow over a black isothermal plate in the presence of thermal radiation has not been studied . So, in this paper, we have studied the effects of thermal radiation on the flow over a black isothermal plate for an optically thin gray micropolar fluid. The flowing medium absorbs and emits radiation, but scattering is not included.

2. Mathematical analysis

We consider a two-dimensional steady flow of a micropolar fluid over α black isothermal plate. The x-axis is along the plate and the y-axis is normal to the plate. The radiative heat flux at the x-direction is considered negligible in comparison to the y-direction. Under the above assumptions, the flow in the presence of thermal radiation is governed by the following equations [21, 22]:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0\,, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+k_1\frac{\partial \sigma}{\partial y}\,, \tag{2.2}$$

$$G_1\frac{\partial^2 \sigma}{\partial y^2}-2\sigma-\frac{\partial u}{\partial y}=0\,, \tag{2.3}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial u}{\partial y}=k\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_{r,y}}{\partial y}\,. \tag{2.4}$$

The boundary conditions are

$$\begin{aligned} &u=0, \quad \upsilon=0, \quad T=T_w, \quad \sigma=0, \quad \text{at} \quad y=0,\\ &u\to U_0\,, \quad T\to T_\infty\,, \quad \sigma\to 0, \quad \text{as} \quad y\to\infty, \end{aligned} \tag{2.5}$$

here u, υ are the components of the velocity parallel and perpendicular to the plate, $\nu=\frac{\mu+S}{\rho}$ is the apparent kinematic viscosity, μ is the coefficient of dynamic viscosity, S is the vortex viscosity, ρ is the fluid density, σ is the microrotation component, $k_1=\frac{S}{\rho}$, $k_1>0$ is the coupling constant, G_1 is the microrotation constant, T is the fluid temperature, T_w is the temperature of the plate, T_∞ is the fluid temperature at infinity where $T_\infty<T_w$, k is the thermal conductivity, c_p is the specific heat at constant pressure, $q_{r,y}$ is the thermal radiative heat flux at the y-direction and U_0 is the free stream velocity.

For the case of an optically thin gray fluid the local radiation over a black isothermal plate is expressed by [22]

$$-\frac{\partial q_{r,y}}{\partial y} = 2a\sigma^{*}\left(T_w^4 + T_\infty^4 - 2T^4\right) \tag{2.6}$$

where a is the absorption coefficient and σ^* is the Stefan-Boltzman constant.
We put

$$2T_r^4 = T_w^4 + T_\infty^4 \tag{2.7}$$

where $T_\infty < T_r < T_w$
By using Eqs (2.6) and (2.7), Eq.(2.4) gives

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{4a\sigma^*}{\rho c_p}\left(T_r^4 - T^4\right). \tag{2.8}$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature T. This is accomplished by expansion in a Taylor series about T_r and neglecting higher-order terms, thus

$$T^4 \cong 4T_r^3 T - 3T_r^4 . \tag{2.9}$$

Substituting (2.9) in equation (2.8) gives

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16a\sigma^*}{\rho c_p}\left(T_r - T\right). \tag{2.10}$$

We introduce the following transformations

$$\eta = \sqrt{\frac{U_0}{\nu x}}y, \qquad u = \frac{\partial\psi}{\partial y}, \qquad \upsilon = -\frac{\partial\psi}{\partial x}, \qquad \psi = \sqrt{U_0\nu x}\, f\left(\eta\right),$$

$$\sigma = \sqrt{\frac{U_0}{\nu x}}U_0 g(\eta), \qquad K = \frac{K_1}{\nu}, \qquad G = G_1\frac{U_0}{\nu x}, \tag{2.11}$$

$$\theta = \frac{T - T_r}{T_w - T_r}, \quad m = \frac{T_\infty - T_r}{T_w - T_r}, \quad P = \frac{\rho\nu c_p}{k}, \quad S^* = \frac{16a\sigma^*\nu T_r^{\,3}x}{kU_0}$$

where η is a similarity variable, ψ is the stream function, $f\left(\eta\right)$ is the dimensionless stream function, K is the coupling parameter, $g\left(\eta\right)$ is the dimensionless microrotation component, G is the dimensionless microrotation parameter, θ is the dimensionless temperature, m is the dimensionless temperature parameter, P is the Prandtl number and S^* is the radiation parameter.

Clearly u, υ satisfy the continuity equation (2.1) identically. Using Eq.(2.11) the Eqs (2.2), (2.3) and (2.10) become

$$f''' + \frac{1}{2} ff'' + Kg' = 0, \tag{2.12}$$

$$Gg'' - 2g - f'' = 0, \tag{2.13}$$

$$\theta'' + \frac{1}{2} Pf\theta' - S^{*}\theta = 0. \tag{2.14}$$

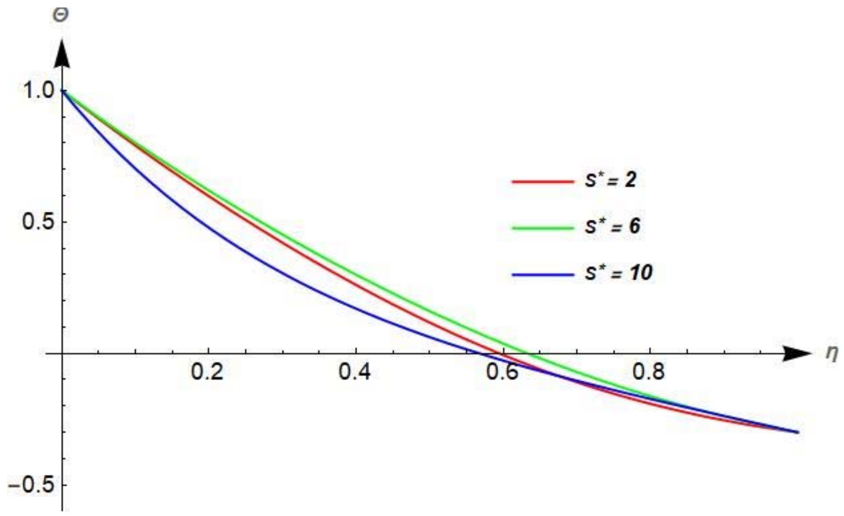

Fig.1. Temperature profiles for different values of the radiation parameter S^{*}.

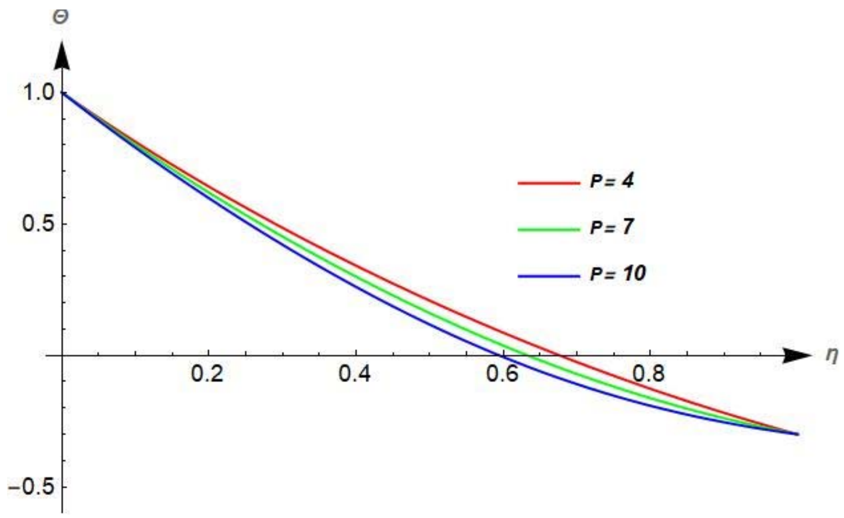

Fig.2. Temperature profiles for different values of the Prandtl number P.

The corresponding boundary conditions are

$$\begin{aligned} &f'(0) = 0, \quad f(0) = 0, \quad g(0) = 1, \quad \theta(0) = 1, \\ &f'(\infty) = 1, \qquad g(\infty) = 0, \quad \theta(\infty) = m. \end{aligned} \tag{2.15}$$

Equations (2.12)-(2.14), subject to the boundary conditions (2.15), constitute a nonlinear system of differential equations, which is solved numerically by using two boundary value problem.

Figure 1 shows the effect of the radiation parameter S^* on the non-dimensional temperature θ when $K = 0.2$, $G = 2$, $P = 10$, $m = -0.3$. It is observed that the non-dimensional temperature decreases with the increase of the radiation parameter S^*.

Figure 2 shows the effect of the Prandtl number P on the non-dimensional temperature θ, when $K = 0.2$, $G = 2$, $S^* = 2$, $m = -0.3$. It is observed that the non-dimensional temperature decreases with the increase of the Prandtl number P.

3. Conclusions

In this paper, we have studied the effects of thermal radiation on the flow over a black isothermal plate for an optically thin gray micropolar fluid. The flowing medium absorbs and emits radiation, but scattering is not included.

- An increase in the radiation parameter S^* leads to a decrease of the temperature.
- An increase in the Prandtl number P leads to a decrease of the temperature.

Nomenclature

c_p – specific heat at constant pressure
$f(\eta)$ – dimensionless stream function
$g(\eta)$ – dimensionless microrotation component
G – microrotation parameter
k – thermal conductivity
K – coupling parameter
K_1 – coupling constant
m – temperature parameter
P – Prandtl number
$q_{r,y}$ – thermal radiative heat flux at y - direction
S – vortex viscosity
S^* – radiation parameter
T – fluid temperature
T_w – fluid temperature on the plate
T_∞ – fluid temperature at infinity
u – velocity parallel to the plate
υ – velocity perpendicular to the plate
U_0 – free stream velocity
x – axis along of the plate
y – axis normal to the plate
α – absorption coefficient

η – similarity variable

θ – dimensionless temperature

ν – kinematic viscosity

ρ – fluid density

σ – microrotation component

σ^* – Stefan-Boltzmann constant

ψ – stream function

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