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DEVELOPMENT OF THE MODELLING AND NUMERICAL SIMULATION OF THE THERMAL PROPERTIES OF MACHINE TOOLS

This paper highlights the need for the holistic modelling of the thermal properties of machine tools to provide a good basis for the effective aiding of machine tool design and optimization. A brief history of this modelling and the idea of accurate modelling and its experimental verification are presented. Examples of the holistic modelling of drive units both isolated from the machine tool and interconnected with its structure are provided.

1. INTRODUCTION

In order to keep up with potential users' demands which machine tools are expected to meet it is necessary to intensively develop software tools aiding the design of machine tools and the experimental verification of their behaviour in complex operating conditions. The development of machine tools is shaped by the continuously growing demands for higher price-competitiveness efficiency of machining characterized by low energy consumption and not disturbing human friendly or of the natural/production environment. The determinants of high machining efficiency are basically high spindle speeds and feed rates in the controllable axes as well as appropriately high accelerations and jerks. This means that the dynamics of: the motions in the controllable axes, the thermal processes like, the power losses generation and the changes dynamics in the heat transmission conditions increase. As a result, it becomes difficult to ensure machine tool thermal stability and it is necessary to identify accurately the internal thermal processes and reduce their intensity. The effective way of achieving this is through modelling as the basis for the optimal shaping of the operational properties of machine tools.

Efficiency can be increased through multitasking which necessitates a more complex machine tool structure, entailing a complex spatial arrangement of heat sources and heat

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transmission. Consequently, thermal error generation models and minimization procedures become highly complex.

Another major direction in the development of machine tools is the necessary increase in their accuracy and in the accuracy of the machining processes executed by them. The higher the required accuracy, the smaller the thermal errors must be, i.e. there must be great thermal symmetry, small heat loading, optimal heat transmission, small inter-actions between the heat sources and high resistance to changes in ambient temperature.

In order to effectively influence the natural and forced thermal processes and errors the machine tool behaviour must be holistically modelled and predicted, whereby a basis for the effective compensation of errors will be provided. Higher machine tool productivity also requires the prevention of chatter, including the adverse interaction between thermal phenomena and vibrations. The determinants of these processes must be included in the holistic model integrating the machine tool, the production fixture and the machining process in the whole machining system with natural interactions.

Having in mind the development of machine tools, it is worth devoting special attention to modelling with regard to its foundations, history and possibilities for improvement.

2. HISTORY OF MODELLING AND SIMULATION

From the historical perspective, machine tool modelling began when kinematic and strength calculations started to be used in designing. This led to the creation of accurate computational models of the stiffness and dynamical characteristics of machine tools. The leaders in this field were the famous Levina and Rieszetov school and the Figatner school specializing in stiffness models of rolling bearing units of spindles. Substantial knowledge on the shape deformations of housings, the phenomena taking place in spindle bearing units and contact deformations was then accumulated. With advancements in computer technology came numerical modelling based on the finite difference method and the finite element method, using computational systems developed for the design of (especially) planes and spacecrafts.

Large computing systems, which offered the advantage of the automatic discretization of objects (reducing in this way the time needed to build geometrical models), but which were difficult to integrate with models of disturbances (particularly the ones due to thermal phenomena) in geometry, were adapted for the design of machine tools.

In the years 1965-1975 numerous studies (including PhD dissertations) were published, forming an experimental basis for the modelling and numerical simulation of machine tool heating and thermal deformations. Also papers presenting the foundations of numerical calculations (FEM) were published [1],[2],[30]. The studies mostly dealt with simple load-bearing structure components and spindle units of machine tools. In Germany the research was concentrated mainly: in Aachen under H. Opitz’s direction [3],[4],[5],[6], in Berlin under G. Spur’s direction [7],[8], in Magdeburg under H. Wiele’s direction [9] and in Dresden [10]. In Poland, significant research was then conducted under J. Jedrzejewski’s direction in Wroclaw [11],[12],[13]. An important contribution to modelling was made by

T. Sata (Japan) who initiated research into both the measurement and modelling of the distributions of temperature and thermal displacements [14],[15]. One should also mention the research conducted then in Manchester in the UK [16] and in Moscow in Russia [17].

The intensively developed modelling of the thermal properties of machine tools increasingly better mapped the processes naturally occurring in them and against the background of ever higher requirements was the subject of many studies periodically summing up the knowledge in this field. To such keynote papers belong papers by H. Wiele [18], M.H. Attia and L. Kops [19], J. Jedrzejewski [20], M. Weck et al [21], P.K. Sinha [31] and recent papers by J. Mayr, J. Jedrzejewski et al [22], highlighting the need for holistic modelling.

3. IDEA OF ACCURATE MODELLING

The improvement of highly efficient and accurate machine tools necessitates particularly accurate identification of the factors affecting their static, thermal and dynamical properties. A reliable source of knowledge about these interactions can be a machine tool model taking into account the natural variability of machine tool operating conditions. This means that such a model should take into account the dependence of the operating conditions on momentary static, thermal and dynamic loads, variation in the interactions within the machine tool load-bearing structure, momentary internal loads and momentary loads generated by the machining process.

In multitasking machine tools also the couplings between the simultaneously performed machining processes (including hybrid processes), and their control can be important.

Only such comprehensive and complex holistic modelling gives a full picture of the error generating processes in the controllable axes. It is very difficult to create such a model and it requires profound knowledge and great skill in the use of systems based on FDM and FEM, which make it possible (not without limitations) to integrate models (in their whole complexity) with the machine tool geometric structure. Only an integrated computing system, in-house Software or based on a commercial FDM/FEM program, with great computing power and automated discretization and input data feeding can constitute a truly useful tool for machine tool designers.

The modelling of the behaviour of machine tool operating properties should take into account the instantaneous thermal condition of the friction pairs, which includes, among other things, the effect of temperature on the variation in the friction coefficients and forces, the coupling between the thermal displacements and the ones produced by dynamic forces, and the moments of friction, simultaneously for all the friction pairs in each of the controllable axis. All the components of the energy consumed by the drive of each of the controllable axis, which determine the torques that the motors must provide, must be modelled. Consistence between the changes in the forces needed to overcome the motion resistances specified by the model and the changes in the energy drawn by the drive motor is the best evidence of the correct response of the model. Such a verification of a model through experiment is shown in Fig. 1, in which the character of the changes in the forces appearing in the model during the operation of a lathe centre with a ball screw is compared with the measured changes in the current/torque drawn by the motor. The good agreement between the shapes of the two curves (torque and friction) indicates that the phenomena are correctly and fully modelled in the FEM program.

Attempts at evaluating model correctness, usually through laser measurements of positioning accuracy for selected thermal states of the machine tool or through measurements of temperature and displacements in its selected points, do not provide an answer if and how the model takes the above mentioned thermal and force interactions into account. Since positioning measurements are performed after the machine has heated up to a certain thermal state and its rotational or linear motion has been stopped most of the friction forces and inertial forces no longer occur and so their impact on the properties of the machine tool cannot be assessed. Neither can this be done on the basis of temperature nor displacement measurements.

If intelligent functions of the active influence on internal force and heat loads are used in the machine tool, this must be taken into account in the model. Moreover, highly complex models are difficult to present graphically in a way that the variation in their particular components is rendered. An example here is an attempt to show the modelled variation in power loss sources in a ball screw gear (Fig. 2).

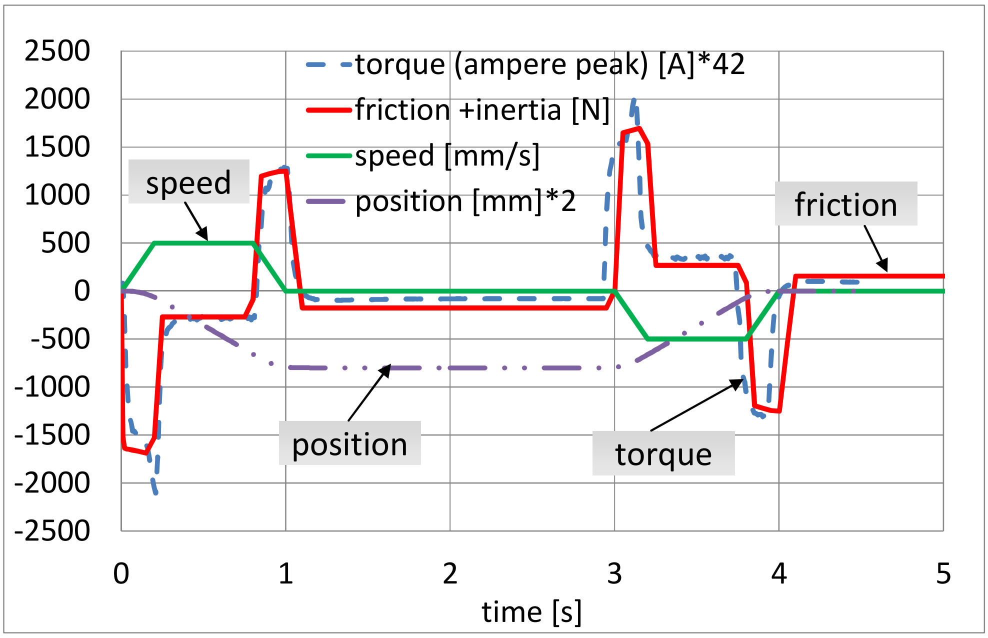

Fig. 1. Comparison of variation in resistance during reciprocating motion, with motor torque

The friction forces occurring on the sliding guides change as a function of the speed of motion in accordance with the working cycle, the friction coefficient characteristic and the pressure distribution determined by the inertial forces of the moving masses. The power losses in the bearings change depending on the screw rotational speed, the instantaneous value of the force tensioning the screw, which, in turn, depends on the degree to which the latter has heated up (i.e. depends on the running time). Similarly, the power losses in the nut change in accordance with the working cycle, depending on the motion resistances due to the friction and inertia of the moving masses.

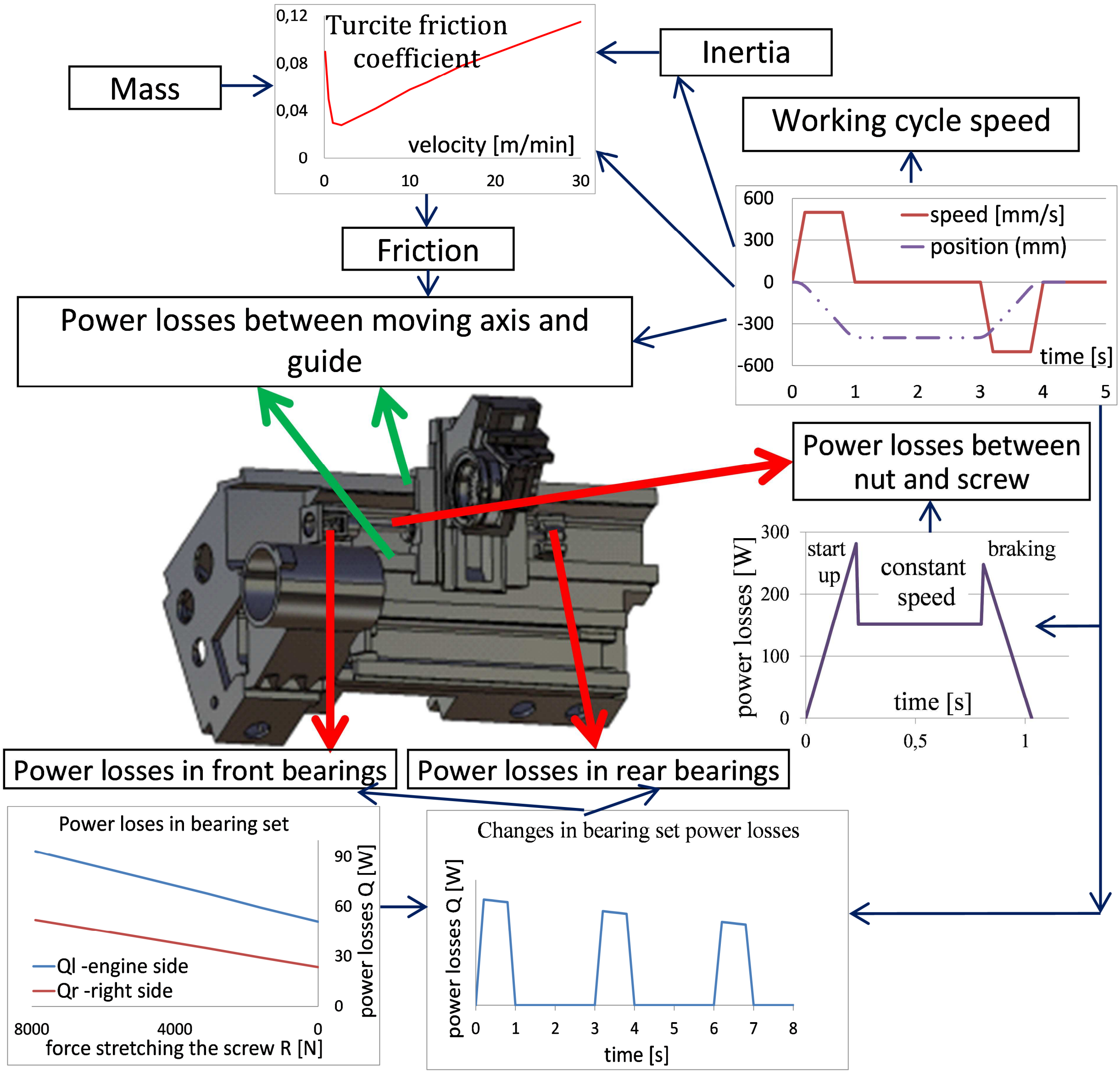

Fig. 2. Sources of power losses in feed axis Z

4. HOLISTIC MODELS

The full holistic model of the machine tool machining system comprises a model of the machine tool itself, a production fixture model (which integrates the workpiece with the machine tool structure) and a model of the machining processes together with the tool assembly (Fig. 3). The model of the static, thermal and dynamic properties of the system completely integrates all the influences on workpiece machining accuracy and machining process productivity [23]. Its division into three partial models (machine, fixture and process) is due to the still insufficient capability of the computing systems to model and integrate the above models.

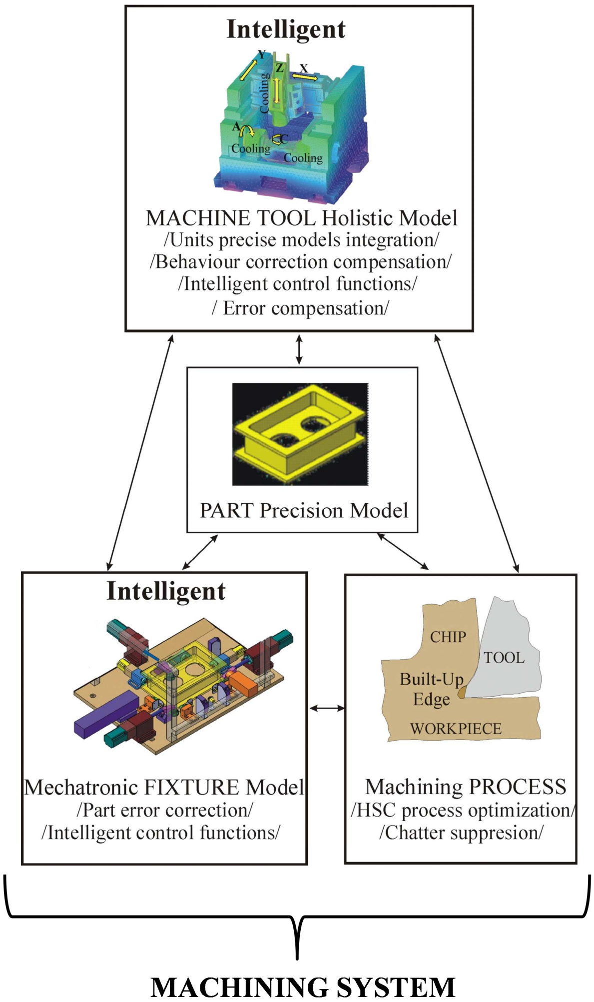

Fig. 3. Holistic model of machining system

The machine tool (as the main component) and the tool assembly are the subject of intensive studies and modelling attempts, stimulated by the development of micro–and nanomachining and by the necessity to minimize and compensate errors. Considering their influence on machining accuracy, fixtures are given relatively little attention. But in the case of the machining of slender precision workpieces, especially aircraft parts, the influence of the fixtures, being intelligent complex modules of the machining system, on the machining process is considerable. The production fixture assembly has its own static, thermal and dynamic properties but as the interface of the workpiece it takes over the impact of the properties of the machine tool and the machining process. Because of its relatively small heat capacity in comparison with that of the machine tool, the thermal interactions in this case are particularly complex and variable. From the point of view of the analysis of chatter prevention possibilities, also the dynamic model of the fixture is very important. The supporting and fixing of the workpiece play a key role here.

In order to holistically model machine tool themselves it is necessary to model their assemblies which occur in the controllable axes and form a chain of individual heat sources, internal loads, stiffness, couplings, heat transmission and thermal deformations and elongations. Basic requirement for accurate modelling, described in detail in sect. 3, is the integration of the thermal and stiffness behaviours and the interactions affecting the naturally proceeding these processes.

The SATO computing system (Fig. 4) developed by J. Jedrzejewski et al. [23] is based on the idea of holistic modelling. The system was created on the basis of accumulated knowledge relating to the modelling and numerical simulation of the thermal behaviour of machine tool spindle units (of various types) with rolling bearings. The system can be used to analyse running clearance in bearings, temperature distributions and power losses as a function of: rotational speed and internal and external load generated by cutting forces, and the conditions of natural and forced heat transmission. It enables one to model the cooling of bearings, electrospindle motors, spindles themselves, ball screws and nuts, toothed gears and housings. The power loss generation models take into account the dynamics of the loads and their connection with the dynamically changing heat load, including the mutual interactions and the interdependence between the power losses, the thermal deformations and the phenomena taking place in the base oil of the lubricating media.

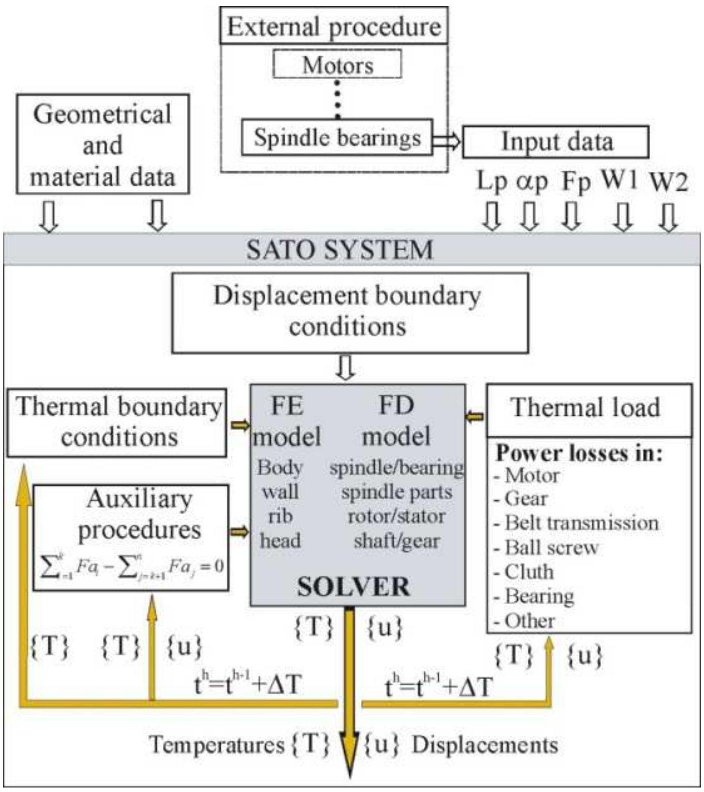

Fig. 4. General structure of SATO system [24]

Thus the SATO system enables the holistic modelling of spindle assemblies characterized by different degrees of complexity – from structurally simple electrospindles to complex high-torque headstocks.

In the case of high-torque headstocks used in the machining of large and heavy elements with high cutting forces, the motor is usually located outside the headstock. Then the driving force is transmitted to the spindle by several toothed gears, there are many bearings supporting the shafts and abundant lubrication needs to be provided (Fig. 5). Consequently, models of heat generation in the toothed gears and in the bearings of their shafts as well as models of the heat exchange between the flowing lubricating-cooling oil and the surfaces of the gears, the bearings and the housing walls are needed. Fig. 6 shows how the heat exchange inside the headstock body and in the cooling channel is modelled by means of additional elements called "Air" and "Fluid" [24]. The symbols in the figure denote:

$Q_{S\text{-}C}$ – the heat flux exchanged between the wall and the fluid,
Q_W – the flux of heat generated in the fluid as a result of its flow resistances in the cooler channel,
Q_U – the flux of heat carried by the fluid or by air,
Q_A – the flux of heat accumulated in the fluid or in the air,
$Q_{P\text{-}C}$ – the heat flux exchanged between the air inside the headstock and the fluid,
$Q_{S\text{-}P}$ – the heat flux exchanged between the wall and the air inside the housing.

Machine tools have a lot of fully and partially closed spaces accumulating heat and taking part in heat transfer, which must be included in a model. Such spaces influence on control axes thermal behaviour , because of accumulation heat.

Since large forces must be carried it is necessary to use such a spindle bearing system which will ensure high operating stiffness and at the same time will not cause excessive changes in temperature and thermal displacements, i.e. will not result in deterioration in machining accuracy or in a reduction in the lifetime of the expensive bearings. In order to achieve the above properties one needs a thermoelastic model of the spindle bearing unit, which will make it possible to determine the properties of the special bearings characterized by great stiffness.

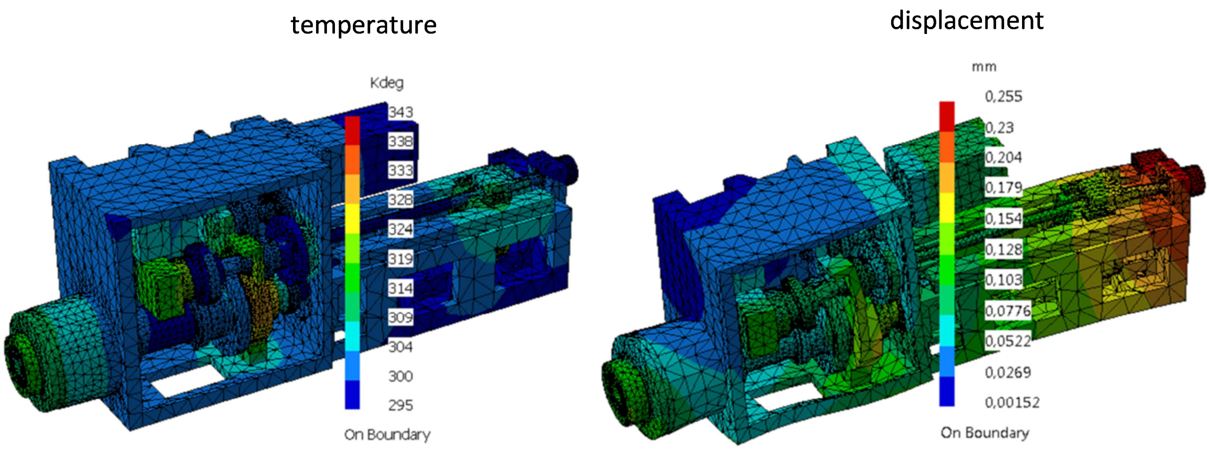

Fig. 5. Distribution of steady temperatures and thermal displacements in heavy headstock idle running at 2500rpm (oil cooling: front bearings, spindle and gears)

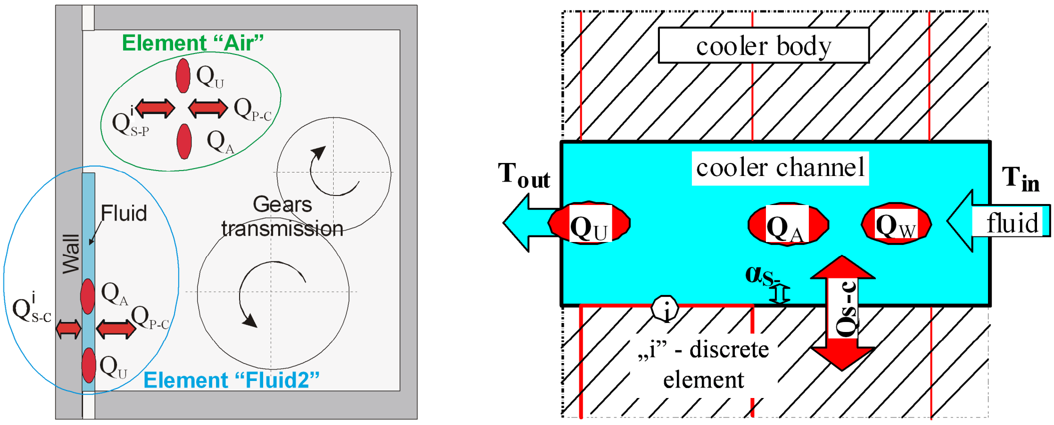

Fig. 6. Modelling of heat exchange inside headstock and in cooler channel [24]

Only then one will be able to analyse such design conditions and parameters as: the kind of lubrication, the mounting fits and preload for the spindle bearings as well as the spindle and housing cooling parameters which strongly influence the performance characteristic of the headstock. An example of such an analysis with regard to the influence of bearing preload and the influence of negative clearance on mounting surfaces W1, W2 and W3 on the lifetime of the bearings is shown in Fig. 7.

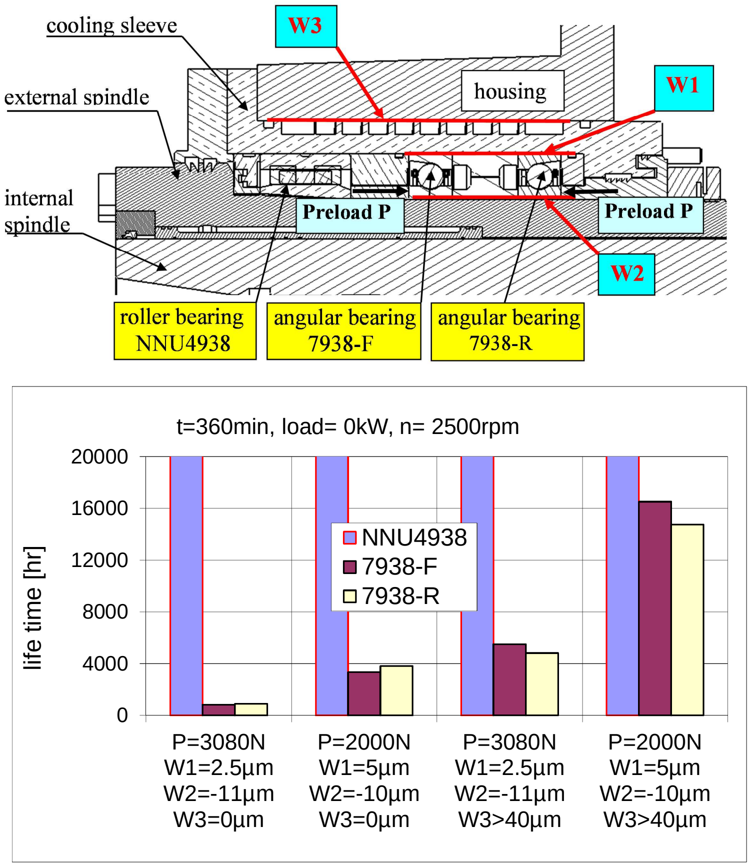

Fig. 7. Lifetime of front spindle bearings for variants of preload and after mounting clearances

Software tools aiding (through simulation) the shaping of the operationalproperties of machine tools are continuously developed whereby the problems and phenomena which previously (by necessity) were modelled in a much simplified way or simply neglected are now successively taken into account in computational models. This applies to both commercial FEM tools, such as FEMAP, NASTRAN, CATIA and ABAQUS and in-house tools, such as SATO. Besides their greater computing power and efficiency the tools are enriched with increasingly more detailed knowledge of the boundary conditions, which opens up new possibilities for the users. In-house software is also updated and extended with new in-house or acquired knowledge. An example here are spindle assemblies which due to the limitations of the model of the internal loads in rolling bearings could be modelled practically to speeds no higher than 20000rpm. As models of high-speed bearings were developed [25],[26],[27] and were included in in-house software tools, it became possible to model the operation of high-speed headstocks [28] with spindle speeds of 50000rpm and higher (Fig. 8).

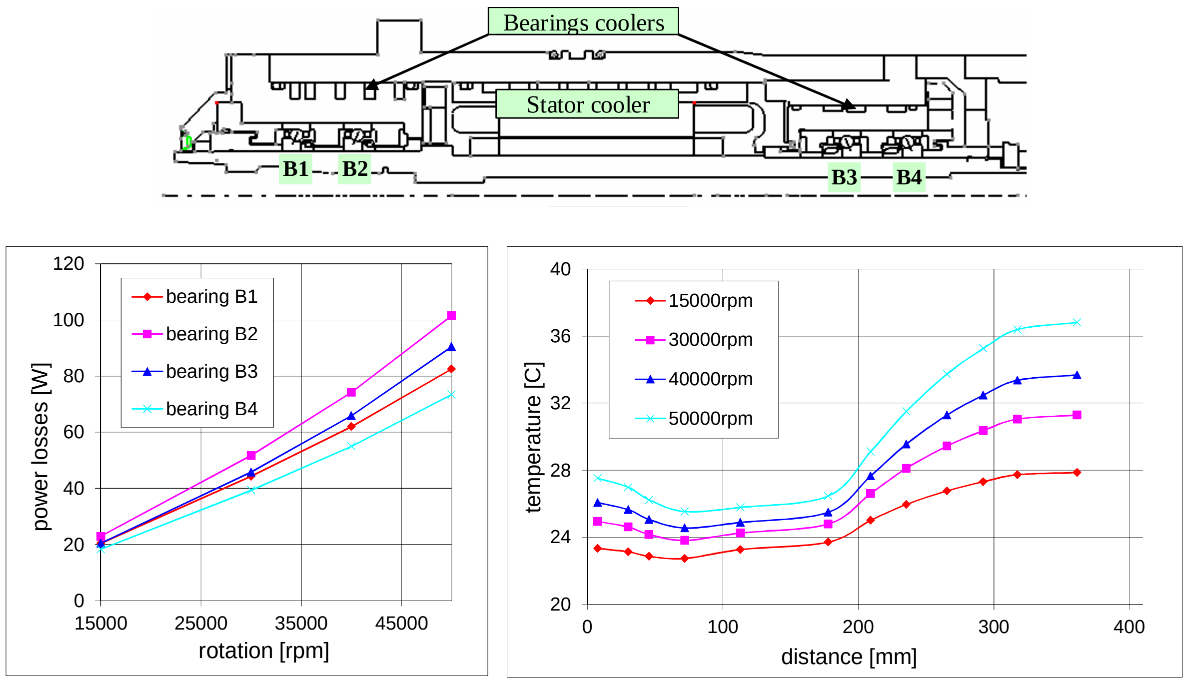

Fig. 8. Changes in power losses in electrospindle bearings as function of rotational speed, and distribution of temperature along spindle

However, for the above purpose still more phenomena, such as changes in and the diversification of the contact angle of the oblique bearing as a function of rotational speed, had to be included in the existing models of power losses in the bearing units (Fig. 9).
The differences in the expectations and requirements relating to models become clearly apparent if the high-speed headstock structures are compared with the low-speed (high-torque) ones. In order to model the former one needs, besides heat generation models, models of the heat exchange between the motor’s stator, the bearing assemblies and the

liquid in the forced circulation circuit. The amount of heat generated in such structures is usually so large that stable and accurate operation is impossible without intensive cooling.

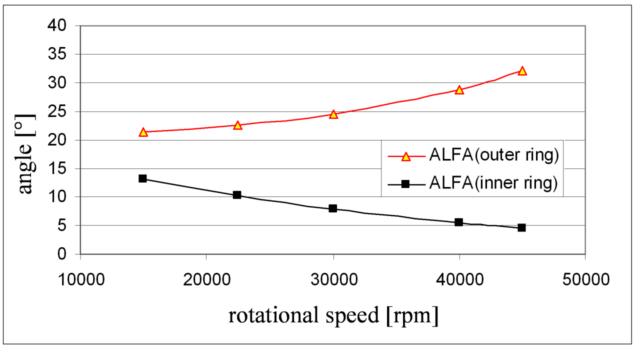

Fig. 9. Changes in contact angle in bearing 50BNR19 (ALFA$_o$ 18°, preload 500N)

Sample calculations for such an intensively cooled electrospindle and the temperature verification of the modelling are shown in Fig. 10.

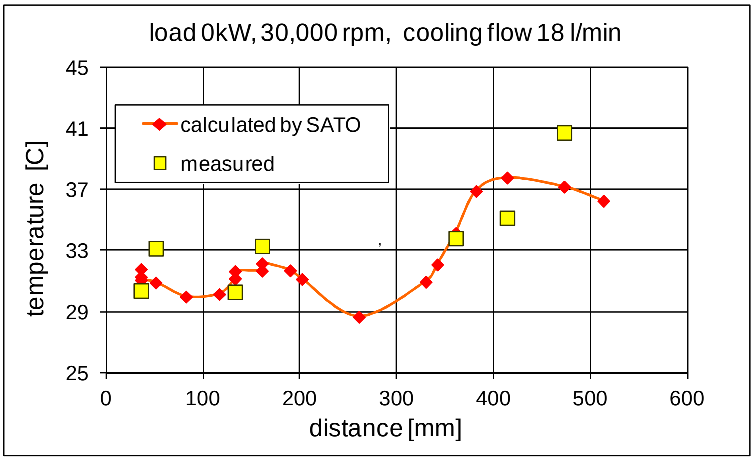

Fig. 10. Calculated temperature distribution and its experimental verification along electrospindle wall for 30,000rpm in thermally steady state by intensive cooling [28]

The computations were performed by SATO. As part of the computations within the SATO module for high-speed units, bearing power loss calculation procedures proper for the type of bearing mounting are added to the discrete object (Fig. 11). Thanks to the use of FDM for the description of cylindrical components this part of the model could be integrated with the analytical relations for power losses and thermal deformations, previously developed for the set of models contained in the procedure library. The bearing assembly model defines the instantaneous power losses on the basis of: the bearing manufacturer's data, the adopted bearing unit structure, the lubrication conditions, the rotational speed and the calculated temperatures and deformations. In addition, at each stage of the analysis one can calculate not only the temperatures in the bearings, but also the running clearances in the bearings and in the fit joints, which determine the lifetime of the bearings, the spindle stiffness and its thermal, static and dynamic properties.

SATO's great capabilities for modelling the behaviours of high-speed bearing assemblies can be demonstrated using as example an analysis of the most advantageous operating conditions for an assembly of spring-preloaded bearings (Fig. 12). The durability of a spindle bearing unit is equal to the durability of the most loaded bearing and each variation in the latter changes the durability of the whole unit. Therefore such assembly conditions for the unit of preloaded spindle bearings should be sought which will ensure an identical internal load in them.

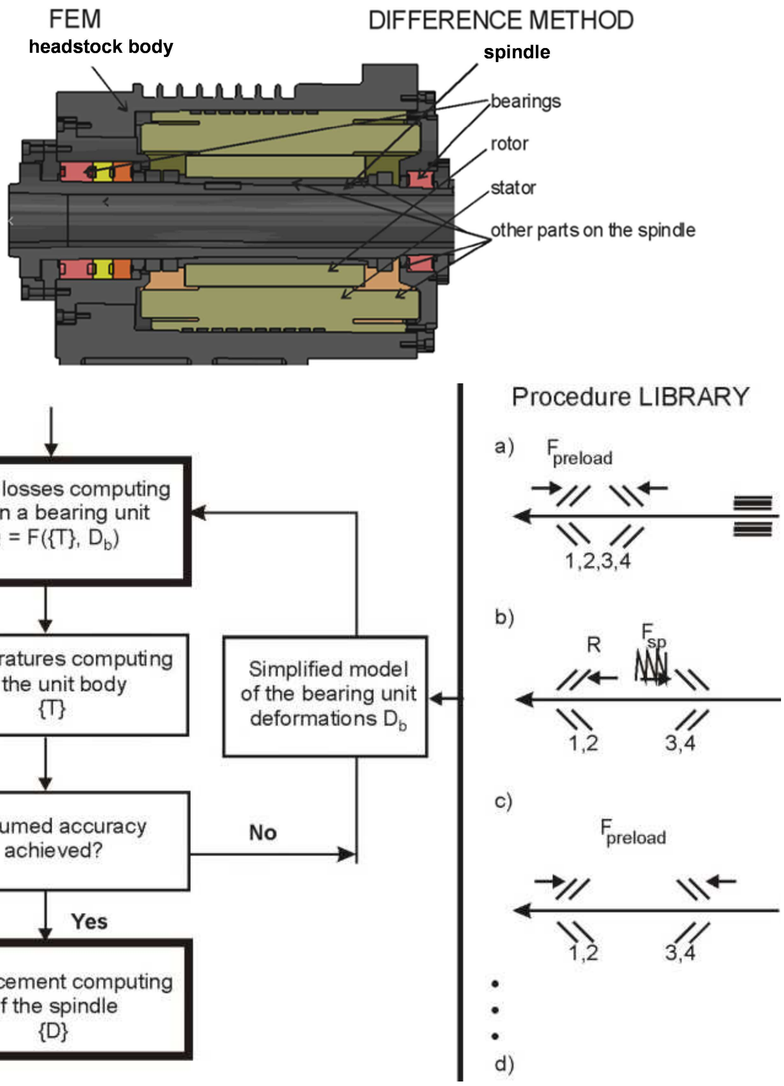

Fig. 11. Chart showing how SATO uses library of procedures connected with spindle bearing unit structure

This can be done through the matching of the lengths of the inner and outer distance sleeves so that the difference in dimensions B and A after mounting the bearings is equal to assumed value D12. In order to determine the proper value of D12 it must be iteratively computationally sought until the condition of identical durability of bearing 1 and 2 is satisfied, i.e. the point of intersection of the two lifetime curves needs to be found. For the example shown in Fig. 12 the sought value is -6μm.

As the speeds of the feed units increase so does the share of the latter in the heat balance of the machine tool [29]. The simplified models, which do not take into account changes in the location and intensity of heat sources and the dynamics connected with the motion and mass of the assemblies are insufficient today. An example of computations performed using FEM software (new model) enabling the modelling of moving heat sources is shown in Fig. 13. The computations are for an isolated feed unit in the X axis of a lathe centre. It means that the model not include a bed and headstock but the calculation time is short.

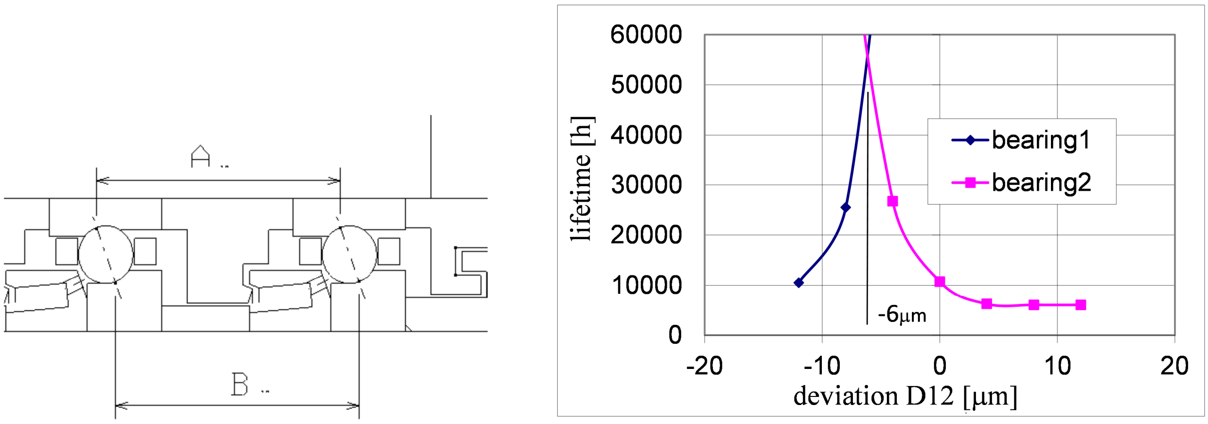

Fig. 12. Bearing lifetime versus deviation D12

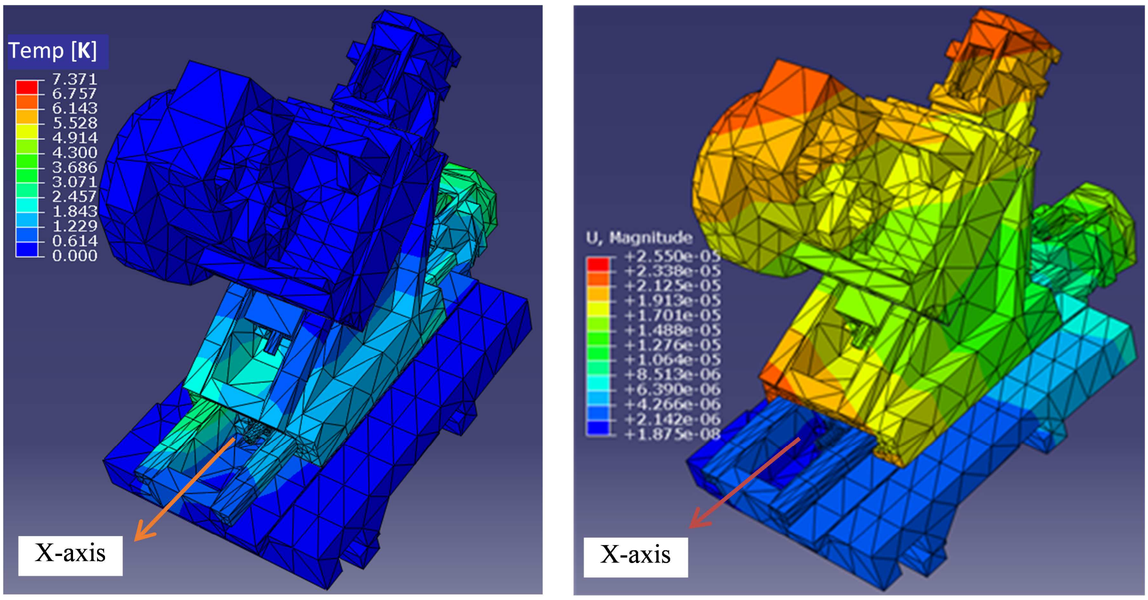

Fig. 13. Axis-X temperature change and displacement distribution during 30minutes of operation in cycle: < Pause 2(s) ▶ Rapid movement (distance -200mm) ▶ Pause 2(s) ▶ Rapid movement (distance +200mm) > at speed of 0.5m/s

Through computations as above one can analyse the behaviour of the drive at any instant in the working cycle and assess the effect of design or assembly factors on such drive characteristics as thermal stability, axial stiffness and displacements of selected points. Sample computations aimed at assessing the effect of screw preload on the displacements of the screw tip (point n335) are shown in Fig. 14. The force stretching the screw was matched so that the preliminary displacement of its tip (right end of the screw), including the deformations of the screw supports, would amount to 30μm.

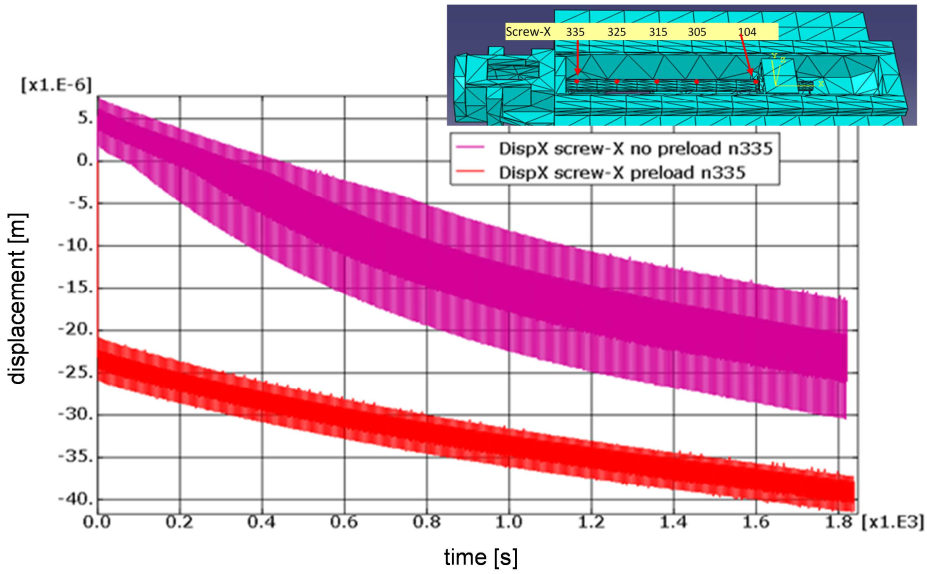

Fig. 14. Influence of preload on transient axial displacement for boundary point (n335) of operating screw-X with reciprocating movement

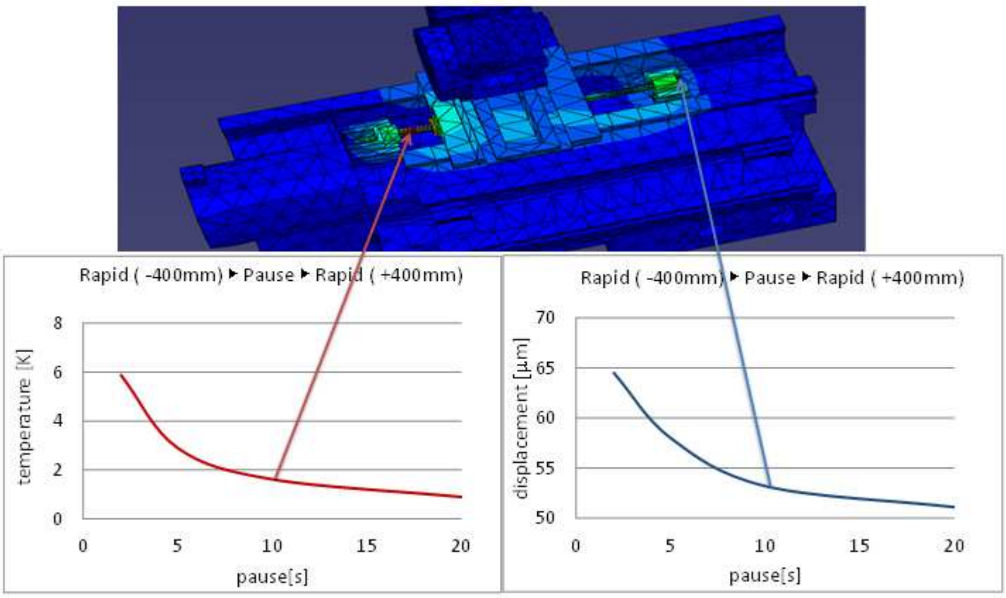

Fig. 15. FEM computations for lathe centre during drive work of Z axis with reciprocating movement < Pause 2(s) ▶ Rapid movement (distance -400mm) ▶ Pause 2(s) ▶ Rapid movement (distance +400mm) > at axis speed 0.5m/s

The initial displacement of the points on the screw (for time t=0) is caused by both the weight of the units, transferred from the nut to the screw (the guideways are inclined) and the force stretching the screw. The oscillations of the displacements of the points on the ball screw, visible in the diagram are mainly due to the inertial forces, which being transmitted from the nut to the screw load and deform the screw. The amplitudes of the oscillations and changes in the values of the displacements are much greater for a screw which has not been preloaded, which adversely affects dynamic stability and positioning accuracy.

By modelling assemblies isolated from the machine tool one can map the geometric structure and the internal couplings more precisely than in the case of the whole machine. But then the mutual interactions with the other assemblies, which are of essential importance in some analyses (e.g. of positioning errors), cannot be taken into account. An example of modelling and computing the drive in axis Z, interconnected with the machine tool structure, is shown in Fig. 15. The screw bearing system in the left support is of the “FIX” type whereby the screw’s right end elongates most. The right screw’s supporting bearings are pre-tensioned (screw elongation 50μm) see Fig. 2.The presented holistic modelling opens up great possibilities for the analysis of the thermal, elastic and dynamic properties of machine tools.

5. CONCLUSION

It has been shown that the SATO software, used to directly calculate power losses from analytical relations combined with FDM elements and operational phenomena, is the effective holistic modelling tool. Also models of isolated assemblies are essential since the computations based on them supply machine tool designers with many important data on the behaviour of the particular assemblies in the operating conditions whereby the most advantageous design and technological parameters, relating to forced cooling, bearing preload, balls screw tension, positioning errors, etc., can be determined.

It emerges from the presented considerations that the development of modelling aims on the most detailed representation of the phenomena which determine the behaviour of the particular machine tool units, in the model and on the integration of these factors and their mutual interactions.

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