

OPTIMAL DESIGN OF COMPRESSED COLUMNS WITH CORROSION TAKEN INTO ACCOUNT

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This study is devoted to the optimal design of compressed columns with a circular cross section under axial compressive forces and exposed to a corrosive environment. The initial volume of the structure is taken as an optimality parameter. The main constraint is the buckling of a loaded column at the final time of its operation. Gutman-Zaynullin's exponential stress corrosion model is adopted for the analysis. Analytical and numerical results are derived for optimal variation of the cross-sectional area of the bar along its axis.

Key words: corrosion, optimization, stability

1. Introduction

Elements of many engineering structures are exposed not only to loads and temperatures, but also to various corrosive environments. These factors often appear in highly unfavorable combinations, reducing the load carrying capacity and service life of the structure. Neglecting corrosive environments in the analysis may lead to premature and often emergent halting of the system operation, causing great damage to both the environment and economy.

Based on experimental results, we conclude that the corrosive process of a structure in an aggressive environment is determined by the temperature, stress-strain state, nature of the aggressive environment and time span during with the structure resides in the corrosive environment. In certain circumstances, the governing parameters may also include fluid pressure, speed of the fluid or of aggressive gas, characteristic location of elements in the structure and other factors. Different researchers offered various models to describe the same process.

The influence of stress on corrosion speed, known as the corrosion rate, was apparently first considered by Dolinsky (1967), who dealt with the strength of thin-walled pipes subjected to a continuous corrosion rate as a linear function of stress. An exponential dependence of corrosion in the stress of the structure was proposed by Gutman and Zaynullin (1984).

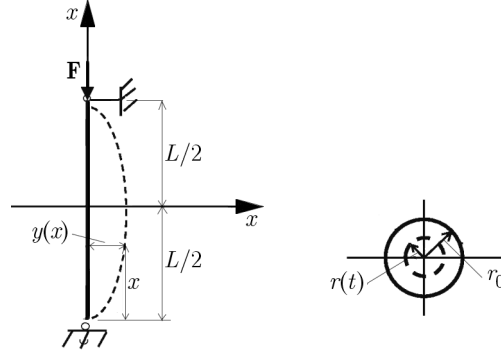
Papers by Potchman and Fridman (1995-1997), Fridman and Życzkowski (2001), Fridman (2002) utilized Dolinsky's model for the optimization study under corrosion. This study extends the above papers to the exponential stress corrosion model by Gutman and Zaynullin to study the stability optimization in the corrosive environment.

2. Problem statement

We consider the problem of optimal design employing the criterion of minimum weight of columns with a circular cross section loaded with an axial compressive F force (Fig. 1) and subjected to uniform corrosion.

We adopt the corrosion model introduced by Gutman and Zaynullin, which reads

$$\frac{dr}{dt} = -\alpha \exp(\eta t) \exp(\gamma \sigma) \quad (2.1)$$

Fig. 1. Bar under axial compressive load P

where α , η , γ are constant coefficients, $\sigma(t) = F/\pi[r(t)]^2$ is the stress, r_0 and $r(t)$ is the initial and current radius of the circular column, respectively. It is assumed that the rack is subject to uniform corrosion on the perimeter of its cross-section.

Separation of variables in Eq. (2.1) leads, with the notation $a = \gamma F/\pi$, to

$$\int \exp\left(-\frac{a}{r^2}\right) dr = -\alpha \int \exp(\eta t) dt \quad (2.2)$$

Taking into account the familiar series expansion

$$e^x = \sum_0^{\infty} \frac{x^n}{n!} \quad (2.3)$$

we get from Eq. (2.2)

$$\int \left(1 - \frac{a}{r^2} + \sum_2^{\infty} \frac{(-a)^n}{n!} r^{-2n}\right) dr = r + \frac{a}{r} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{r^{1-2n}}{1-2n} = -\frac{\alpha}{\eta} \exp(\eta t) + C \quad (2.4)$$

Invoking the initial conditions at $t = 0$, we obtain

$$C = r_0 + \frac{a}{r^2} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{r_0^{1-2n}}{1-2n} + \frac{\alpha}{\eta} \quad (2.5)$$

When $t = T$, with T denoting the durability, while r_T – radius of the section at the time instant T , we have

$$\frac{\eta}{\alpha} \left(r_0 - r_T + a \left(\frac{1}{r_0} - \frac{1}{r_T} \right) + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{r_0^{1-2n} - r_T^{1-2n}}{1-2n} \right) = \exp(\eta T) - 1 \quad (2.6)$$

3. Lagrange function and optimality conditions

The minimum weight of the column is achieved by the optimal distribution of the initial radius of the cross section r_0 along the columns length. In terms of the volume

$$V = 2 \int_0^{l/2} A dx = 2\pi \int_0^{l/2} r_0^2(x) dx \quad (3.1)$$

The conditions of optimality are expressed as the Euler-Lagrange equations

$$\delta_y f = f_y - \frac{d}{dx} f_{y'} + \frac{d^2}{dx^2} f_{y''} = 0 \quad \delta_{r_0} f = \frac{df}{dr_0} = 0 \quad (3.2)$$

where f is the so-called Lagrange function with an additional condition in form of equation (2.6) as follows

$$f = \pi r_0^2 + \lambda(x) \left[-\frac{\eta}{\alpha} \left(r_0 - r_T + a \left(\frac{1}{r_0} - \frac{1}{r_T} \right) + \sum_{n=2}^{\infty} \frac{(-a)^n r_0^{1-2n} - r_T^{1-2n}}{n! (1-2n)} \right) + \exp(\eta T) - 1 \right] \quad (3.3)$$

The governing equation for buckling of the column reads

$$EIy'' + Fy = 0 \quad (3.4)$$

Assuming that the loss of stability occurs at time $t > T$ (and considering that $I = \pi r^4/4$) from the equation of buckling, r_T is defined as

$$r_T = \left(-\frac{4Fy}{\pi E y''} \right)^{\frac{1}{4}} \quad (3.5)$$

In these circumstances, Eq. (3.3) becomes

$$f = \pi r_0^2 + \lambda(x) \left\{ -\frac{\eta}{\alpha} \left[r_0 - \left(-\frac{4Fy}{\pi E y''} \right)^{\frac{1}{4}} + \frac{a}{r^2} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{1}{4}} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{1}{1-2n} \left[r_0^{1-2n} - \left(\frac{-4Fy}{\pi E y''} \right)^{\frac{1-2n}{4}} \right] \right] + \exp(\eta T) - 1 \right\} \quad (3.6)$$

Evaluation of the derivatives yields

$$f_y = \lambda(x) \left[-\frac{\eta}{\alpha} \left(\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} \frac{F}{\pi E y''} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} \frac{F}{\pi E y''} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(-\frac{4Fy}{\pi E y''} \right)^{-\frac{3}{4}-\frac{n}{2}} \frac{F}{\pi E y''} \right) \right] \\ f_{y'} = 0 \quad (3.7)$$

$$f_{y''} = \lambda(x) \left[-\frac{\eta}{\alpha} \left(\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} \frac{Fy}{\pi E (y'')^2} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} \frac{Fy}{\pi E (y'')^2} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(-\frac{4Fy}{\pi E y''} \right)^{-\frac{3}{4}-\frac{n}{2}} \frac{Fy}{\pi E (y'')^2} \right) \right]$$

Substituting these expressions into Eq. (3.2)₁ results in

$$\delta_y f = \lambda(x) \left[\left(\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} \frac{F}{\pi E y''} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} \frac{F}{\pi E y''} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(-\frac{4Fy}{\pi E y''} \right)^{-\frac{3}{4}-\frac{n}{2}} \frac{F}{\pi E y''} \right) \right] \\ + \left\{ \lambda(x) \left[\frac{y}{y''} \left(\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} \frac{F}{\pi E y''} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} \frac{F}{\pi E y''} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(-\frac{4Fy}{\pi E y''} \right)^{-\frac{3}{4}-\frac{n}{2}} \frac{F}{\pi E y''} \right) \right] \right\} = 0 \quad (3.8)$$

4. Chentsov's method

Hereinafter, we utilize the method proposed by Chentsov (1936), Rzhantsyn (1955) to solve Eq. (3.8). We introduce the following notation

$$k = \frac{\lambda y}{y''} \left[\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} \frac{F}{\pi E y''} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} \frac{F}{\pi E y''} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}-\frac{n}{2}} \frac{F}{\pi E y''} \right] \quad (4.1)$$

Multiplying Eq. (3.8) by y , we represent it in the form

$$ky'' - yk'' = 0 \quad (4.2)$$

One can check by direct differentiation that the integral of the latter equation is

$$ky' - yk' = C \quad (4.3)$$

where C is an arbitrary constant. For further integration, one should take into account that $C = 0$ for the adopted boundary conditions. Indeed, assuming the buckling mode as being symmetric with respect to the middle cross-section of the column, we establish that $y, y'', y/y''$ and k are even functions of x . Hence, the derivatives of y as well as k would be odd functions of x and thus vanish at $x = 0$. Therefore, letting in Eq. (4.3) $x = 0$, we get $C = 0$. Thus, Eq. (4.3) becomes

$$ky' - yk' = 0 \quad (4.4)$$

Its integral is evaluated by separation of variables

$$k = C_1 y \quad (4.5)$$

where C_1 is a new arbitrary constant. Returning to the original notation in Eq.(4.1), we find

$$\frac{\lambda y}{y''} \left[\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} \frac{F}{\pi E y''} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} \frac{F}{\pi E y''} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4} - \frac{n}{2}} \frac{F}{\pi E y''} \right] = C_1 y \quad (4.6)$$

Additionally,

$$\delta_{r_0} f = 2\pi r_0 + \lambda \left[-\frac{\eta}{\alpha} \left(1 - \frac{a^2}{r_0^2} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} r_0^{-2n} \right) \right] = 0 \quad (4.7)$$

from which we determine the Lagrange multiplier

$$\lambda = \frac{2\pi r_0 \alpha}{\eta \left(1 - \frac{a}{r_0} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} r_0^{-2n} \right)} \quad (4.8)$$

Substituting λ into (4.6), we obtain

$$\begin{aligned} & \frac{2r_0 \alpha F}{\eta E} \left[\left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4}} - a \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{5}{4}} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \left(\frac{-4Fy}{\pi E y''} \right)^{-\frac{3}{4} - \frac{n}{2}} \right] \\ & = \left(1 - \frac{a}{r_0^2} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} r_0^{-2n} \right) C_1 (y'')^2 \end{aligned} \quad (4.9)$$

Or, given that r_T (Eq. (3.5)), we finally have

$$\frac{2r_0 \alpha F}{\eta E} \left(\frac{1}{r_T^3} - \frac{a}{r_T^5} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{1}{r_T^{3+2n}} \right) = \left(1 - \frac{a}{r_0^2} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} r_0^{-2n} \right) C_1 (y'')^2 \quad (4.10)$$

5. Particular cases

First let us consider case 1 when in the expansion of exponent in Eq. (2.3) the first two terms are retained. In this case, we get a model of corrosion similar to the model by Dolinsky.

The equations (2.6) and (4.10) become, respectively

$$\begin{aligned} \frac{\eta}{\alpha} \left[r_0 - r_T + a \left(\frac{1}{r_T} - \frac{1}{r_0} \right) \right] &= \exp(\eta T) - 1 \\ \frac{2r_0 \alpha F}{\eta E} \left(\frac{1}{r_T^3} - \frac{a}{r_T^5} \right) &= \left(1 - \frac{a}{r_0^2} \right) C_1 (y'')^2 \end{aligned} \quad (5.1)$$

We introduce the following non-dimensional quantities

$$\begin{aligned} A &= \exp(\eta T) - 1 & B &= \frac{\gamma F}{\pi} \frac{\eta^2}{\alpha^2} & R_0 &= \frac{r_0 \eta}{\alpha} \\ R_T &= \frac{r_T \eta}{\alpha} = \frac{\eta}{\alpha} \left(-\frac{4Fy}{\pi E y''} \right)^{\frac{1}{4}} & &= \frac{\eta}{\alpha} \left(-\frac{4F}{\pi E} \frac{l^2}{4} \frac{V}{V''} \right)^{\frac{1}{4}} & &= C \left(-\frac{V}{V''} \right)^{\frac{1}{4}} \end{aligned} \quad (5.2)$$

In view of

$$\frac{y}{y''} = \frac{l^2}{4} \frac{V}{V''} \quad (5.3)$$

we get

$$C = \frac{\eta}{\alpha} \left(\frac{Fl^2}{\pi E} \right)^{\frac{1}{4}} \quad V = \sqrt{\frac{\alpha E C_1}{\eta F}} \frac{4}{l^2} y \quad V_{\xi\xi}'' = \sqrt{\frac{\alpha E C_1}{\eta F}} y'' \quad \xi = \frac{2x}{l} \quad (5.4)$$

In this case, Eqs. (5.1) become, respectively

$$R_0 - R_T + B \left(\frac{1}{R_0} - \frac{1}{R_T} \right) = A \quad \frac{2R_0^3 R_T^2 - B}{R_T^5 R_0^2 - B} = V''^2 \quad (5.5)$$

From Eq. (5.5)₁, we find

$$R_0 = \frac{D + \sqrt{D^2 - 4R_T^2 B}}{2R_T} \quad (5.6)$$

where $D = R_T^2 + AR_T + B$.

Substituting the expression into Eq. (5.6) with (5.2), we arrive at

$$f_1(A, B, C, V, V'') = 0 \quad (5.7)$$

where $f_1(A, B, C, V, V'')$ depend on A, B, C, V, V'' .

Now we turn to numerical implementation of the procedure. Taking into account symmetry of the buckling mode with respect to the y axis, we divide half-length of the column into elementary parts of the length $\Delta\xi$ as shown in Fig. 2.

To determine the shape of the column in this case, in addition to Eq. (5.7), we use a relationship between the buckling mode and its second derivative in the central finite difference setting

$$V_{i-1} = \Delta\xi^2 V_i'' + 2V_i - V_{i+1} \quad (5.8)$$

Starting from the arbitrary negative value of V_{11} and $V_{10} = V_{10}'' = 0$ (i.e. $V_9 = -V_{11}$), we determine the value V_9'' from Eq. (5.7) by using a random search algorithm (Gurvich *et al.*,

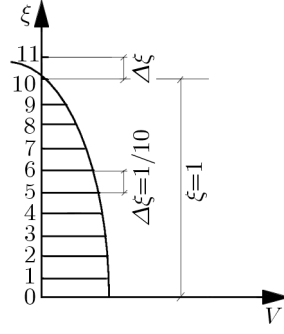


Fig. 2. Discretization of the half-column

1979). Then, from Eq. (5.8) we find V_8 . The process is repeated until values V_0 and V_0'' are determined. The solution is validated by evaluating V_0' , since the latter must vanish due to $V_1 \approx V_0$.

After the values of V_i and V_i'' are found, values of R_{Ti} and R_{0i} can be directly evaluated.

Let us consider particular case 2 when $n = 2$. In this case, equations (2.6) and (4.10) lead to the following

$$\begin{aligned} \frac{\eta}{2\alpha} \left[r_0 - r_T + a \left(\frac{1}{r_0} - \frac{1}{r_T} \right) - \frac{a^2}{6} \left(\frac{1}{r_0^3} - \frac{1}{r_T^3} \right) \right] &= \exp(\eta T) - 1 \\ \frac{2r_0\alpha F}{\eta E} \left(\frac{1}{r_T^3} - \frac{1}{r_T^5} + \frac{a^2}{2r_T^7} \right) &= \left(1 - \frac{a}{r_0^2} + \frac{a^2}{2r_0^4} \right) C_1 (y'')^2 \end{aligned} \quad (5.9)$$

or the non-dimensional quantities

$$\begin{aligned} R_0 - R_T + B \left(\frac{1}{R_0} - \frac{1}{R_T} \right) - \frac{B^2}{6} \left(\frac{1}{R_0^3} - \frac{1}{R_T^3} \right) &= A \\ \frac{2R_0^5 R_T^4 - BR_T^2 + \frac{B^2}{2}}{R_T^7 R_0^4 - BR_T^2 + \frac{B^2}{2}} &= V''^2 \end{aligned} \quad (5.10)$$

We find out R_0 in Eq. (5.10)₁. We obtain the following equation

$$R_0^4 + \left(\frac{B^2}{6R_T^3} - \frac{B}{R_T} - R_T - A \right) R_0^3 + BR_0^2 - \frac{B^2}{6} = 0 \quad (5.11)$$

To solve, it we apply the method by Ferrari.

Then

$$R_0 = y - b \quad (5.12)$$

where

$$\begin{aligned} b &= \frac{1}{4} \left(\frac{B^2}{6R_T^3} - \frac{B}{R_T} - R_T - A \right) & y &= \sqrt{\frac{t}{2}} - \sqrt{-p - \frac{t}{2} - \frac{q}{\sqrt{2t}}} & p &= 3c - 3b^2 \\ q &= 4b^3 - 6cb & c &= \frac{B}{6} & t &= z_1 - \frac{r_1}{3} \\ z_1 &= \sqrt[3]{-\frac{q_1}{2} + \sqrt{D_*}} + \sqrt[3]{-\frac{q_1}{2} - \sqrt{D_*}} & D_* &= \left(\frac{p_1}{3} \right)^3 + \left(\frac{q_1}{2} \right)^3 & p_1 &= \frac{3S_1 - r_1^2}{3} \\ q_1 &= \frac{2r_1^3}{27} - \frac{r_1 s_1}{3} + t_1 & s_1 &= p^2 - r & r &= 6cb^2 + e - 3b^4 \\ e &= -\frac{B^2}{6} & t_1 &= -\frac{q^2}{2} \end{aligned}$$

Defining R_0 and substituting into Eq. (5.10)₂ with (5.2), we arrive at

$$f_2(A, B, C, V, V'') = 0 \quad (5.13)$$

where $f_2(A, B, C, V, V'')$ depend on on A, B, C, V, V'' .

A more accurate analytical solution for $n = 3$ is not possible because of obvious reasons.

6. Numerical results

The optimum shape of the initial radius of the cross section bar $R_0(\xi)$ and its form at $t \approx T$: $R_T(\xi)$ was derived for alloy D16T with the following rates of corrosion models (1): $\alpha = 4.8 \cdot 10^{-4}$ m/year, $\gamma = 0.588 \cdot 10^{-5}$ m²/kN, $\eta = 0.091$ 1/year, $E = 7 \cdot 10^{10}$ Pa. The parameters were fixed at $h = 10^{-2}$ m, $F = 10$ kN, $T = 10$ years, $l = 1$ m, $C_1 = 2$ m³. In this case, the non-dimensional quantities are $A = 1.484$, $B = 0.6727$, $C = 2.768$. In all cases, the optimized shapes turn out to be almost identical (Fig. 3).

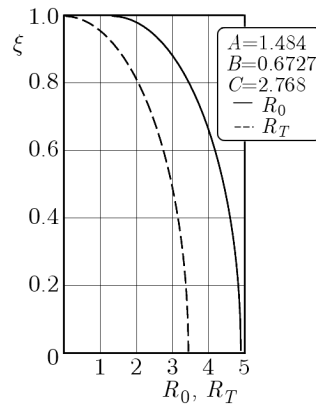


Fig. 3. Optimum initial shape and its final form for $B = 0.6727$

Similar results were obtained with the $F = 14.86$ kN, $T = 7.62$ years, $l = 1$ m. In this case, the associated non-dimensional quantities are $A = 1$, $B = 1$, $C = 3.06$. The optimum shape of the initial radius of the cross section bar $R_0(\xi)$ and its form at $t \approx T$: $R_T(\xi)$ are shown in Fig. 4a.

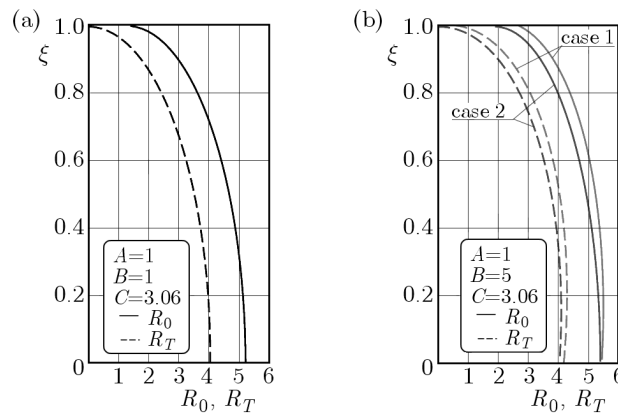


Fig. 4. Optimum initial and final shapes for (a) $B = 1$ and (b) $B = 5$

Figure 4b shows the numerical results obtained for the following data: $F = 14.86$ kN, $T = 7.62$ years, $l = 1$ m, $A = 1$, $C = 3.057$, $B = 5$. The new value of B with remaining

parameters kept constant is associated with a dramatic increase in the corrosion rate, i.e. in the parameter $\gamma = (0.588 \cdot 10^{-5}) \cdot 5 = 2.94 \cdot 10^{-5} \text{ m}^2/\text{kN}$. The optimum shape of the initial radius of the cross section bar $R_0(\xi)$ and its form when $t \approx T$, namely $R_T(\xi)$, are shown in Fig. 4b.

7. Conclusion

In this paper, the general and particular optimization solutions are obtained for the Gutman-Zaynullin corrosion model.

The results of numerical evaluation shown in Fig. 3 and Fig. 4a show that the increase in the number of terms in the expansion in Eq. (2.3) does not produce significant changes for the chosen sets of parameters. In all cases, the optimal shape of the initial radius of the cross section bar $R_0(\xi)$ and of its counterpart for $t \approx T$, namely $R_T(\xi)$, are almost identical. This closeness in the results may have the following explanation. By substituting the values obtained for R_0, R_T in Eq. (5.10)₁, one observes that the terms in front of B^2 tend to zero. The increasing of the value of factor B (with other values fixed at $\alpha = 4.8 \cdot 10^{-4} \text{ m/year}$, $\gamma = 0.588 \cdot 10^{-5} \text{ m}^2/\text{kN}$, $\eta = 0.091 \text{ 1/year}$, $E = 7 \cdot 10^{10} \text{ Pa}$) should lead also to an increase in the force F , which – in turn – leads to a sharp increase in R_0, R_T . Since these terms appear in the denominators in (5.10)₁ then, respectively, the contribution of terms containing B^2 is decreased.

An increased slight difference (between 3.7-21%) in all cases occurs when the corrosion rate depends strongly on B , as shown in Fig. 4b.

Summing up the above, the use of the corrosion model by Dolinsky allows one to get a sufficiently accurate optimization problem for bars under axial compression in a corrosive environment, when the corrosion rate does not depend strongly on B . However, when the corrosion rate strongly depends on B , one is recommended to utilize the model by Gutman and Zaynullin. The results of his study include, as a particular case, the analysis associated with Dolinsky's corrosion model.

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