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AN EXPERIMENTAL INVESTIGATION INTO THE POSITIONING ACCURACY OF LOW-COST GPS RECEIVERS IN LABVIEW ENVIRONMENT

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Abstract: A major problem in the application of satellite navigation systems is the accuracy of position determination. The manufacturers of navigation receivers use a variety of indicators to describe their accuracy of positioning, which causes difficulty in comparing the parameters of different receivers. This study presents a measurement system that allows for experimental determination of satellite navigation systems positioning accuracy. The main element of the implemented system is an application developed in the LabVIEW which receives navigation data from the receiver, collects the appropriate amount of data, and then calculates selected positioning accuracy indicators and presents them in tabular and graphical form. Then relevant calculation formulas and sample measurement results for selected GPS receiver are presented.

Keywords: GNSS, GPS receiver, positioning accuracy, LabVIEW.

1. INTRODUCTION

Global navigation satellite systems (GNSS) have numerous military and civil applications. The most popular system of this kind is the American GPS [1, 2]. The navigation receivers of these systems designed for military and some civil purposes (e.g. for geodesy, cartography and professional navigation) guarantee the highest accuracy. Nowadays most applications, however, use low-cost receivers available on the market for all consumers. The comparison of parameters of receivers produced by different manufacturers is difficult, because the manufacturers use a lot of different indicators of positioning quality [3, 4]. Different testing conditions also influence the results of the measurements what makes them difficult to compare. This study presents a measurement system that allows for an experimental study of positioning accuracy with any GPS receiver providing navigation data in the popular NMEA standard [5]. The developed application allows for reliable assessment of the accuracy of different receivers and the direct comparison between them.

2. BASICS OF THE GPS

The GPS, illustrated schematically in Figure 1, consists of three segments [1, 2]: the space segment, the control segment and the user segment. Today, the space segment consists of 31 satellites orbiting at 6 orbits at an altitude of about 20 000 km from the Earth. The control segment is composed of the main control station (MCS) and a network of monitoring stations (MS) located near the equator around the globe, and four corrective stations with ground antennas (GA). The user segment represents all users of the GPS receivers.

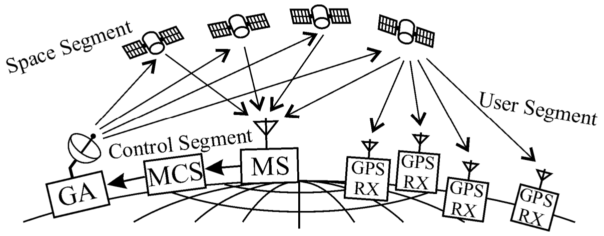


Fig. 1. Configuration of the GPS: Space, Control and User segment

The information about the parameters of the satellites orbit, the current GPS time, and the time of the radio signal propagation from the satellite to the user receiver are required to determine the user position. In the GPS, both the satellites and user positions are determined in Cartesian coordinate system centered relative to the center of the Earth and associated with the zero meridian and the equator of the Earth. This is the so-called World Geodetic System 1984 (WGS-84) [1]. It is presented in Figure 2. WGS-84 is anchored to the Earth, i.e. it rotates together with it at the angular velocity Ω_e=7.2921151467×10^{-5} rad/sec.

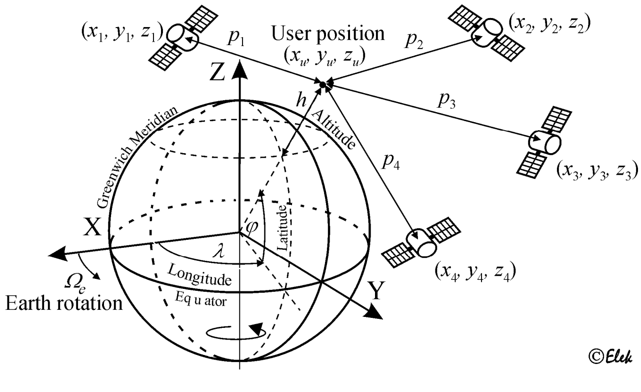


Fig. 2. The principle of determining the user's position in WGS-84

The position of each satellite is described by three coordinates (x_i, y_i, z_i), and the user position has the following coordinates: (x_u, y_u, z_u). User's position determined in Cartesian coordinate system must be converted into the polar coordinate system commonly used for modern maps. It requires calculation of the latitude φ_u and longitude λ_u as well as height h_u above the sea level:

$$\varphi_c = \arctan\left(\frac{z_u}{\sqrt{x_u^2 + y_u^2}} \right) , \tag{1}$$

$$\lambda_u = \arctan\left(\frac{y_u}{x_u} \right) , \tag{2}$$

$$r_u = \sqrt{x_u^2 + y_u^2 + z_u^2} , \tag{3}$$

$$h_u = r_u - r_e , \tag{4}$$

where: r_u is the distance from center of the Earth to the user, r_e is the average radius of the Earth, φ_c is the geocentric latitude, assuming that the Earth is a perfect sphere of radius r_e. However, since the surface of the Earth is not a perfect sphere, geocentric latitude φ_c and height h_u must be corrected [1]:

$$\varphi_u = \varphi_c + e_p \sin 2\varphi_u , \tag{5}$$

$$r_0 = a_e(1 - e_p \sin^2 \varphi_u) , \tag{6}$$

$$h_u \approx r_u - r_0 , \tag{7}$$

where φ_u is the latitude, e_p=0.00335281066474 is the Earth's flattening, r_0 is the distance from the center of the Earth to the point on surface of the Earth below the user's position. The determined longitude λ_u does not require correction.

The easiest way to assess the accuracy of the positioning in the GPS is to perform a series of measurements and to calculate root mean squared (RMS) errors of determined longitude σ_λ and latitude σ_φ:

$$\sigma_\lambda = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(\bar{\lambda} - \lambda_k\right)^2} , \tag{8}$$

$$\bar{\lambda} = \frac{1}{n}\sum_{k=1}^{n}\lambda_k , \tag{9}$$

$$\sigma_\varphi = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(\bar{\varphi} - \varphi_k\right)^2} , \tag{10}$$

$$\bar{\varphi} = \frac{1}{n}\sum_{k=1}^{n}\varphi_k , \tag{11}$$

where: φ_k, λ_k - are subsequent values of the longitude and latitude, k - is the number of measurement.

Calculated RMS errors σ_λ, σ_φ are expressed in the same units as longitude and latitude, that is in degrees. This is very impractical and it is much more natural to use meter as the unit of error in the position determination. Figure 3 presents the principle of converting the longitude σ_λ and latitude σ_φ errors in degrees to the errors in latitudinal direction σ_x and longitudinal direction σ_y expressed in meters. Earth model in the form of a perfect sphere of an average radius of r_e=6368 km is adopted [1]. For carrying out calculations on the Earth's sphere, a surface tangent to the parallel and the meridian passing through the user position $\bar{\varphi}$, $\bar{\lambda}$, should be imagined. On this surface there is an x-y coordinate system which is in line with the direction of the parallel and the meridian. Taking into account the formulas to calculate the length of parallel and the meridian, we can calculate the errors expressed in meters:

$$\sigma_x = \sigma_\lambda \frac{2\,\pi\, r_e \cos\varphi}{360°} , \tag{12}$$

$$\sigma_y = \sigma_\varphi \frac{2\,\pi\, r_e}{360°} . \tag{13}$$

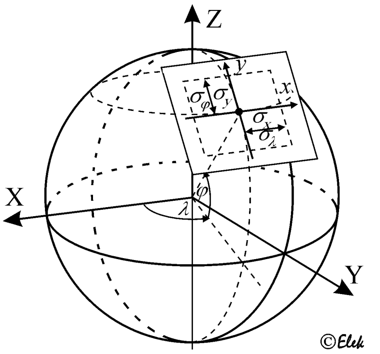


Fig. 3. The principle of conversion of positioning errors from degrees to meters in the latitudinal and meridional directions

RMS errors σ_x, σ_y describe one-dimensional errors in the latitudinal and meridional directions. In practice, the DRMS (Distance Root Mean Squared) error is more reliable, describing the error of distance in any direction [3, 4]:

$$DRMS = \sqrt{\sigma_x^2 + \sigma_y^2} . \tag{14}$$

CEP (Circular Error Probability) error is defined as the radius of the circle comprising half of the position determining results and is calculated with the formula:

$$CEP = 0{,}62\sigma_y + 0{,}56\sigma_x , \tag{15}$$

which is valid for $\sigma_y / \sigma_x > 0{,}3$. Similarly defined is the R95 error, which is the radius of the circle comprising 95% of the position determining results. R95 error is calculated with the following formula:

$$R95 = 2{,}08\left(0{,}62\sigma_y + 0{,}56\sigma_x\right), \tag{16}$$

which is valid for $\sigma_y / \sigma_x = 1$. Knowing the actual position of the receiver (reference position) φ_{ref}, λ_{ref} (e.g. read from a detailed map) we can determine the actual errors of latitude Δ_φ and longitude Δ_λ expressed in degrees:

$$\Delta_\varphi = \bar{\varphi} - \varphi_{ref} , \tag{17}$$

$$\Delta_\lambda = \bar{\lambda} - \lambda_{ref} , \tag{18}$$

which can then be converted in accordance with the equation (12), (13) to the actual positioning errors Δ_x and Δ_y expressed in meters. Finally, the actual radius of error Δ_R expressed in meters is calculated:

$$\Delta_R = \sqrt{\Delta_x^2 + \Delta_y^2} . \tag{19}$$

3. EXPERIMENTAL DETERMINATION OF THE POSITIONING ACCURACY

In order to allow experimental determination of the positioning accuracy of a GPS receiver, a measurement system shown in Figure 4 was built. The tested GPS receiver (Rx) is powered from a power supply (PS). Through the interface system (IS) the receiver transmits navigation data in NMEA standard [5] to a PC equipped with appropriate software developed in LabVIEW environment. The program reads data transmitted from the GPS receiver, decodes out of them the current geographical position, the altitude, the GPS time, the speed and azimuth of movement of the receiver.

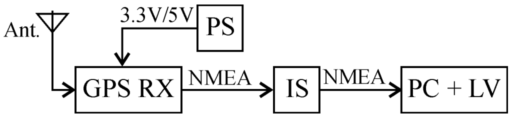
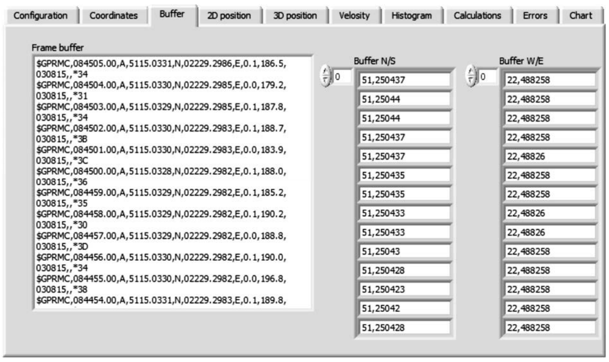


Fig. 4. Block diagram of the system for experimentally determining the positioning accuracy with GPS receiver



Frame buffer

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030815,,*34
$GPRMC,084504.00,A,5115.0330,N,02229.2985,E,0.0,179.2,
030815,,*31
$GPRMC,084503.00,A,5115.0329,N,02229.2985,E,0.1,187.8,
030815,,*34
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030815,,*3B
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030815,,*3C
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030815,,*35
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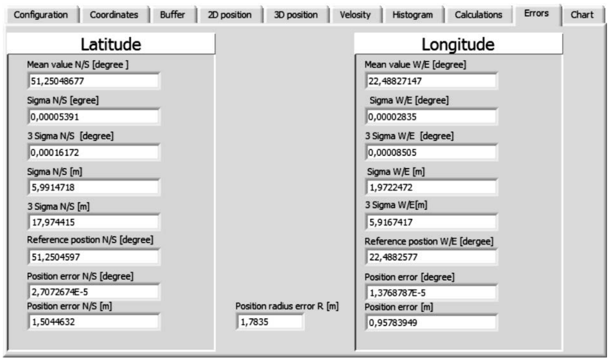
030815,,*38
$GPRMC,084454.00,A,5115.0331,N,02229.2983,E,0.1,189.8,
```

Buffer N/S	Buffer W/E
0	0
51,250437	22,488258
51,25044	22,488258
51,25044	22,488258
51,250437	22,488258
51,250437	22,48826
51,250435	22,488258
51,250435	22,488258
51,250433	22,48826
51,250433	22,48826
51,25043	22,488258
51,250428	22,488258
51,250423	22,488258
51,25042	22,488258
51,250428	22,488258

Fig. 5. Navigation data retrieved from the GPS receiver in NMEA sentences standard and decoded position: latitude and longitude



Latitude

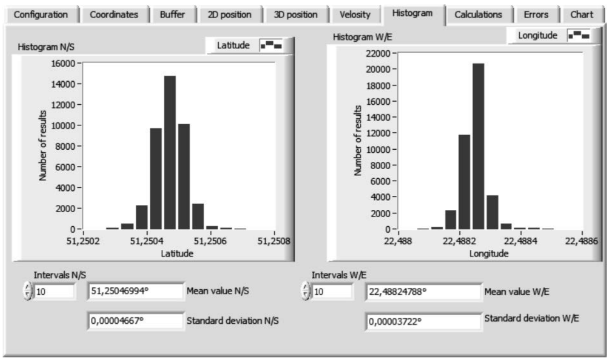
Mean value N/S [degree]: 51,25048677
Sigma N/S [egree]: 0,00005391
3 Sigma N/S [degree]: 0,00016172
Sigma N/S [m]: 5,9914718
3 Sigma N/S [m]: 17,974415
Reference postion N/S [degree]: 51,2504597
Position error N/S [degree]: 2,7072674E-5
Position error N/S [m]: 1,5044632

Position radius error R [m]: 1,7835

Longitude

Mean value W/E [degree]: 22,48827147
Sigma W/E [degree]: 0,00002835
3 Sigma W/E [degree]: 0,00008505
Sigma W/E [m]: 1,9722472
3 Sigma W/E[m]: 5,9167417
Reference postion W/E [dergee]: 22,4882577
Position error [degree]: 1,3768787E-5
Position error [m]: 0,95783949

Fig. 6. Results of the statistical calculations on a set of data



Intervals N/S: 10
Mean value N/S: 51,25046994°
Standard deviation N/S: 0,00004667°

Intervals W/E: 10
Mean value W/E: 22,48824788°
Standard deviation W/E: 0,00003722°

Fig. 7. Latitude and longitude histograms expressed in degrees

Navigation data retrieved from the GPS receiver are shown in Figure 5. These data are used to calculate the mean values of geographical coordinates (9), (11), standard deviations (8), (10), which are then converted into the RMS errors (12), (13), expressed in meters. Finally, DRMS (14), CEP (15), and R95 (16) errors are calculated, as shown in Figure 6. After entering the reference position φ_{ref}, λ_{ref} we can calculate the actual errors of positioning Δ_φ, Δ_λ in degrees (17), (18) and Δ_x, Δ_y in meters as well as the radius of error Δ_R (19). The program constantly shows histograms of longitude and latitude (Fig. 7) and actual errors Δ_x, Δ_y shown in the X-Y coordinate system (Fig. 8). Figures 5 - 10 show a few examples of program tabs with real measurement results.

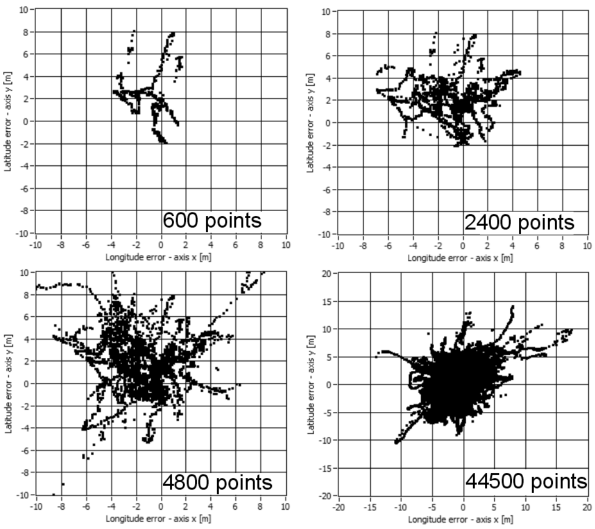


Fig. 8. Positioning errors in measurement series with increased number of measurements from 600 to 44500 points

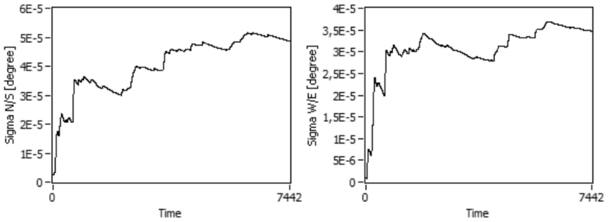


Fig. 9. Standard deviation of latitude σ_φ and longitude σ_λ in degrees versus time of measurement in seconds

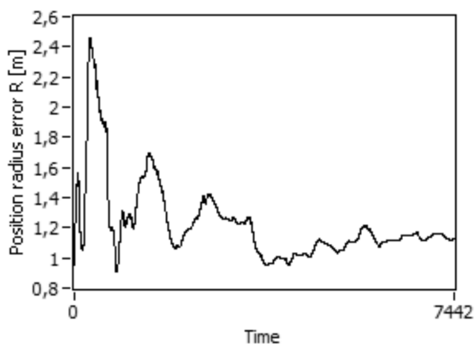


Fig. 10. Position radius error Δ_R in meters versus time of measurement in seconds

Using the implemented measurement system, the accuracy of determining the position was tested with the use of exemplary GPS receiver [6]. It was a 12-channel GPS module which defines the position in three dimensions, the speed and azimuth of movement, and the GPS time. The manufacturer declared the position determination accuracy CEP<5 m provided that no multipath occurs and the GDOP geometric dilution of precision factor is < 2. Several series of measurements of various duration were made. The results are shown in Figures 8, 9, 10 and Table 1. Figure 8 presents errors for series comprising from 600 measurements (duration 10 minutes) up to 44500 measurements (duration over 12 hours). The results is increasing CEP error 2.36 m to 5.14 m. Fig. 9 presents standard deviation of latitude σ_φ and longitude σ_λ in degrees versus time of measurement in seconds for the first two hours. It can be observed that standard deviation σ_φ, σ_λ increase all the time. Figure 10 presents Δ_R errors in meters versus time of measurement in seconds for the first two hours. The results indicate decreasing Δ_R error from 2.48 m to 1.15 m.

Table 1. Results obtained for series with the increasing number of measurements

k	$\sqrt{k}$	σ_x	σ_y	CEP	DRMS	Δ_R
-	-	m	m	m	m	m
600	24,5	1,98	2,01	2,36	2,82	1,85
1200	34,6	2,88	2,97	3,45	4,14	1,28
1800	42,4	2,97	3,55	3,86	4,63	1,36
2400	49,0	3,45	3,62	4,18	5,00	1,26
3000	54,8	3,16	3,35	3,85	4,61	1,31
3600	60,0	2,95	3,76	3,98	4,78	1,87
4200	64,8	2,86	3,95	4,05	4,88	0,98
4800	69,3	3,21	4,46	4,56	5,50	1,13
5400	73,5	3,36	4,52	4,68	5,63	1,15
6000	77,5	3,61	4,68	4,92	5,91	1,22
43200	207,8	3,82	4,84	5,14	6,17	1,15

Table 1 presents the number of measurements k, $\sqrt{k}$ and σ_x, σ_y, CEP, DRMS, Δ_R errors for these measurements. It can be observed that increasing the number of measurements k causes the increase of standard deviation σ_x, σ_y and increase of CEP and DRMS errors, and at the same time the Δ_R error for longer series stays at the same level. It can be assumed that this effect is associated with switching the receiver to subsequent satellites, which can be seen in the X-Y error graphs (Fig. 8) in the form of appearance of new error areas distant from each other.

It should be noted, however, that the standard deviation for the average value from the series of measurements decreases proportionally to the square of the number of measurements. Standard deviation values σ_x, σ_y in Table 1 show that increasing the length of the measurement series eventually improves the accuracy, although this effect is weaker than for the most of other measurement types.

4. SUMMARY

This paper presents a method and system for experimental evaluation of the accuracy of determining position with the use of a GPS receiver. Error types and relevant calculation formulas are described hereinabove. The implemented measurement system is designed for static testing of a single receiver and can be expanded to simultaneously test a few receivers and to support dynamic testing. The conducted measurements for a low-cost GPS receiver show that increasing the measurement series results in significant increase in the standard deviation of determined geographical coordinates and CEP, DRMS and other errors calculated from them. This increase is smaller than the square root of the number of measurements. Thus it can be concluded that the increase and averaging of the measurement series allows us to improve the accuracy of GPS positioning system receiver, however, this requires a larger number of measurements than in most other cases.

5. REFERENCES

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EKSPERYMENTALNE BADANIE DOKŁADNOŚCI POZYCJONOWANIA TANICH ODBIORNIKÓW GPS W ŚRODOWISKU LABVIEW

Istotnym problemem w stosowaniu systemów nawigacji satelitarnej jest ich dokładność wyznaczania pozycji geograficznej. Niestety producenci odbiorników GPS stosują różne wskaźniki opisujące dokładność pozycjonowania, co powoduje trudności w porównywaniu parametrów odbiorników różnych firm. Problem ten dotyczy zwłaszcza tanich i łatwo dostępnych odbiorników przeznaczonych do innych celów niż profesjonalna nawigacja, geodezja i zastosowania militarne. Dzięki niskiej cenie i coraz lepszym parametrom odbiorniki takie znajdują liczne zastosowania komercyjne w różnych dziedzinach nauki i techniki oraz w wielu wyrobach powszechnego użytku. W pracy przedstawiono system pomiarowy umożliwiający eksperymentalne wyznaczanie dokładności pozycjonowania takich odbiorników. Głównym jego elementem jest aplikacja opracowana w środowisku LabVIEW, która odbiera dane nawigacyjne w standardzie NMEA z badanego odbiornika, gromadzi ich odpowiednią ilość w pamięci komputera, oblicza wybrane wskaźniki dokładności pozycjonowania oraz przedstawia je w postaci tabelarycznej i graficznej. Zaprezentowano odpowiednie wzory obliczeniowe oraz wyniki pomiarów dla przykładowego odbiornika systemu GPS.

Słowa kluczowe: GNSS, GPS, WGS-84, odbiornik nawigacji satelitarnej, dokładność pozycjonowania, LabVIEW.