AXISYMMETRIC VIBRATION FOR MICROPOLAR POROUS THERMOELASTIC CIRCULAR PLATE

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The present investigation is concerned with a two dimensional axisymmetric problem in a homogeneous isotropic micropolar porous thermoelastic circular plate by using the eigen value approach. The Laplace and Hankel transform are used to solve the problem. The expression of displacements, microrotation, volume fraction field, temperature distribution and stresses are obtained in the transformed domain subjected to thermomechanical sources. A computer algorithm is developed for numerical computations. To obtain the resulting quantities in a physical domain, a numerical inversion technique is used. The resulting quantities are depicted graphically for a specific model. Some special cases are also deduced.

Key words: micropolar porous thermoelasticity, eigen value, Laplace and Hankel transforms, thermomechanical sources.

1. Introduction

The theory of micropolar elasticity developed by Eringen [1] aroused much interest because of its possible utility in investigating the deformation properties of solids for which the classical theory is inadequate. The micropolar theory is significantly useful for investigating materials consisting of bar-like molecules which exhibit microrotation effects and support body and surface couples. A special micropolar material was fabricated in which a uniformly distributed rigid aluminium shot was cast in an elastic epoxy matrix by Gauthier [2] and the values of the relevant parameters based on specimen of aluminium-epoxy composite were investigated.

The linear theory of micropolar thermoelasticity was developed by Eringen [3] and Nowacki [4] to include thermal effects. Touchert *et al*. [5] developed the linear theory of micropolar thermoelasticity in which Duhamel-Neumann analogy is extended to micropolar materials and the thermoelastic problem is reduced to a corresponding isothermal one with body forces and couples. Boschi and Iesan [6] investigated

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the linear theory of generalized micropolar thermoelasticity. Passeralla [7] established some results in micropolar thermoelasticity.

Cowin and Nunziato [8] investigated the linear theory of elastic materials with voids. This theory differs significantly from the classical linear elasticity in that the volume fraction corresponding to the void volume is taken as an independent kinematical variables. Several applications of the theory are developed, including the response to homogeneous deformations, pure bending of a beam, and small amplitude acoustic waves. In each of these applications, the change in the volume fraction field induced by the deformation is determined. Iesan [9] studied the shock waves in micropolar elastic materials with voids. Iesan [10] developed the linear theory of thermoelastic materials with voids. Scarpetta [11] studied the fundamental solution for the differential system of micropolar elasticity with voids for the steady vibration case and reciprocal properties are also explored. Marin [12] studied the mixed boundary value problem in elastostatic micropolar materials with voids. Marin [13] applied the general results from the theory of elliptic equations to obtain the existence and uniqueness of the generalized solutions for the boundary value problems in micropolar elasticity with voids.

Ciarletta *et al.* [14] constructed the fundamental solutions of the systems of equations of the steady oscillation of the linear theory of micropolar thermoelasticity for materials with voids. Kumar and Panchal [15] studied the propagation of circular crested waves in a micropolar porous medium possessing cubic symmetry. Ailawalia and Kumar [16] studied the thermomechanical deformation of micropolar generalized thermoelastic materials with voids under the influence of various sources. Othman and Youssef [17] studied the deformation of a micropolar thermoelastic solid with voids considering the influence of various sources acting on the plane interface. Sharma and Marin [18] studied the reflection of plane waves at the free surface of a micropolar generalized thermoelastic solid with distinct conductive and thermodynamic temperatures. Sharma *et al.* [19] studied the propagation of Lamb waves in a homogeneous, isotropic thermoelastic micropolar solid with two temperatures bordered with layers of inviscid liquid. Sharma [20] studied the effect of two temperatures on the reflection coefficient for a micropolar thermoelastic solid. Sharma and Kumar [21] studied the propagation of plane waves in a thermoviscoelastic medium with voids. Kumar *et al.* (2015) constructed the fundamental solution to a system of micropolar viscothermoelastic solids with voids in terms of elementary functions. Marin [23] formulated a heat flux dependent theory for micropolar porous materials.

2. Basic equations

Following Kumar and Partap [24], the constitutive relations and the field equations in a micropolar porous thermoelastic medium with body forces, body couples, heat sources and extrinsic equilibrated body force are given by

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \varepsilon_{ijk}\phi_k\right) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T\delta_{ij} + b\delta_{ij}\phi^*, \tag{2.1}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{2.2}$$

$$(\lambda + \mu)\nabla(\nabla.\boldsymbol{u}) + (\mu + K)\nabla^2\boldsymbol{u} + K\nabla\times\phi + b\nabla\phi^* - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla T = \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{2.3}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla.\phi) - \gamma\nabla\times(\nabla\times\phi) + K\nabla\times\boldsymbol{u} - 2K\phi = \rho j\frac{\partial^2\phi}{\partial t^2}, \tag{2.4}$$

$$\alpha_1 \nabla^2 \phi^* - b(\nabla . \boldsymbol{u}) - \xi_1 \phi^* - \omega_0 \frac{\partial \phi^*}{\partial t} + m\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T = \rho \chi \frac{\partial^2 \phi^*}{\partial t^2}, \tag{2.5}$$

$$K_1^* \nabla^2 T - \nu T_0 \left(\frac{\partial}{\partial t} + \eta_0 \tau_0 \frac{\partial^2}{\partial t^2}\right)(\nabla . \boldsymbol{u}) - m T_0 \left(\frac{\partial}{\partial t} + \eta_0 \tau_0 \frac{\partial^2}{\partial t^2}\right)\phi^* = \rho C^* \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)T \tag{2.6}$$

where $\boldsymbol{u}$ is the displacement vector, ϕ is the microrotation vector, ρ is the density, j is the micro inertia, K_1^* is the coefficient of thermal conductivity, T is the change in temperature of the medium at any time, C^* is the specific heat at constant strain, ϕ^* is the change in volume fraction field, $\lambda, \mu, K, \alpha, \beta, \gamma$ are micropolar constants, $\alpha_1, b, \xi_1, \omega_0, m$ and χ are the elastic constants due to the presence of voids, $\nu = (3\lambda + 2\mu + K)\alpha_t$, α_t is the coefficient of linear thermal expansion, τ_0, τ_1, are the thermal relaxation times, t_{ij}, m_{ij} are the stress tensor and couple stress tensor, δ_{ij} is the Kronecker delta and $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator. For L-S theory, $\tau_1 = 0, \tau_0 > 0,$ and $\eta_0 = 1.$ For G-L theory, $\tau_1 \geq \tau_0 > 0$ and $\eta_0 = 0.$

3. Formulation of the problem

Consider an infinite, homogeneous isotropic micropolar porous thermoelastic circular plate of thickness *2h* occupying the region defined by $0 \leq r \leq \infty, -h \leq z \leq h.$ The cylindrical polar coordinates (r, θ, z) are introduced. The plate is axisymmetric with the *z*-axis as the axis of symmetry. The origin of the co-ordinate system (r, θ, z) is taken as the middle surface of the plate and the *z*-axis normal to it along the thickness. We take the $r - z$ plane as the plane of incidence. The initial temperature in the thick plate is given by a constant temperature T_0.

For a two dimensional problem, we take

$$\boldsymbol{u} = (u_r, 0, u_z), \qquad \phi = (0, \phi_\theta, 0). \tag{3.1}$$

The dimensionless quantities are given by

$$r' = \frac{\omega^* r}{c_1}, \quad z' = \frac{\omega^* z}{c_1}, \quad u_r' = \frac{\rho c_1 \omega^* u_r}{\nu T_0}, \quad u_z' = \frac{\rho c_1 \omega^* u_z}{\nu T_0}, \quad \phi_\theta' = \frac{\rho c_1^2 \phi_\theta}{\nu T_0}, \quad \phi^{*\prime} = \frac{\rho c_1^2 \phi^*}{\nu T_0},$$

$$T' = \frac{T}{T_0}, \quad t' = \omega^* t, \quad \tau_0' = \omega^* \tau_0, \quad \tau_1' = \omega^* \tau_1, \quad t_{ij}' = \frac{t_{ij}}{\nu T_0}, \quad m_{ij}' = \frac{\omega^*}{c_1 \nu T_0} m_{ij} \tag{3.2}$$

where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, \qquad \omega^* = \frac{K}{\rho j}.$$

We define the Laplace and Hankel transform as

$$\bar{f}(r,z,s) = L\{\bar{f}(r,z,t)\} = \int_0^{\infty} f(r,z,t)e^{-st}dt, \tag{3.3}$$

$$\tilde{f}(\xi,z,s) = H\{\bar{f}(x,z,s)\} = \int_0^{\infty} r\bar{f}(x,z,s)J_n(\xi r)dr . \tag{3.4}$$

Equations (2.1)-(2.4) with the aid (3.1)-(3.4) recast into the form

$$\tilde{u}_r'' = a_{11}\tilde{u}_r + a_{14}\tilde{\phi}^* + a_{15}\tilde{T} + b_{12}\tilde{u}_z' + b_{13}\tilde{\phi}_\theta', \tag{3.5}$$

$$\tilde{u}_z'' = a_{22}\tilde{u}_z + a_{23}\tilde{\phi}_\theta + b_{21}\tilde{u}_r' + b_{24}\tilde{\phi}^{*'} + b_{25}\tilde{T}', \tag{3.6}$$

$$\tilde{\phi}_\theta'' = a_{32}\tilde{u}_z + a_{33}\tilde{\phi}_\theta + b_{31}\tilde{u}_r', \tag{3.7}$$

$$\tilde{\phi}^{*''} = a_{41}\tilde{u}_r + a_{44}\tilde{\phi}^* + a_{45}\tilde{T} + b_{42}\tilde{u}_z', \tag{3.8}$$

$$\tilde{T}'' = a_{51}\tilde{u}_r + a_{54}\tilde{\phi}^* + a_{55}\tilde{T} + b_{52}\tilde{u}_z' \tag{3.9}$$

where

$$a_{11} = \left(\frac{\xi^2 + s^2}{\delta^2}\right), \quad a_{14} = \frac{p_0\xi}{\delta^2}, \quad a_{15} = -\frac{\xi}{\delta^2}(1 + \tau_1 s), \quad a_{22} = \left(\xi^2\delta^2 + s^2\right), \quad a_{23} = -p\xi,$$

$$a_{32} = -\xi\delta^{*2}, \quad a_{33} = \left(\xi^2 + \frac{s^2}{\delta_1^2} + 2\delta^{*2}\right), \quad a_{41} = p_0\delta_1^*\xi, \quad a_{44} = \left(\xi^2 + \delta_3^* s^2 + p_1\delta_1^* + \delta_2^* s\right),$$

$$a_{45} = -\bar{v}\delta_1^*(1 + \tau_1 s), \quad a_{51} = \xi\epsilon\left(s + \eta_0\tau_0 s^2\right), \quad a_{54} = \bar{v}\epsilon\left(s + \eta_0\tau_0 s^2\right),$$

$$a_{55} = \left(\xi^2 + Q^*\left(s + \tau_0 s^2\right)\right), \quad b_{12} = \frac{\xi\left(1 - \delta^2\right)}{\delta^2}, \quad b_{13} = \frac{p}{\delta^2}, \quad b_{21} = -\xi\left(1 - \delta^2\right),$$

$$b_{24} = -p_0, \quad b_{25} = \left(1 + \tau_1 s\right), \quad b_{31} = -\delta^{*2}, \quad b_{42} = p_0\delta_1^*, \quad b_{52} = \epsilon\left(s + \eta_0\tau_0 s^2\right),$$

$$c_2^2 = \frac{\mu + K}{\rho}, \quad \delta^2 = \frac{c_2^2}{c_1^2}, \quad p = \frac{K}{\rho c_1^2}, \quad p_0 = \frac{b}{\rho c_1^2}, \quad \delta^{*2} = \frac{Kc_1^2}{\gamma\omega^{*2}}, \quad \delta_1^2 = \frac{c_3^2}{c_1^2},$$

$$c_3^2 = \frac{\gamma}{\rho j}, \quad \delta_1^* = \frac{\rho c_1^4}{\alpha_1\omega^{*2}}, \quad \bar{v} = \frac{m}{v}, \quad p_1 = \frac{\xi_1}{\rho c_1^2}, \quad \delta_2^* = \frac{\omega_0 c_1^2}{\alpha_1\omega^*}, \quad \delta_3^* = \frac{\rho\chi c_1^2}{\alpha_1},$$

$$Q^* = \frac{\rho C^* c_1^2}{K_1^*\omega^*}, \quad \epsilon = \frac{v^2 T_0}{\rho K_1^*\omega^*}.$$

The system of Eqs (3.5)-(3.9) can be written as

$$\frac{d}{dz}W(\xi,z,s) = A(\xi,s)W(\xi,z,s) \tag{3.10}$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \quad U = \begin{bmatrix} \tilde{u}_r \\ \tilde{u}_z \\ \tilde{\phi}_\theta \\ \tilde{\phi}^* \\ \tilde{T} \end{bmatrix}, \quad D = \frac{d}{dz}, \tag{3.11}$$

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} a_{11} & 0 & 0 & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 & 0 \\ a_{41} & 0 & 0 & a_{44} & a_{45} \\ a_{51} & 0 & 0 & a_{54} & a_{55} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & b_{12} & b_{13} & 0 & 0 \\ b_{21} & 0 & 0 & b_{24} & b_{25} \\ b_{31} & 0 & 0 & 0 & 0 \\ 0 & b_{42} & 0 & 0 & 0 \\ 0 & b_{52} & 0 & 0 & 0 \end{bmatrix}.$$

To solve Eq.(3.10), we take

$$W(\xi,z,s) = X(\xi,s)e^{qz}, \tag{3.12}$$

with

$$A(\xi,s)W(\xi,z,s) = qW(\xi,z,s), \tag{3.13}$$

which leads to the eigen value problem. The characteristic equation corresponding to Eq.(3.13) on expansion, yields

$$q^{10} - \lambda_1 q^8 + \lambda_2 q^6 - \lambda_3 q^4 + \lambda_4 q^2 - \lambda_5 = 0 \tag{3.14}$$

where λ_1, λ_2, λ_3, λ_4 and λ_5 are given in Appendix I and $\pm q_i$, $(i = 1,2,3,4,5)$ are the roots of Eq.(3.14).

The eigenvectors $X_i(\xi,s)$ corresponding to the eigenvalues q_i may be obtained by solving

$$[A - qI]X_i(\xi,s) = 0.$$

The set of eigen vector $X_i(\xi,s)$ can be written as

$$X_i(\xi,s) = \begin{bmatrix} X_{i1}(\xi,s) \\ X_{i2}(\xi,s) \end{bmatrix}$$

where

$$X_{i1}(\xi,s) = \begin{bmatrix} a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{i2}(\xi,s) = \begin{bmatrix} a_i q_i^2 \\ b_i q_i \\ -\xi q_i \\ d_i q_i \\ e_i q_i \end{bmatrix}, \qquad q = q_i; i = 1,2,3,4,5,$$

$$X_{j1}(\xi,s) = \begin{bmatrix} -a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{j2}(\xi,s) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ \xi q_i \\ -d_i q_i \\ -e_i q_i \end{bmatrix}, \quad j = i+5, \ q = -q_i; i = 1,2,3,4,5$$

Where $a_i, b_i, d_i, e_i, \Delta_i, r_1, r_2, r_3, r_4$ and r_5 are given in Appendix II

We assume the solution of Eq.(3.12) as

$$W(\xi,z,s) = \sum_{i=1}^{5} K_i X_i(\xi,s) \cosh(q_i z)$$

where K_1, K_2, K_3, K_4 and K_5 are arbitrary constants.

4. Boundary conditions

The boundary conditions at the surface $z = \pm h$ of the plate are given by

$$\frac{dT}{dz} = z^2 H(a-r), \tag{4.1}$$

$$t_{zz} = p_0 \frac{\delta(r)H(t)}{2\pi r}, \tag{4.2}$$

$$t_{zr} = 0, \tag{4.3}$$

$$m_{z\theta} = 0, \tag{4.4}$$

$$\frac{d\phi^*}{dz} = 0 \,. \tag{4.5}$$

(δ) is a Dirac delta function and (H) is the Heavy side unit step function and t_{zz}, t_{zr} and $m_{z\theta}$ are given by

$$t_{zz} = \left(\lambda + 2\mu + K\right)\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) - \nu\left(1 + \tau_1\frac{\partial}{\partial r}\right)T + b\phi^*, \tag{4.6}$$

$$t_{zr} = \left(\mu + K\right)\frac{\partial u_r}{\partial z} + \mu\frac{\partial u_z}{\partial r} - K\phi_\theta, \tag{4.7}$$

$$m_{z\theta} = \gamma\frac{\partial \phi_\theta}{\partial z} \,. \tag{4.8}$$

From Eqs (3.2)-(3.4) and (3.15)-(4.8), after some algebraic calculations, we get

$$\left(\tilde{u}_r, \ \tilde{u}_z, \ \tilde{\phi}_\theta, \tilde{\phi}^*, \tilde{T}\right) = \frac{1}{\Delta}\sum_{i=1}^{5}\left(a_i q_i, \ b_i, -\xi, \ d_i, \ e_i\right)\Delta_i \cosh\left(q_i z\right), \tag{4.9}$$

$$\left(\tilde{t}_{zz}, \ \tilde{t}_{zr}, \ \tilde{m}_{z\theta}\right) = \frac{1}{\Delta}\sum_{i=1}^{5}\left(L_i, \ M_i, \ P_i\right)\Delta_i \cosh\left(q_i z\right) \tag{4.10}$$

where

$$\Delta = \begin{vmatrix} S_1 & S_2 & S_3 & S_4 & S_5 \\ T_1 & T_2 & T_3 & T_4 & T_5 \\ U_1 & U_2 & U_3 & U_4 & U_5 \\ V_1 & V_2 & V_3 & V_4 & V_5 \\ W_1 & W_2 & W_3 & W_4 & W_5 \end{vmatrix},$$

and $\Delta_i\,(i = 1,2,3,4,5)$ are obtained from Δ by replacing i^{th} column of Δ with $\left|Q, R, 0, 0, 0\right|^{tr}$,
also $\quad S_i = e_i q_i \cosh\left(q_i h\right), \ T_i = L_i \cosh\left(q_i h\right), \ U_i = M_i \cosh\left(q_i h\right), \ V_i = P_i \cosh\left(q_i h\right),$

$$W_i = d_i q_i \cosh\left(q_i h\right), \qquad Q = \frac{a z^2 J_1(\xi a)}{\xi}, \qquad R = \frac{p_0}{2\pi s},$$

$$L_i = \left[\frac{\lambda\xi a_i q_i}{\rho c_1^2} + p_0 d_i - \left(1 + \tau_1 s\right)e_i + \left(\frac{\lambda + 2\mu + K}{\rho c_1^2}\right)b_i q_i\right], \quad i = 1,2,3,4,5\,,$$

$$M_i = \left[-\frac{\mu\xi b_i}{\rho c_1^2} + \frac{\xi K}{\rho c_1^2} + \left(\frac{\mu + K}{\rho c_1^2}\right)a_i q_i^2\right], \qquad i = 1,2,3,4,5\,,$$

$$P_i = \frac{-\gamma\xi\omega^{*2}q_i}{\rho c_1^4}, \quad i = 1,2,3,4,5 .$$

Particular cases

(i)   Without the thermal effect, the boundary conditions (4.1)-(4.5) for a micropolar porous medium reduce to the form

$$t_{zz} = p_0 \frac{\delta(r)H(t)}{2\pi r}, \quad t_{zr} = 0, \quad m_{z\theta} = 0, \quad \frac{d\phi^*}{dz} = 0,$$

and following the same procedure, the corresponding expressions for displacements, microrotation, volume fraction field and stresses for a micropolar porous are obtained as

$$\left(\tilde{u}_r, \ \tilde{u}_z, \ \tilde{\phi}_\theta, \tilde{\phi}^* \right) = \frac{1}{\Delta^*} \sum_{i=1}^{4} (a_i q_i, \ b_i, -\xi, \ d_i) \Delta_i^* \cosh(q_i z),$$

$$\left(\tilde{t}_{zz}, \ \tilde{t}_{zr}, \ \tilde{m}_{z\theta} \right) = \frac{1}{\Delta^*} \sum_{i=1}^{4} (L_i, \ M_i, \ P_i) \Delta_i^* \cosh(q_i z)$$

where

$$\Delta^* = \begin{vmatrix} T_1^* & T_2^* & T_3^* & T_4^* \\ U_1^* & U_2^* & U_3^* & U_4^* \\ V_1^* & V_2^* & V_3^* & V_4^* \\ W_1^* & W_2^* & W_3^* & W_4^* \end{vmatrix},$$

and $\Delta_i^* (i = 1,2,3,4)$ are obtained from Δ^* by replacing i^{th} column of Δ^* with $|R, 0, 0, 0|^{tr}$, also

$$T_i^* = L_i^* \cosh(q_i h), \quad U_i^* = M_i^* \cosh(q_i h), \quad V_i^* = P_i^* \cosh(q_i h), \quad W_i^* = d_i q_i \cosh(q_i h),$$

$$L_i^* = \left[\frac{\lambda\xi a_i q_i}{\rho c_1^2} + p_0 d_i + \left(\frac{\lambda + 2\mu + K}{\rho c_1^2} \right) b_i q_i \right], \quad i = 1,2,3,4,$$

$$M_i^* = \left[-\frac{\mu\xi b_i}{\rho c_1^2} + \frac{\xi K}{\rho c_1^2} + \left(\frac{\mu + K}{\rho c_1^2} \right) a_i q_i^2 \right], \quad i = 1,2,3,4,$$

$$P_i^* = \frac{-\gamma\xi\omega^{*2}q_i}{\rho c_1^4}, \quad i = 1,2,3,4.$$

(ii)   Neglecting the porous effect, i.e., $\alpha_1, b, \xi_1, \omega_0, \chi$ and $\phi^* \to 0$, yields the boundary conditions for micropolar thermoelastic medium as

$$\frac{dT}{dz} = z^2 H(a-r), \quad t_{zz} = p_0 \frac{\delta(r)H(t)}{2\pi r}, \quad t_{zr} = 0, \quad m_{z\theta} = 0,$$

and corresponding expressions are given by

$$\left(\tilde{u}_r, \tilde{u}_z, \tilde{\phi}_\theta, \tilde{T}\right) = \frac{1}{\Delta^{**}} \sum_{i=1}^{4} (a_i q_i, b_i, -\xi, e_i) \Delta_i^{**} \cosh(q_i z),$$

$$\left(\tilde{t}_{zz}, \tilde{t}_{zr}, \tilde{m}_{z\theta}\right) = \frac{1}{\Delta^{**}} \sum_{i=1}^{4} (L_i, M_i, P_i) \Delta_i^{**} \cosh(q_i z)$$

where

$$\Delta^{**} = \begin{vmatrix} S_1^{**} & S_2^{**} & S_3^{**} & S_4^{**} \\ T_1^{**} & T_2^{**} & T_3^{**} & T_4^{**} \\ U_1^{**} & U_2^{**} & U_3^{**} & U_4^{**} \\ V_1^{**} & V_2^{**} & V_3^{**} & V_4^{**} \end{vmatrix},$$

and $\Delta_i^{**} (i = 1,2,3,4)$ are obtained from Δ^{**} by replacing i^{th} column of Δ^{**} with $|Q, R, 0, 0|^{tr}$, also

$$S_i^{**} = e_i q_i \cosh(q_i h), \quad T_i^{**} = L_i^{**} \cosh(q_i h), \quad U_i^{**} = M_i^{**} \cosh(q_i h), \quad V_i^{**} = P_i^{**} \cosh(q_i h),$$

$$L_i^{**} = \left[\frac{\lambda \xi a_i q_i}{\rho c_1^2} - (1 + \tau_1 s)e_i + \left(\frac{\lambda + 2\mu + K}{\rho c_1^2}\right) b_i q_i \right], \quad i = 1,2,3,4$$

$$M_i^{**} = \left[-\frac{\mu \xi b_i}{\rho c_1^2} + \frac{\xi K}{\rho c_1^2} + \left(\frac{\mu + K}{\rho c_1^2}\right) a_i q_i^2 \right], \quad i = 1,2,3,4,$$

$$P_i^{**} = \frac{-\gamma \xi \omega^{*2} q_i}{\rho c_1^4}, \quad i = 1,2,3,4.$$

(iii)   Taking $\tau_1 = 0$, $\eta_0 = 1$, in Eqs (4.9)-(4.10), yields the corresponding expressions for micropolar porous thermoelastic with one relaxation time.

(iv)   The corresponding expressions for a micropolar porous thermoelastic with two relaxation times are obtained by taking $\tau_1 > 0$, $\eta_0 = 0$ in Eqs (4.9)-(4.10).

5.   Inversion of transforms

The transformed displacements, microrotation vector, volume fraction field, temperature distribution and stresses are of the form $\tilde{f}(\xi, z, s)$ and to obtain the function $f(r, z, t)$, the inversion of the Hankel transform is of the form

$$\tilde{f}(\xi, z, s) = \int_0^\infty \xi \bar{f}(\xi, z, s) J_n(\xi r) d\xi .$$ (5.1)

The inversion formula for the Laplace transforms is given as

$$f(r, z, t) = \frac{1}{2\iota\pi} \int_{c-\iota\infty}^{c+\iota\infty} \bar{f}(r, z, s) e^{-st} ds$$ (5.2)

where c is an arbitrary constant greater than all real parts of the singularities of $\bar{f}(r, z, t)$.

6. Numerical results and discussion

The values of micropolar parameters for numerical computations are given by Eringen [25]

$$\lambda = 9.4 \times 10^{10} \, Nm^{-2}, \quad \mu = 4.0 \times 10^{10} \, Nm^{-2}, \quad K = 1.0 \times 10^{10} \, Nm^{-2},$$

$$\rho = 1.74 \times 10^3 \, Kgm^{-3}, \quad j = 0.2 \times 10^{-19} \, m^2, \quad \gamma = 0.779 \times 10^{-9} \, N,$$

Following Dhaliwal and Singh [26], we take the values of thermal parameters as

$$C^* = 1.04 \times 10^3 \, JKg^{-1}K^{-1}, \quad K_1^* = 1.7 \times 10^6 \, Jm^{-1}s^{-1}K^{-1}, \quad \alpha_t = 2.33 \times 10^{-5} \, K^{-1},$$

$$\tau_0 = 6.131 \times 10^{-13} \, sec, \quad \tau_1 = 8.765 \times 10^{-13} \, sec, \quad m = 1.13849 \times 10^{10} \, N / m^2,$$

$$T_0 = 0.298 \times 10^3 \, K.$$

The values of void parameters are taken as

$$\alpha_1 = 3.688 \times 10^{-9} \, N, \quad b = 1.138494 \times 10^{10} \, N / m^2, \quad \xi_1 = 1.1475 \times 10^{10} \, N / m^2,$$

$$\chi = 1.1753 \times 10^{-19} \, m^2, \quad \omega_0 = 0.0787 \times 10^{-1} \, N \times sec / m^2.$$

The variations of displacements, microrotation, volume fraction field, temperature distribution and stresses with distance r in the case of a micropolar thermoelastic porous medium (MTPM), micropolar thermoelastic medium (MTM) and micropolar porous medium (MPM) are shown in Figs 1-8, respectively. In all these figures, a solid line (——), a small dash line (- - - - -) and a dash line with centred symbol ($-\cdot-\cdot-$) are used for MTPM, MPM and MTM, respectively.
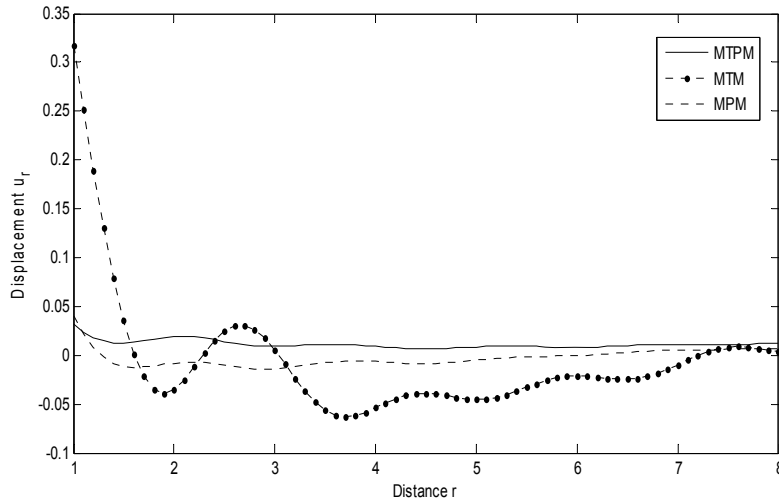
Fig.1. Variations of displacement of u_r.

Figure 1 depicts that the value of u_r initially decreases for $1 \le r \le 1.8$ and then oscillates for $1.8 \le r \le 8$ with large amplitude for MTPM, MTM and MPM. For MTM, u_r has a maximum value at the beginning and a minimum value for $3.4 \le r \le 7.4$ in comparison with MTPM and MPM. The oscillation behavior is not uniform for the three cases.
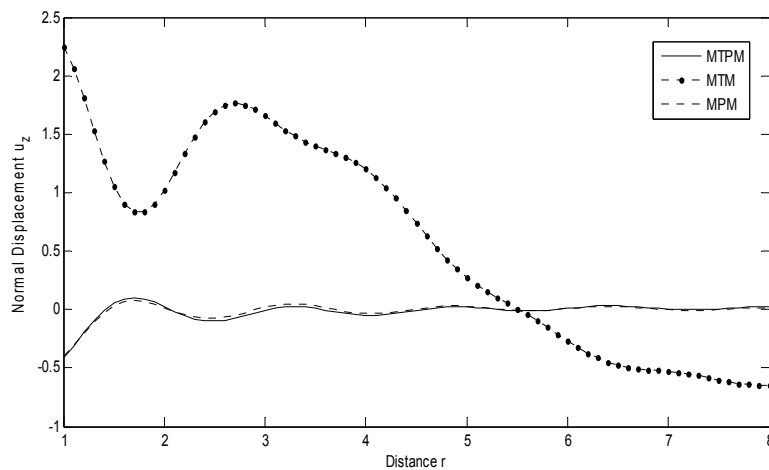

Fig.2. Variations of normal displacement u_z.

Figure 2 shows that the value of u_z which initially increases for $1 \le r \le 1.8$ and then oscillates for the whole range with large amplitude for MTPM and MPM. The value of u_z for MTM decreases for $1 \le r \le 1.7$, increases for $1.7 \le r \le 2.8$ and then rapidly decreases for $2.8 \le r \le 8$. The variation and behavior for MTPM and MPM are similar with a slightly different magnitude. The values of normal displacement for MTPM and MPM are small compared to MTM for $1 \le r \le 5.6$ and the values approach zero.
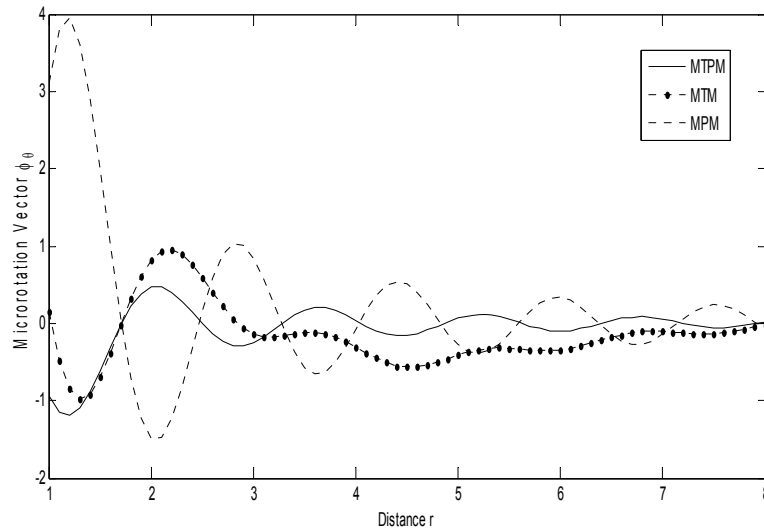
Fig.3. Variations of the microrotation vector ϕ_θ.

In Fig.3, the value of ϕ_θ for MTPM initially decreases for $1 \le r \le 1.4$ and oscillates for $1.4 \le r \le 8$, whereas for MPM, its value initially increases for $1 \le r \le 1.3$, sharply decreases for $1.3 \le r \le 2$ and then oscillates for $2 \le r \le 8$. For the range $1 \le r \le 8$, the behavior of ϕ_θ for MTPM and MTM is similar and opposite for MPM.
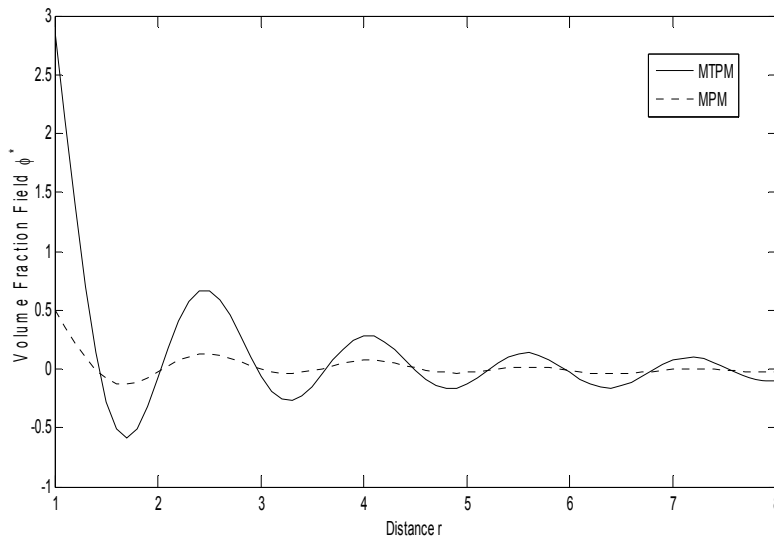

Fig.4. Variations of volume fraction field ϕ^*.

Figure 4 shows that the value of ϕ^* sharply decreases for $1 \le r \le 1.6$ and then oscillates with increasing in amplitude about the origin for MTPM, whereas for MPM, its value decreases for $1 \le r \le 1.6$, oscillates for $1.6 \le r \le 5.4$ with a large amplitude and further oscillates with a constant amplitude $5.4 \le r \le 8$.

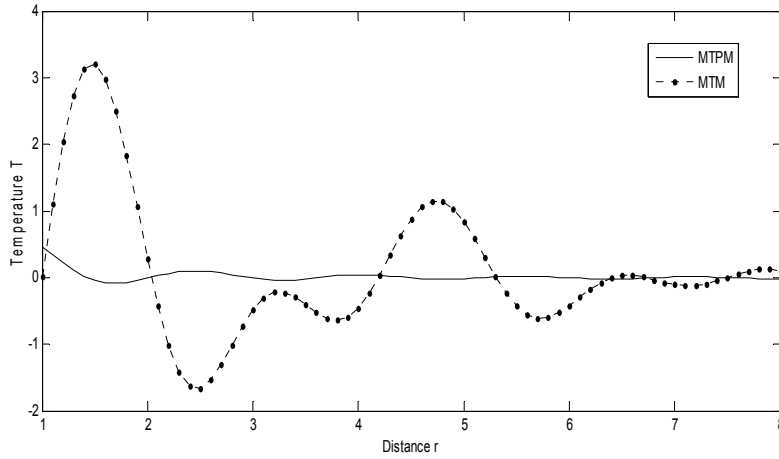Fig.5. Variations of temperature distribution T .

Figure 5 represents that the value of T initially decreases and then oscillates with a large amplitude for the whole range for MTPM. For MTM, its value initially increases for $1 \leq r \leq 1.6$, sharply decreases for $1.6 \leq r \leq 2.5$ and then oscillates for $2.5 \leq r \leq 8$ with different amplitude. The value is large for MTM for $1.2 \leq r \leq 2.2, 4.4 \leq r \leq 5.4$ and $7.6 \leq r \leq 8$ and small for $2.2 \leq r \leq 4.4$, $5.4 \leq r \leq 6.4$ and $6.8 \leq r \leq 7.6$.
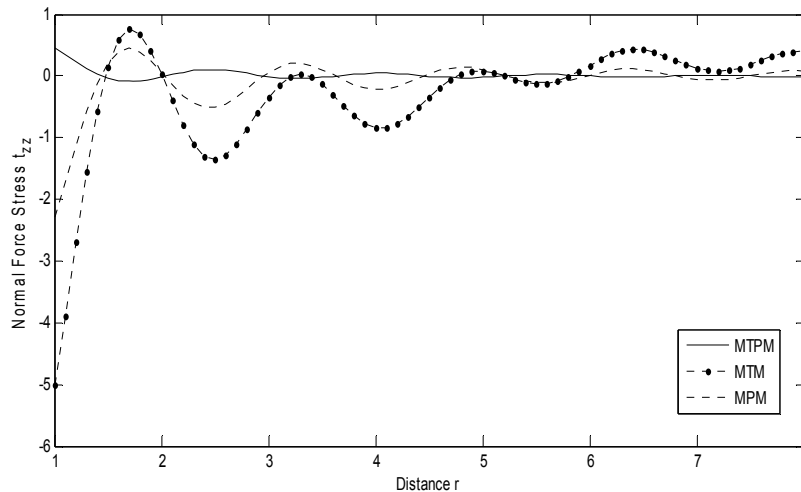

Fig.6. Variations of normal force stress t_{zz} .

Figure 6 shows that the value of t_{zz} initially decreases for MTPM for $1 \leq r \leq 1.6$ and then oscillates about the origin with a large amplitude. The value of t_{zz} increases in the beginning for $1 \leq r \leq 1.7$, decreases for $1.7 \leq r \leq 2.6$ and then oscillates for $2.6 \leq r \leq 8$ with r for MTM and MPM. The behavior of t_{zz} for MTPM is opposite to MTM and MPM for $1 \leq r \leq 6.6$. The variation in t_{zz} is oscillatory for MTPM, MTM and MPM.
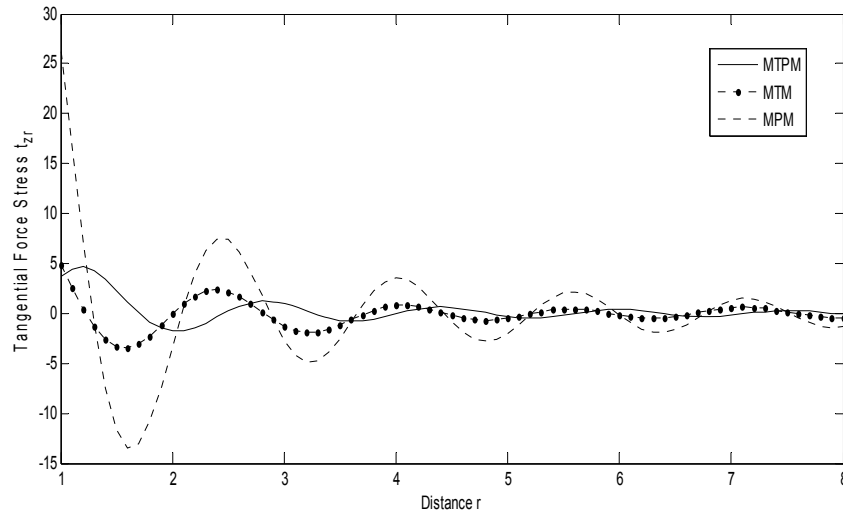
Fig.7. Variation of tangential force stress t_{zr}.

Figure 7 shows that for MTPM, the value of t_{zr} increases initially for $1 \leq r \leq 1.4$ and then oscillates with r. The value of t_{zr} starts with a sharp decrease for MPM as compared to MTM for $1 \leq r \leq 1.7$ and oscillates about the origin for $1.7 \leq r \leq 8$. The value of t_{zr} for MTPM and MTM is similar near the application of the source and the value is also the same away from the source for the three cases.

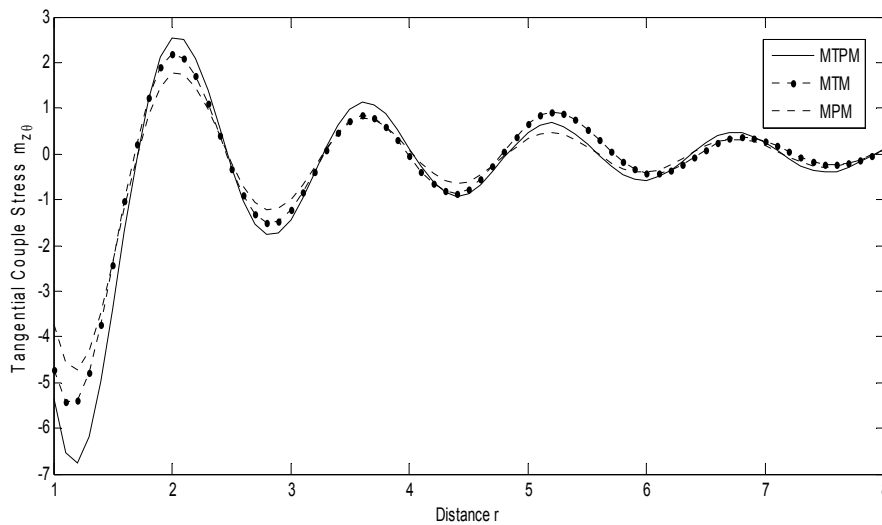Fig.8. Variations of tangential couple stress $m_{z\theta}$.

Figure 8 shows that the value of $m_{z\theta}$ initially decreases for $1 \leq r \leq 1.4$, sharply increases for $1.4 \leq r \leq 2$ and then oscillates for $2 \leq r \leq 8$ for MTPM, MTM and MPM. The variation and behavior of $m_{z\theta}$ is uniformly oscillatory for the three cases for the whole range. The values for MTPM, MTM and MPM approach zero.

7. Conclusions

The present investigation is concerned with the eigen value approach for a micropolar porous thermoelastic circular plate due to the application of thermo mechanical source. From the figures, it is observed that all the physical quantities have non zero value in considered domain and approache the boundary surface away from the sources. The appreciable effects of the porosity and thermoelasticity are observed on displacements, microrotation, volume fraction field, temperature distribution and stresses. The variation pattern of displacements for MTPM and MPM are the same. The variations patterns are also similar for normal and tangential stresses for MTM and MPM. However, a similar behavior is also observed for tangential couple stress for MTPM, MTM and MPM. The behavior of temperature distribution gets oscillatory due to the porosity effect. Due to the thermal effect, the values of the volume fraction field are initially decreased but afterwards its they increase and oscillate.

Appendix I

$$\lambda_1 = -\left(a_{11} + a_{22} + a_{33} + a_{44} + a_{55} + b_{12}b_{21} + b_{13}b_{31} + b_{25}b_{52} + b_{24}b_{42}\right),$$

$$\begin{aligned}
\lambda_2 =\ & -a_{14}a_{41} + a_{33}a_{55} + a_{44}a_{55} + a_{11}a_{55} + a_{22}a_{55} + a_{33}a_{44} + a_{11}a_{33} + a_{22}a_{33} + \\
& + a_{11}a_{44} + a_{22}a_{44} + a_{11}a_{22} - a_{15}a_{51} - a_{45}a_{54} - a_{23}a_{32} + \left(a_{33} + a_{44} + a_{55}\right)b_{12}b_{21} + \\
& - \left(a_{14}b_{42} + a_{15}b_{52} + a_{32}b_{13}\right)b_{21} + \left(a_{11} + a_{33} + a_{55}\right)b_{42}b_{24} + \left(a_{11} + a_{33} + a_{44}\right)b_{25}b_{52} + \\
& - \left(a_{41}b_{24} + a_{23}b_{31} + a_{51}b_{25}\right)b_{12} + \left(a_{22} + a_{44} + a_{55} + b_{42}b_{24} + b_{25}b_{52}\right)b_{31}b_{13} + \\
& - a_{45}b_{52}b_{24} - a_{54}b_{42}b_{25},
\end{aligned}$$

$$\begin{aligned}
\lambda_3 =\ & (a_{11}a_{22} + a_{22}a_{55})(a_{33} + a_{44}) - a_{23}a_{32}\left(a_{11} + a_{44} + a_{55}\right) + \\
& + a_{11}a_{55}\left(a_{22} + a_{33} + a_{44}\right) + a_{33}a_{44}\left(a_{11} + a_{22} + a_{55}\right) - a_{45}a_{54}\left(a_{11} + a_{22} + a_{33}\right) + \\
& - a_{14}a_{41}\left(a_{22} + a_{33} + a_{55}\right) - a_{15}a_{51}\left(a_{22} + a_{33} + a_{44}\right) + \\
& + b_{42}b_{25}(a_{14}a_{51} + a_{11}a_{54} + a_{33}a_{54}) + b_{52}b_{25}(-a_{14}a_{41} + a_{11}a_{33} + a_{14}a_{44} + a_{33}a_{44}) + \\
& + b_{52}b_{24}\left(a_{15}a_{41} - a_{11}a_{45} + a_{33}a_{45}\right) - b_{12}b_{25}(a_{33}a_{51} + a_{44}a_{51} - a_{41}a_{54}) + \\
& - b_{12}b_{24}(a_{33}a_{41} - a_{45}a_{51} + a_{41}a_{55}) + b_{42}b_{24}(-a_{15}a_{51} + a_{11}a_{33} + a_{11}a_{55} + a_{33}a_{55}) + \\
& - a_{32}b_{21}b_{13}\left(a_{44} + a_{55}\right) + b_{31}b_{13}(a_{22}a_{44} + a_{22}a_{55} + a_{44}a_{55} - a_{45}a_{54}) + \\
& + b_{21}b_{42}(a_{15}a_{54} - a_{14}a_{33} - a_{14}a_{55}) + b_{21}b_{52}(-a_{14}a_{45} - a_{15}a_{33} - a_{15}a_{44}) + \\
& + b_{12}b_{21}\left(-a_{45}a_{54} + a_{33}a_{44} + a_{44}a_{55} + a_{33}a_{55}\right) - b_{12}b_{31}\left(a_{23}a_{44} + a_{23}a_{55}\right) + \\
& - b_{31}b_{13}\left(a_{54}b_{42}b_{25} - a_{44}b_{52}b_{25} + a_{45}b_{52}b_{24}\right) + a_{32}b_{13}\left(a_{51}b_{25} + a_{41}b_{24}\right) + \\
& + a_{15}a_{41}a_{54} + a_{14}a_{45}a_{51} + a_{14}a_{23}b_{31}b_{42},
\end{aligned}$$

$$\lambda_4 = \left(a_{44}a_{55} - a_{45}a_{54}\right)\left(a_{23}b_{12}b_{31} - a_{33}b_{12}b_{21}\right) +$$

$$+\left(a_{15}a_{54} - a_{14}a_{55}\right)\left(a_{33}b_{21}b_{42} - a_{23}b_{31}b_{42}\right) + \left(a_{14}a_{45} - a_{15}a_{44}\right)\left(a_{33}b_{21}b_{52} - a_{23}b_{31}b_{52}\right) +$$

$$+\left(a_{45}a_{54} - a_{44}a_{55}\right)\left(a_{32}b_{21}b_{13} - a_{22}b_{31}b_{13}\right) + a_{33}b_{12}b_{24}\left(a_{45}a_{51} + a_{33}a_{41}a_{55}\right) +$$

$$+a_{33}b_{42}b_{24}(-a_{15}a_{51} + a_{11}a_{55}) + a_{32}b_{13}b_{24}(a_{41}a_{55} - a_{45}a_{51}) +$$

$$+ \left(a_{41}a_{54} - a_{44}a_{51}\right)\left(a_{33}b_{12}b_{25} - a_{32}b_{13}b_{25}\right) + a_{33}b_{42}b_{25}\left(a_{14}a_{51} - a_{11}a_{54}\right) +$$

$$+a_{33}b_{52}b_{25}\left(a_{11}a_{44} - a_{14}a_{41}\right) + a_{15}a_{51}(a_{23}a_{32} - a_{22}a_{33} - a_{22}a_{44} - a_{33}a_{44}) +$$

$$+a_{14}a_{45}a_{51}\left(a_{22} + a_{33}\right) + a_{15}a_{41}a_{54}\left(a_{22} + a_{33}\right) + a_{45}a_{54}(a_{23}a_{32} - a_{11}a_{22} - a_{11}a_{33} +$$

$$-a_{22}a_{33}) + a_{14}a_{41}\left(a_{23}a_{32} - a_{22}a_{33} - a_{22}a_{55} - a_{33}a_{51}\right) - a_{11}a_{23}a_{32}(a_{44} + a_{55}) +$$

$$+a_{55}\left(a_{11}a_{22}a_{33} - a_{23}a_{32}a_{44}\right) + a_{11}a_{22}a_{44}\left(a_{33} + a_{55}\right) + a_{33}a_{44}a_{55}\left(a_{11} + a_{22}\right) +$$

$$+(a_{15}a_{41} - a_{11}a_{45})a_{33}b_{24}b_{52},$$

$$\lambda_5 = \left(a_{22}a_{33} - a_{23}a_{32}\right)\left(a_{11}a_{44}a_{55} - a_{11}a_{45}a_{54} + a_{14}a_{45}a_{51}\right) +$$

$$+\left(a_{15}a_{54} - a_{14}a_{55}\right)\left(a_{22}a_{33}a_{41} - a_{23}a_{32}a_{41}\right) + \left(a_{32} + a_{33}\right)a_{15}a_{23}a_{44}a_{51}.$$

Appendix II

$$a_i = \frac{\xi}{\Delta_i}\left[r_1^2\left\{r_2\left(r_3\left(1 - \delta^2\right) + p\delta^{*2}\right) - p_0^2\delta_1^*r_3\right\} +$$

$$+\epsilon r_1 r_5\left\{r_3\left(r_2 - \bar{v}^2\delta_1^*\left(1 - \delta^2\right) - 2\bar{v}p_0\delta_1^* - \bar{v}\delta_1^*\right) + p\bar{v}^2\delta_1^*\delta^{*2}\right\}\right],$$

$$b_i = \frac{-1}{\Delta_i}\left[r_1^2\left\{r_2\left(r_3 r_4 + p\delta^{*2}q_i^2\right) - p_0^2\delta_1^*\xi^2 r_3\right\} +$$

$$+\epsilon r_1 r_3 r_5\left(\xi^2\ r_2 + \bar{v}^2\delta_1^*r_4 - 2\bar{v}p_0\delta_1^*\xi^2\right) + \epsilon p\bar{v}^2\delta_1^*\delta^{*2}q_i^2 r_1 r_5\right],$$

$$d_i = \delta_1^*\left(p_0 r_1 + \epsilon\bar{v}r_5\right)\left(\xi a_i + b_i\right)q_i / \left(-r_1 r_2 - \epsilon\bar{v}^2\delta_1^*r_5\right),$$

$$e_i = \epsilon\left(s + \eta_0\tau_0 s^2\right)\left\{\left(r_1 r_2 + \epsilon\bar{v}^2\delta_1^*r_5\right) - \epsilon\bar{v}\delta_1^*\left(p_0 r_1 + \epsilon\bar{v}r_5\right)\right\}\left(\xi a_i + b_i\right)q_i\Big/\left\{r_1\left(-r_1 r_2 - \epsilon\bar{v}^2\delta_1^*r_5\right)\right\},$$

$$\Delta_i = \delta^{*2}\left[r_1^2\left\{r_2\left(\xi^2 + s^2 - q_i^2\right) + p_0^2\delta_1^*\left(q_i^2 - \xi^2\right)\right\} +$$

$$+\epsilon r_1 r_2 r_5\left(\xi^2 - q_i^2\right) + \epsilon\delta_1^*r_1 r_5\left\{\bar{v}^2\left(\xi^2 + s^2 - q_i^2\right) - 2p_0\bar{v}\left(\xi^2 - q_i^2\right)\right\}\right],$$

$$r_1 = \left(\xi^2 + Q^*\left(s + \tau_0 s^2\right) - q_i^2\right), \qquad r_2 = \left(\xi^2 + \delta_3^*s^2 + p_1\delta_1^* + \delta_2^*s - q_i^2\right),$$

$$r_3 = \left(\xi^2 + \frac{s^2}{\delta_1^2} + 2\delta^{*2} - q_i^2\right), \qquad r_4 = \left(\xi^2 + s^2 - \delta^2 q_i^2\right), \qquad r_5 = \left(1 + \tau_1 s\right)\left(s + \eta_0\tau_0 s^2\right).$$

Nomenclature

C^* – specific heat at constant strain

j – micro inertia

K_1^* – coefficient of thermal conductivity

m_{ij} – couple stress tensor

T – change in temperature of the medium at any time

t_{ij} – stress tensor

$\boldsymbol{u}$ – displacement vector

α_t – coefficient of linear thermal expansion

$\alpha_1, b, \xi_1, \omega_0, m$ and χ – elastic constants due to the presence of voids

δ_{ij} – Kroneckor delta

$\lambda, \mu, K, \alpha, \beta, \gamma$ – micropolar constants

ρ – density

τ_0, τ_1 – thermal relaxation times

ϕ – microrotation vector

ϕ^* – change in volume fraction field

∇^2 – Laplacian operator

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