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On the damping intensity of the odd Fourier impulse loading the boundary of the periodic composite*

Key words: heat transfer, Fourier series, tolerance averaging, micro-macro hypothesis, surface localization, effective conductivity

Introduction

In this paper, we intend to investigate what factors affect the intensity of suppression of a single periodic temperature impulse charging the boundary of the periodic composite. To this end, we use surface localized heat transfer equations, cf. Woźniak, Wierzbicki and Woźniak (2002), Kula (2015), Kula and Wierzbicki (2015), Kula (2016), Wodzyński, Kula and Wierzbicki (2018), Kula, Wierzbicki, Witkowska-Dobrev and Wodzyński (2018), Wierzbicki, Kula and Wodzyński (2018a, 2018b), Wierzbicki (2019), which allows for such analysis without the need to introduce any correctors to ensure the possibility of satisfy related boundary conditions in homogenization approach, cf. Ariault (1983), Bensoussan, Lions and Papanicolaou (2011). Surface localised heat transfer equations are obtained by the applying the modelling technique based on micro-macro hypothesis, cf. Woźniak and Wierzbicki (2000) as well as Woźniak, Łacińska and Wierzbicki (2005), Woźniak (2009), Jędrysiak (2010), Michalak (2010), Woźniak (2010).

Model equations described in the subsequent considerations (are developed by Wierzbicki, 2019) equivalent reformulation of heat transfer equations (HTE) in which a Fourier expansion as a certain representation of the temperature field is used. They consist of the single equation for average temperature with additional terms through which the average temperature, as the first term of the mentioned expansion, is ad-

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ditionally controlled via Fourier coefficients together with the finite number of tolerance amplitudes. Fourier basis taken into account in the proposed approach includes the changing of the composite periodicity along directions perpendicular to the periodicity directions, c.f. Kula et al. (2018). The resulted interaction of the composite media with the boundary impulse imposed on the average temperature is known as a boundary effect behaviour. The tolerance description of this phenomenon takes into account only near-boundary exponential damping, which is subject to the moving thermal impulse, c.f. Woźniak (2009), Woźniak (2010) and continuators, Szlachetka & Wągrowska (2011), Witkowska-Dobrev & Wągrowska (2015), Woźniak et al. (2005). The reason of this situation is to use a description that takes into account a single tolerance shape function. The sum of Fourier fluctuating terms (excepting the first equal to the average temperature) using in the presented in this paper modelling approach can be treated as the analytical formula for the error in using of approximate solutions of HTE proposed in tolerance averaging technique (TAT) approach. On the other hand proposed description of boundary effect behaviour is a certain complement to the mentioned tolerance description for that including a richer collection of shape functions. The aim of this paper is to describe one-impulse boundary effect behaviour in the framework of surface localized HTE.

The starting point of considerations is the well-known parabolic heat transfer equation.

$$\nabla^T (K\nabla\theta) - c\dot{\theta} = b \tag{1}$$

in which the region $\Omega \subset R^d$, $2 \le D \le 3$, occupied by the composite is restricted to the form

$$\Omega = \Omega_d \times \Omega_{D-d} \tag{2}$$

where: $1°\Omega_d = (0, L)$, $\Omega_{D-d} = (0, \delta_1) \times (0, \delta_2)$, while $(d, D) = (1, 3)$, $2°\Omega_d = (0, L_1) \times (0, L_2)$, $\Omega_{D-d} = (0, \delta)$, while $(d, D) = (2, 3)$, and $3°\Omega_d = (0, L)$, $\Omega_{D-d} = (0, \delta)$, while $(d, D) = (1, 2)$ for $L_1, L_2, L, \delta_1, \delta_2, \delta > 0$. In equation (2) $\theta = \theta(y, z, t)$, $y \in \Omega_d \subset R^d$, $z \in \Omega_{D-d} \subset R^{D-d}$, $t \ge 0$, denotes the temperature field, d is a specific heat field and k is the heat conductivity constant matrix. Moreover, $\nabla \equiv \nabla_d + \nabla_{D-d}$ for $\nabla_d \equiv [\partial / \partial y^1, \ldots, \partial / \partial y^d, 0, \ldots, 0\,]^T$ with zeros placed in $D - d$ positions and $\nabla_{D-d} \equiv [0, \ldots, 0, \partial / \partial z^1, \ldots, \partial / \partial z^{D-d}]^T$ with zeros placed in d positions. Fields $c = c(\cdot)$ and $k = k(\cdot)$ take S values denoted by $c^1, \ldots, c^S$ and $k^1, \ldots, k^S$, respectively, do not depend on the temperature field θ and both are restrictions to Ω_d of a certain periodic fields defined in R^d. Considerations of the paper are restricted to Δ-periodic composites. Diameter $diam(\Delta)$ of repetitive cell is not necessary small where compared to the characteristic length dimension L of the region Ω. With dimensionless scale parameter $\lambda = diam(\Delta) / L$ we control the analysed equations in the subsequent considerations. The Δ-periodicity of the composite means that there exists d-tuple $(\mathbf{v}^1, \ldots, \mathbf{v}^d)$ of independent vectors $\mathbf{v}^1, \ldots, \mathbf{v}^d \in R^d$ determining σ directions of periodicity such that: (i) points $x + k_1\mathbf{v}_1 + \ldots + k_d\mathbf{v}_d$, $-0.5 < k_1$, $k_d < 0.5$, cover for the interior of the cell $\Delta(x)$; (ii) $\Delta = \Delta(x_0)$ for fixed $x_0 \in R^3$ and (iii) $c(x + \mathbf{v}) = c(x)$, $K(x + \mathbf{v}) = K(x)$ for an arbitrary $\mathbf{v} \in \{\mathbf{v}_1, \ldots, \mathbf{v}_d\}$, $x \in R^3$. The averaging $\langle f\rangle(x)$, $x \equiv (z, y)$, of an arbitrary integrable field f is defined by:

$$\langle f\rangle(x) = \frac{1}{|\Delta|}\int_{\Delta} f(\xi)d\xi \tag{3}$$

and is a constant field provided that is Δ-periodic.

Tolerance micro-macro hypothesis

Considerations take into account two fundamental assumptions. The first modelling assumption is a certain extension of the micro-macro hypothesis introduced in the framework of the tolerance averaging technique, cf. equations (1)–(6). In accordance with the mentioned hypothesis, the temperature field θ can be approximated with an acceptable accuracy

$$\theta_M(z) = \vartheta(z) + h^A(x)\psi_A(z) \tag{4}$$

The slowly varying fields $\vartheta(\cdot)$ and $\psi_A(\cdot)$ are referred here to as tolerance averaging of the temperature field and as fluctuation amplitudes fields, respectively. Here and in the sequel the summation convention holds with respect to indices $A = 1, \ldots, N$. Symbols h^A, $A = 1, \ldots, N$, used in equation (5) denote tolerance shape functions which should be periodic and satisfy conditions

$$h^A \in \mathrm{o}(\lambda), \quad \lambda\nabla_y h^A \in o(\lambda), \quad \langle ch^A\rangle = 0, \quad \langle Kh^A\rangle = 0 \tag{5}$$

Usually RHS of equation (4) is called micro-macro decomposition of the temperature field. For particulars the reader is referred to equations (1)–(6). In equation (4) we suggest to interpret terms $\theta_{long} = \vartheta$ and $\theta_{short} = h^A(x)\psi_A(z)$ as the short- and the long-wave approximations of θ, respectively. The tolerance-micro macro hypothesis can be formulated in the following form.

Micro-macro hypothesis. The residual part of the temperature field θ_{res} being the difference between the temperature field θ and its tolerance part θ_M given by equation (5) can be treated as zero, $\theta_{res} \equiv \theta - \theta_M \approx 0$, i.e. it vanish with an acceptable "tolerance approximation".

The tolerance temperature part θ_M is debarked from the temperature field θ by the micro-macro hypothesis as an approximation of this field leading to the equation for the average temperature controlled by the finite number of fluctuation amplitudes $\psi_A(\cdot)$.We intend to supplement this micro-macro approximation to the total temperature field θ interpreting decomposition

$$\theta \equiv \theta_M + \theta_{res} \tag{6}$$

as a certain temperature field representation in which θ_{res} is added as the error made while micro-macro decomposition (eq. 4) is used as tolerance approximation of the temperature field.

Modified micro-macro hypothesis

Taking into account the intention of adapting the idea implemented in the theory of signals, where we are dealing with the "overlap" of many signals controlled by various parameters, we will try to impose, following Wierzbicki (2019), onto decomposition (eq. 6) a new interpretation referred to as modified micro-macro hypothesis.

Modified micro-macro hypothesis. The composite temperature field θ awards LS-decomposition onto the sum

$$\theta \equiv \theta_L + \theta_S \tag{7}$$

of the long-wave part θ_L (L-part) and short-wave part θ_S (S-part), both sufficiently regular, which determine disappearing heat flux vector component

$$(q_S)_n \equiv k(\nabla \theta_S)_n = 0 \tag{8}$$

normal to Γ. Corresponding oscillation part

$$\theta(y,z,t) - \theta_L(y,z,t) = a_p(z,t)\phi^p(y,z) \tag{9}$$

of a certain orthogonal Fourier expansion $a_0 + a_p\phi^p$ represents S-part θ_S of θ.

In Equation (8) $n = n(x)$ denotes the unit vector field normal to discontinuity surfaces Γ in regular points x placed on Γ. In equation (9) summation convention holds with respect to positive integer p. Tolerance temperature approximation (eq. 4) is interpreted here as a temperature L-part θ_L if the related L-part $(q_M)_n \equiv n^T K \nabla \theta_M$ of heat flux normal component $(q)_n \equiv n^T K \nabla \theta$ is continuous on Γ. In this case expansion (9) is the error made under using $\theta_L = \theta_M$ as an approximation of the temperature θ. The representation

$$\theta = \vartheta + \lambda[g^A \psi_A + a_p(z,t)\varphi^p(y,z)] \tag{10}$$

under rescaling $h^A(x, t) \equiv \lambda g^A(\lambda^{-1}x)$ and $ø^p(x, t) \equiv \lambda\varphi^p(\lambda^{-1}x)$ and under denotation

$$\vartheta = a_0 + \theta_{res} - \lambda g^A \psi_A \tag{11}$$

allows to interpret equation (10) as tolerance micro-macro hypothesis provided that additional conditions:

$$\langle c\varphi^p \rangle = 0, \quad \langle k\varphi^p \rangle = 0, \quad p = 1, 2, ...,$$
$$\langle cg^A \rangle = 0, \quad \langle kg^A \rangle = 0, \quad A = 1, 2, ..., N \tag{12}$$

will be attached.

Onto the *LS*-decomposition a special interpretation will be imposed. So the term θ_L will debarked from the temperature field θ represented by decomposition (eq. 7) as a special field supported on the ε-ribbon surrounding surfaces of material discontinuities of a composite while the part θ_L of $\theta \equiv \theta_L + \theta_S$ should not be affected the presence of a heterogeneous composite structure. That is why mentioned decomposition includes a long-wave part and a short-wave part terms depending on the microstructure size λ and localized inside and outside of the thin the ε-ribbon surrounding mentioned surfaces. Thus decomposition $\theta \equiv \theta_L + \theta_S$ provides the ability to perform tolerance modelling procedure with respect to $u = \langle \vartheta \rangle$ as average temperature field, and to fields $\psi_A(\cdot)$ and $a_p(\cdot)$ as tolerance and Fourier amplitudes, respectively. The parameter ε will be treated as a certain small parameter ε. Substituting equation (10) into HTE instead of the temperature field θ will lead us to the equivalent reformulation of HTE as an effect of asymptotic passage with the parameter ε to zero. To this end the following locality property (firstly formulated in Wierzbicki, 2019) should be taken into account.

Locality property hypothesis. The temperature L-part θ_L is supported on the ε-ribbon surrounding the discontinuity surfaces Γ, i.e. $\theta_L\,(y, z, t) \neq 0$ for $(y, z) \in \Gamma_\varepsilon$ and $\theta_L(y,z,t) = 0$ for $(y, z) \in \Omega \setminus \Gamma_\varepsilon$.

Above hypothesis means that the limit passage $\varepsilon \to 0$ applied to

$$\theta_L = (\theta_L)_\varepsilon \to u = \langle \theta \rangle \tag{13}$$

and equation (10) can be properly realized and arrive at the expansion

$$\theta(y,z,t) = u(z,t) + \lambda[g^{\omega}_{(z)}(y,z)\psi^{(z)}_{\omega}(z,t) + a_p(z,t)\varphi^p(y,z)] + o(\varepsilon) \tag{14}$$

treated in the subsequent considerations as the basic representation of the temperature field.

Surface localization of heat transfer equation

Three steps of reformulation HTE, presented in Wierzbicki (2019), will be applied as a procedure resulting in model equations written here as

$$\langle c \rangle \dot{u} - \nabla^T [\langle k \rangle \nabla u + \langle k \nabla^T \varphi^p \rangle a_p + \langle k \nabla g^{\omega}_{(y)} \rangle \psi^{(y)}_{\omega} + \langle k \nabla g^{\omega}_{(z)} \rangle \psi^{(z)}_{\omega}] = -\langle b \rangle \tag{15a}$$

$$\langle\nabla_y{}^T g_{(z)}^{\nu} k\nabla g_{(z)}^{\omega}\psi_{\omega}^{(z)} + \langle\nabla_y{}^T g_{(z)}^{\nu} k\nabla\varphi^q\rangle a_q + \lambda\langle\nabla_y{}^T g_{(z)}^{\nu} k\varphi^q\rangle\nabla_z a_q + L_g[u] = 0 \quad (15b)$$

together with the infinite system of the second order partial differential equations for Fourier amplitudes

$$\begin{aligned}&\lambda^2\{\langle\varphi^p c\varphi^q\rangle\dot{a}_q - \nabla_z{}^T\langle\varphi^p c\varphi^q\rangle\nabla_z a_q\} + \lambda(\langle\nabla_y{}^T\varphi^p k\varphi^q\rangle - \langle\nabla^T\varphi^q k\varphi^p\rangle)\nabla_z a_q + \\ &+\langle\nabla_y{}^T\varphi^p k\nabla\varphi^q\rangle a_q + \langle\nabla_y{}^T\varphi^p k\nabla g_{(z)}^{\nu}\rangle\psi_{\nu}^{(z)} = L_a^{\lambda}[u]\end{aligned} \quad (16)$$

In equations (15a) and (15b) together with (16) summation convention holds with respect to p, $q = 1, 2, \ldots, \omega$, $\nu = (A, B) \in \pi_S$. A characteristic feature of model equations is that equation (15b) is algebraic and hence we obtain surface localized version of HTE:

$$\langle c\rangle\dot{u} - \nabla^T(k_{surf}[\nabla u] + [k]_{surf}{}^p a_p) = -\langle b\rangle \quad (17a)$$

$$\lambda^2(A_c{}^{pq}\dot{a}_q - \nabla_z{}^T A_k{}^{pq}\nabla_z a_q) + 2\lambda s_{surf}{}^{pq}\nabla_z a_q + \{k\}_{surf}{}^{pq} a_p = L_a^{\lambda}[u] \quad (17b)$$

as a final form of model equation in which coefficients:

$$\begin{aligned}
k_{surf} &= \langle k\rangle - \langle k\nabla^T{}_y g_{(y)}^{\nu}, k\nabla_y g_{(z)}^{\nu}\rangle(H^{-1})_{\nu\mu}\begin{bmatrix}\langle\nabla_y{}^T g_{(y)}^{\mu} k\rangle\\ \langle\nabla_y{}^T g_{(z)}^{\mu} k\rangle\end{bmatrix}\\
[k]_{surf} &= k\langle\nabla^T\varphi^p\rangle - \langle k\nabla^T{}_y g_{(y)}^{\nu}, k\nabla_y g_{(z)}^{\nu}\rangle(H^{-1})_{\nu\mu}\begin{bmatrix}\langle\nabla_y{}^T g_{(y)}^{\mu} k\nabla_z\varphi^q\rangle\\ \langle\nabla_y{}^T g_{(z)}^{\mu} k\nabla_z\varphi^q\rangle\end{bmatrix}\\
2s^{pq} &= \langle\nabla_y{}^T\varphi^p k\varphi^q\rangle - \langle\nabla^T\varphi^q k\varphi^p\rangle\\
\{k\}^{pq} &= \langle\nabla_y{}^T\varphi^p k\nabla\varphi^q\rangle, \quad A_c = \langle\varphi^p c\varphi^q\rangle, \quad A_k = \langle\varphi^p k\varphi^q\rangle
\end{aligned} \quad (18)$$

have been used.

Boundary effect equation

Differential equation, homogeneous for equation (17b),

$$\lambda^2(A_c{}^{pq}\dot{a}_q - \nabla_z{}^T A_k{}^{pq}\nabla_z a_q) + 2\lambda s_{surf}{}^{pq}\nabla_z a_q + \{k\}_{surf}{}^{pq} a_p = 0 \quad (19)$$

will be considered as a boundary effect equation since it describes the moving of Fourier fluctuations across the composite media under linearly distributed average temperature $u = k_{surf}{}^{-1} q_0 \nabla_z u(0) z + u_0$ free of temperature sources loadings ($b = 0$) and obtained under boundary conditions $\nabla_z u(z = 0) = k_{surf}{}^{-1} q_0$ and $u(z = 0) = u_0$.

Benchmark problem

Let us consider $D = d = 1$. Hence we deal with two-dimensional composite layer with one-directionally periodicity. In this case boundary effect equations (19) will be treated as two-dimensional mathematical model of a construction wall made of the periodic composite material. At the same time (eq. 19) becomes a system of the second order ordinary differential equations.

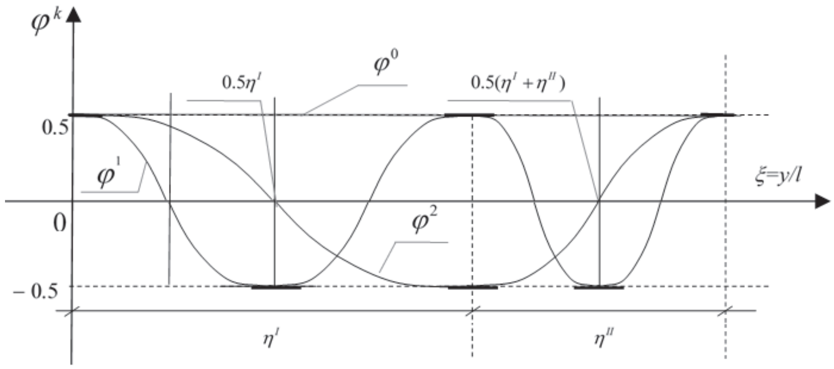

FIGURE 1. Fourier amplitudes: φ^k: ($v = 1$) first odd Fourier amplitude for $k = 2v - 1$ and first ($v = 1$) even Fourier amplitude for $k = 2v$

Impulses illustrated on Figure 1 are one-directionally v-th odd, v-th even left and v-th right one-directional Fourier impulses for v = 1-th. Analytically k-th Fourier impulse φ^k is considered: as odd provided that it is defined by

$$f_{2v-1}(v;y) = \begin{cases} -\dfrac{\lambda}{2}\cos(2v-1)\pi(\dfrac{y}{l^I}+1) & \text{for} \quad -l^I \le y \le 0 \\ -\dfrac{\lambda}{2}\cos(2v-1)\pi(\dfrac{y}{l^{II}}-1) & \text{for} \quad 0 \le y \le l^{II} \end{cases} \tag{20}$$

for $k = 2v - 1$, as even left denoted by $\varphi^{2v}_{(-)}$ provided that it is defined by

$$f^{2v}_{(-)}(v;y) = \begin{cases} \dfrac{\lambda}{2}\{1-\alpha_1[1+\cos 2\pi v(\dfrac{y}{\lambda\eta^I}+1)]\} & \text{for} \quad -\lambda\eta^I \le y \le 0 \\ \dfrac{\lambda}{2}\{1-\alpha_1[1+\cos 2\pi v(\dfrac{\overline{y}}{\lambda\eta^I}+1)]\} & \text{for} \quad 0 \le y \le \lambda\eta^{II}, \overline{y} = 0 \end{cases} \tag{21}$$

for $k = 2v$, and as even right denoted by $\varphi_{(+)}^{2v}$ provided that it is defined by

$$\varphi_{(+)}^{2v}(v;y) = \begin{cases} \frac{\lambda}{2}\{1-\alpha_2[1+\cos 2\pi v(\frac{\bar{y}}{\lambda\eta^{II}}-1)]\} & \text{for} \quad -\lambda\eta^{I} \le y \le 0, \bar{y}=0 \\ \frac{\lambda}{2}\{1-\alpha_2[1+\cos 2\pi v(\frac{y}{\lambda\eta^{II}}-1)]\} & \text{for} \quad 0 \le y \le \lambda\eta^{II} \end{cases} \tag{22}$$

also for $k = 2v$, respectively.

As the benchmark problem we consider boundary value problem for stationary variant of (19) in which boundary of the layer is loading by a single odd fluctuation. It is illustrated on Figure 2. In this case equation (19) reduces here to the single ordinary differential equation with constant coefficients

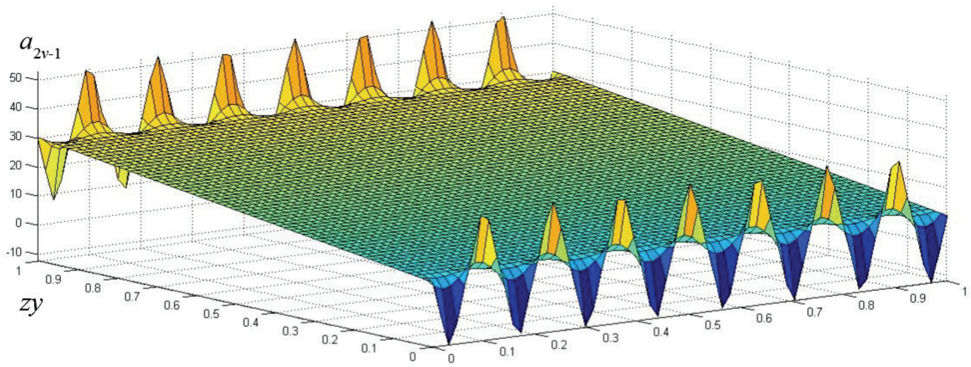

FIGURE 2. Boundary effect behaviour for single odd Fourier amplitude $a = a_{2v\text{-}1}$. Oscillatory dumping is absent

$$A_k \frac{d^2 a(z)}{dz^2} - \{k\}_{surf}\, a(z) = 0 \tag{23}$$

which is satisfied, under boundary conditions $a(z = 0) = a_0$, $a(z = \delta) = a_\delta$, by

$$a(z) = -\frac{\sinh\sqrt{\frac{\{k\}_{surf}}{A_k}}\frac{z-\delta}{\lambda}}{\sinh\sqrt{\frac{\{k\}_{surf}}{A_k}}\frac{\delta}{\lambda}}a_0 + \frac{\sinh\sqrt{\frac{\{k\}_{surf}}{A_k}}\frac{z}{\lambda}}{\sinh\sqrt{\frac{\{k\}_{surf}}{A_k}}\frac{\delta}{\lambda}}a_\delta \tag{24}$$

Formula (24) describes the odd single amplitude boundary layer behaviour in the case one-directionally periodic composite layer. Parameter $\omega = \sqrt{\frac{\{k\}_{surf}}{A_k}}$ will be considered as exponential damping factor. Since characteristic equation for formula (23) has no complex roots solution (eq. 23) has no parts responsible for oscillatory damping along the *z* variable direction. In the case of two-phased layer we have

$$A_k = \langle k\varphi^2 \rangle = \frac{\langle k \rangle}{8} \langle k \rangle = \frac{1}{8}(\eta_{\mathrm{I}} k^{\mathrm{I}} + \eta_{\mathrm{II}} k^{\mathrm{II}}), \quad \{k\}_{surf} = \frac{(2\nu - 1)^2}{8}(\frac{k^{\mathrm{I}}}{\eta_{\mathrm{I}}} + \frac{k^{\mathrm{II}}}{\eta_{\mathrm{II}}}) \tag{25}$$

and hence equation (24) takes the form

$$\omega = \sqrt{\frac{\{k\}_{surf}}{A_k}} = (2\nu - 1)\sqrt{\frac{\eta_{\mathrm{II}} k^{\mathrm{I}} + \eta_{\mathrm{I}} k^{\mathrm{II}}}{\eta_{\mathrm{I}} \eta_{\mathrm{II}} (\eta_{\mathrm{I}} k^{\mathrm{I}} + \eta_{\mathrm{II}} k^{\mathrm{II}})}} \tag{26}$$

Replacing $\chi = \frac{k^{\mathrm{II}}}{k^{\mathrm{I}}}$ in equation (26) we arrive at

$$\omega(\chi) = (2\nu - 1)\sqrt{\frac{\eta_{\mathrm{II}} + \eta_{\mathrm{I}} \chi}{\eta_{\mathrm{I}} \eta_{\mathrm{II}} (\eta_{\mathrm{I}} + \eta_{\mathrm{II}} \chi)}} \tag{27}$$

Hence, exponential damping factor $\omega = \omega(\chi)$ increases with the growth of χ when $\eta_{\mathrm{II}} < \eta_{\mathrm{I}}$ and decreases with the increase in χ when $\eta_{\mathrm{II}} > \eta_{\mathrm{I}}$. If $\eta_{\mathrm{I}} = \eta_{\mathrm{II}} = 0.5$ exponential damping factor is equal to a constant value $\omega = \omega(\chi) = 2(2\nu-1)$ regardless of the value of fraction $\chi = \frac{k_{\mathrm{II}}}{k_{\mathrm{I}}}$. Also $\omega(\chi) \to \frac{2\nu - 1}{\eta_{\mathrm{I}}}$ while $\chi \to 0$ and $\omega(\chi) \to \frac{2\nu - 1}{\eta_{\mathrm{II}}}$ while $\chi \to +\infty$. Moreover, $\omega(\chi) \to +\infty$ while $\eta_{\mathrm{I}} \to 0$ or $\eta_{\mathrm{II}} \to 0$.

Equation (27) describes the simplest variant of the formula for the damping exponential intensity according to which it is the square root of the rational function of the variable χ whose limit values for $\chi = 0+$ and for $\chi = +\infty$ are finite while saturations η_{I} and η_{II} take constant values placed between 0 and 1. In such cases damping exponential intensity has a convex support obtained as a result of a double has a convex rim obtained as a result of a double Legendre transformation of $\omega(\chi)$ known from Hamiltonian mechanics and therefore it reaches at least one local minimum for fixed η_{I} and η_{II}. This result is a basic conclusion of the presented paper.

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Summary

On the damping intensity of the odd Fourier impulse loading the boundary of the periodic composite. Investigated in the paper boundary effect behaviour for a single odd amplitude which loads rectangular boundary of the two-phased periodic composite layer confirms the common view through the prism of the expected strong suppression of the

boundary impulses of the physical field near the boundary of the region occupied by the composite. There is no presence of a composite reaction to the boundary loadings mentioned here different than the exponential damping effect. However, the presence in the general equation describing the boundary effect equations the component $2\lambda s_{surf}{}^{pq}\nabla_z \alpha_q$ with the first space derivative responsible for suppression of the solution along the axis Oz should cause not only exponential type of boundary temperature fluctuation damping. This component disappears in principle for the boundary effect analysed for a single impulse.

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