Mathematical simulation of the stressed-deformed condition of reinforced concrete beams in simultaneous influence of aggressive environment and loading

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Abstract. In this paper, the method of determining the stress-strain state of reinforced concrete beams at the simultaneous action of the aggressive medium and the load on the basis of the discrete representation of the of concrete and reinforcementε-σbeam structures using the diagrams with the help of a computer is considered. The proposed methodology allowed to simulate the work of reinforced concrete beams at all stages of loading, including destruction. At the same time, at each stage of loading, deformations and stresses in concrete and reinforcements are obtained along the entire height of the section and along the length of the beam. The method also allows to take into account the previous tension in the valve, which takes place in the manufacturing stage. In addition, it enables to simulate the formation and development of normal and inclined cracks, the beginning of the destruction of reinforced concrete beams, determine their bearing capacity and bends. Performed with the help of the proposed methodology, calculations of experimental models of reinforced concrete beams demonstrated a good coincidence of results.

Key words: reinforced concrete beam, stress-deformed state, hypothesis of plane sections, aggressive environment, corrosion.

THE INTRODUCTION

The reinforced concrete constructions of buildings and structures are often used in complex production conditions (under high positive or low negative temperatures, high humidity, under a complex aggressive environment, etc.)

The intense action of the aggressive environment contributes to the emergence and development of the process of corrosion of concrete and reinforcing steel reinforced concrete structures. And this, in turn, affects the reduction of the bearing capacity of the reinforced concrete structure, the increase of deflections, operational suitability.

Corrosive processes actively change the shape of the cross section of the reinforced concrete element, affect the physical and mechanical characteristics of the corrosive outer layers of the reinforced concrete element.

The development of a numerical computer model for calculating reinforced concrete beams subject to aggressive corrosion effects will provide sufficiently complete information on the stress-strain state of reinforced concrete beam construction at all stages of operation, from the beginning of the application of external load, the emergence and development of normal and inclined cracks, prior to the deteriorating state of the construction and on the very stage of building structure destruction.

EXPERIMENT AND EVALUATION

At the basis of the proposed calculation model for determining the stress-strain state of reinforced concrete beams with corrosion damage lies the hypothesis of plane cross sections and a sequential approximation algorithm that allows taking into account the physical nonlinearity of concrete and reinforcement.

The reinforced concrete beam (Fig. 1) in the calculation model is presented in a discrete form. Concrete is described by rectangular flat elements, reinforcing rods by linear core elements. The beam is divided into sections by length and layers at a section height.
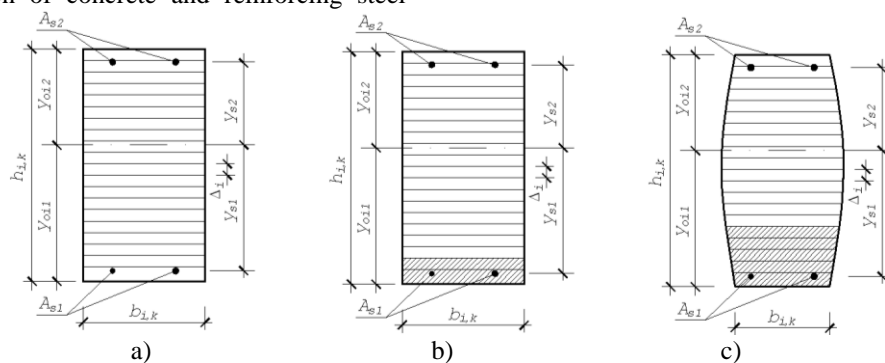

Fig. 1. The cross-section of reinforced concrete beams: a) before the appearance of normal cracks; b) after the occurrence of normal cracks; c) during intense corrosion of concrete

Thus, it has been previously stipulated that the stress-strain state formed in the elementary regions is the same. The number of elementary sections and elementary layers affects the accuracy of the calculation, but is always limited to the available RAM of the PC.

At considerable intensity of loading, and consequently high level of stresses on the side faces of the beam there is an increase in the intensity of corrosion of the concrete, which causes more intensive reduction in the size of the cross-section. Moreover, it has been experimentally discovered that the further we are from the neutral axis of the beam at the height of the cross section, the more this effect occurs. Numerous experimental studies [1-20] have shown that the cross sections of the reinforced concrete beams took the oval shape.

The width b_{ij} and thickness h_{ij} of a plane elementary layer ij taking into account the intensity of corrosion, must be determined from the following conditions:

$$b_{ij} = b_{ij} - 2 \cdot \delta \cdot k_{cij}, \qquad h_{ij} = h_{ij} - 2 \cdot \delta \cdot k_{cij}, \qquad (1)$$

where: k_{cij} – coefficient of intensity of corrosion of concrete, which depends on the magnitude of stress.

The coefficient of intensity of corrosion is proposed to be determined by the formula:
- at stresses not exceeding the limits of microcracking:

$$k_{cij} = \left[1 - k \frac{\sigma_{bij}}{E_{bij} \cdot m} (1 - v_0) \right]^3. \qquad (2)$$

- at stresses exceeding the boundary of microcracking:

$$k_{cij} = \gamma_b \left[1 + k \frac{\sigma_{bij}}{E_{bij} \cdot m} (1 - v_0 + v) \right]^3. \qquad (3)$$

- in tension:

$$k_{cij} = \frac{1}{\gamma_b^c} \left[1 + \frac{\sigma_{bij}}{E_{bij} \cdot m} (1 - v_0) \right]^3. \qquad (4)$$

where: k – the correction factor, which takes into account that during compression there is a partial decrease in the size of concrete pores, through which access to the aggressive environment;

σ_b – tension in concrete;

m – porosity;

E_b – module of deformation of concrete, which it is recommended to define on a diagram $\sigma_b - \varepsilon_b$ of concrete.

The depth of corrosion damages of concrete in the absence of stresses or at the level of the neutral axis of the cross section depending on the time factor are determined by the formula:

$$\delta = \sqrt{\frac{2 \cdot D \cdot c \cdot t}{m_0}} \quad . \qquad (5)$$

Linear rod elements are placed in height cross-section with their own protective layers.

The procedure for calculating the bearing capacity of a reinforced concrete beam with the simultaneous action of the external load and the action of the aggressive medium is as follows. Before the application of the external concentrated load, all elementary plane and linear elements are assigned with initial physical-mechanical characteristics. The given geometrical i characteristics of sections of reinforced concrete beams are determined taking into account the concrete modules E_{bij} and concrete deformation modules E_{si}:

$$(EA)_i = \sum_{j=1}^{j=\max} E_{bij} A_{bij} + E_{si} A_{si}, \quad (ES)_i = \sum_{j=1}^{j=\max} E_{bij} A_{bij} y_{bij} + E_{si} A_{si} y_{si}, \qquad (6)$$

$$y_{oi} = \frac{(ES)_i}{(EA)_i}, \qquad (EI)_i = \sum_{j=1}^{j=\max} E_{bij} A_{bij} (y_{0i} - y_{bij})^2 + E_{si} A_{si} (y_{0i} - a_i)^2 .$$

where: E_{bij}, E_{si} – ordinary modules of deformations of concrete and rod valves, which before the application of external load are taken equal to the initial modules of deformations of the corresponding materials;

A_{bij} – The cross-sectional area of the elementary j - height of the section i -section of the layer along the length of the beam;

A_{si} – Cross-sectional area of the rod armature;

y_{bij} – Distance from the bottom edge of the beam to the center of gravity of the elemental concrete area;

a_i – Distance from the bottom edge of the beam to the center of the weight of the bar reinforcement.

We calculate the bending moments M_i in the i lengths of the beams.

Determination of deformation in concrete ε_{bij} and rod valves ε_{si}:

$$\varepsilon_{bij} = \frac{M_i}{(EI)_i} (y_{bij} - y_{0i}), \quad \varepsilon_{si} = \frac{M_i}{(EI)_i} (a_i - y_{0i}). \qquad (7)$$

We check according to the corresponding diagrams $\sigma - \varepsilon$ the conditions of not exceeding the obtained deformations of limiting deformations in concrete ε_{bu} and reinforcement ε_{sy}:

$$\varepsilon_{btij} < \varepsilon_{bt\max}, \quad \varepsilon_{bij} < \varepsilon_{b\max}, \quad \varepsilon_{si} < \varepsilon_{sy} . \qquad (8)$$

In the case of normal or inclined cracks, that is, if the strain of the tension ε_{bij} in any elementary layer is greater than the maximum permissible $\varepsilon_{bij} > \varepsilon_{bt,max}$ in the concrete tension diagram, the elemental flat section of the concrete should be zeroed. And since the main characteristic of the planar elemental site is the slip modulus of deformation of the concrete E_{bij}, in this case it will be accepted as equal $E_{bij} = 0$.

If the excess of deformations of compression in concrete or tension in the reinforcement of the limiting ε_{bu} and ε_{sy} values is fixed, and it is accordingly assumed that in this section the strength of the normal sections has been exhausted. In this case, to specify the magnitude of the external destructive load, the load initially decreases to the value of the previous value $F = F - \Delta F$, followed by its increase under $\Delta F = \dfrac{\Delta F}{2}$. The calculations continue to provide the necessary precision for determining the destructive load ΔF_{min}.

If the deformations in concrete and reinforcement do not exceed the maximum permissible values ε_{bu} and ε_{sy}, the refinement and correction of the deformation modules of concrete and reinforcement deformations are performed.

$$E_{bij} = \frac{\sigma_{bij}}{\varepsilon_{bij}}, \quad E_{si} = \frac{\sigma_{si}}{\varepsilon_{si}} \qquad (10)$$

According to the specified values of the deformation modules, new geometric characteristics of the beam cross sections are calculated: $(EA)_i$, $(ES)_i$, $(EI)_i$, y_{0i}.

Under the corrected geometrical characteristics of the beam cross-sections, we specify the values of the deformations of the concrete ε_{bij} and the rod fittings ε_{si}. If their values are close to the predetermined accuracy of the calculations (the discrepancy not more than 1% gives satisfactory results of calculations $\Delta \varepsilon_b \leq 0,01\varepsilon_b$, it is considered that at this stage of the load the parameters of the stress-strain state of the cross sections of the reinforced concrete are set and proceed to the definition of deflections):

$$f = \int_0^L \frac{\overline{M_i} \cdot M_i}{(EI)_i} dl, \qquad (11)$$

where: $\overline{M}_i$ – the bending moment in the section i from the action of the unit force applied in the direction of movement in the section i for which the movement was determined.

In order to determine the maximum value of the deflections, the unit force is applied in the middle of the reinforced concrete beam.

On this, the first external cycle of calculation on the magnitude of the external concentrated load is completed. The value of external force is increased by ΔF:

$$F = F + \Delta F \qquad (12)$$

and the calculation starts from the beginning.

It is assumed that from this moment in addition to the external concentrated load and aggressive environment (corrosion) begins to influence on the beam.

Corrosive influences continue in time, and therefore we take the point in the days of simultaneous action of external loading and aggressive environment as the first day.

We calculate the depth of corrosion damage of reinforced concrete beam according to the formulas taking into account the coefficients of intensity of corrosion k_{cij}, which was calculated by the formulas depending on the value of stresses in concrete σ_{bij}:

In the light of the depth of corrosion damage, we determine the width of each elementary layer at the height of the section b_{ij}, the thickness of the extreme (contour) layers t_{i1} and $t_{i\,max}$, calculate new geometric characteristics of the sections $(EA)_i$, $(ES)_i$, $(EI)_i$, y_{0i}. Determine the deformation of creep ε_{crij} and shrinkage ε_{shij}. Determine the total deformation in the concrete $\varepsilon_{bij} = \varepsilon_{bij} + \varepsilon_{crij} + \varepsilon_{shij}$ by taking into account corrosion damage and real concrete concrete modules and fittings.

We check according to the corresponding diagrams the $\sigma - \varepsilon$ conditions of not exceeding the obtained deformations of the limiting deformations in concrete ε_{bu} and reinforcement ε_{sy} (8).

In the case of a normal or inclined cracks, the elemental flat section of the concrete is reset. If the excess of deformations of compression in concrete or tension in the reinforcement of the limiting values ε_{bu} and ε_{sy} is fixed, and it is accordingly assumed that in this section the strength of the normal sections has been exhausted. The calculation is completed.

If deformations in concrete and fittings do not exceed the maximum permissible values ε_{bu}, ε_{sy} the refinement and correction of the deformation modules of concrete and reinforcement deformations (10) are performed.

Under the specified values of the deformation modules, new geometric characteristics of the beam cross sections $(EA)_i$, $(ES)_i$, $(EI)_i$, y_{0i} are calculated.

According to the corrected geometrical characteristics of the beam cross-sections, we specify the values of the deformations of the concrete ε_{bij} and the rod fittings ε_{si}. If their values are close to the predetermined accuracy of the calculations, it is assumed that at this stage of the load the parameters of the stress-strain state of the cross-section of the reinforced concrete are set and proceed to the definition of deflections (11).

On this, the second external counting cycle after the countdown is completed. The magnitude of the day is increased by one day:

$$Day = Day + 1 \qquad (12)$$

and calculation of the second external cycle repeats.

CONCLUSIONS

According to the proposed method for determining the stress-strain state of reinforced concrete beams at the time of the external load and aggressive effects of the environment, the work of the prototypes was simulated at all stages of loading, including destruction. As a result, the distribution of deformations, strains, determining the bearing capacity and deformability of the prototype samples, the day of possible destruction of the structures are established.
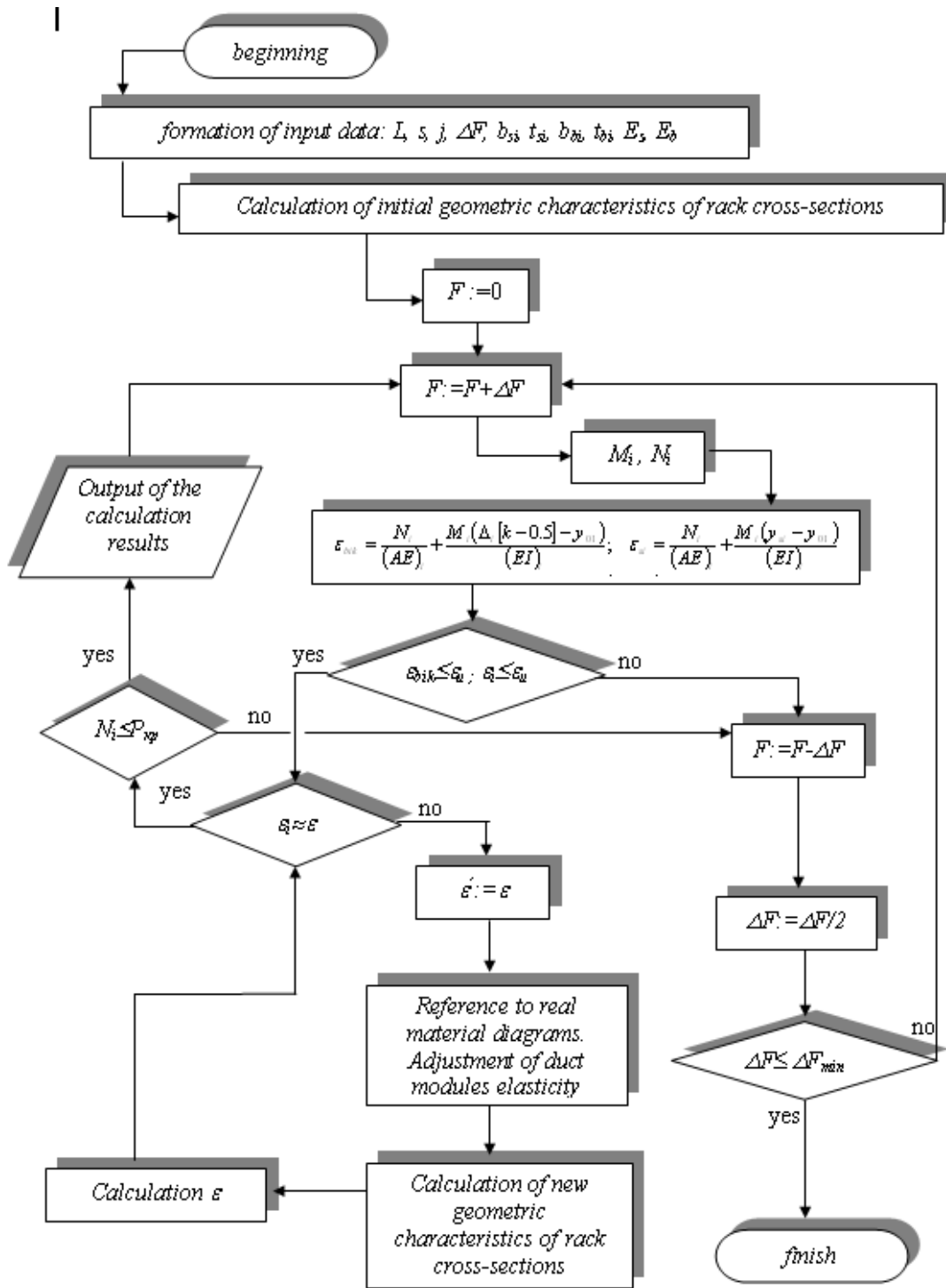

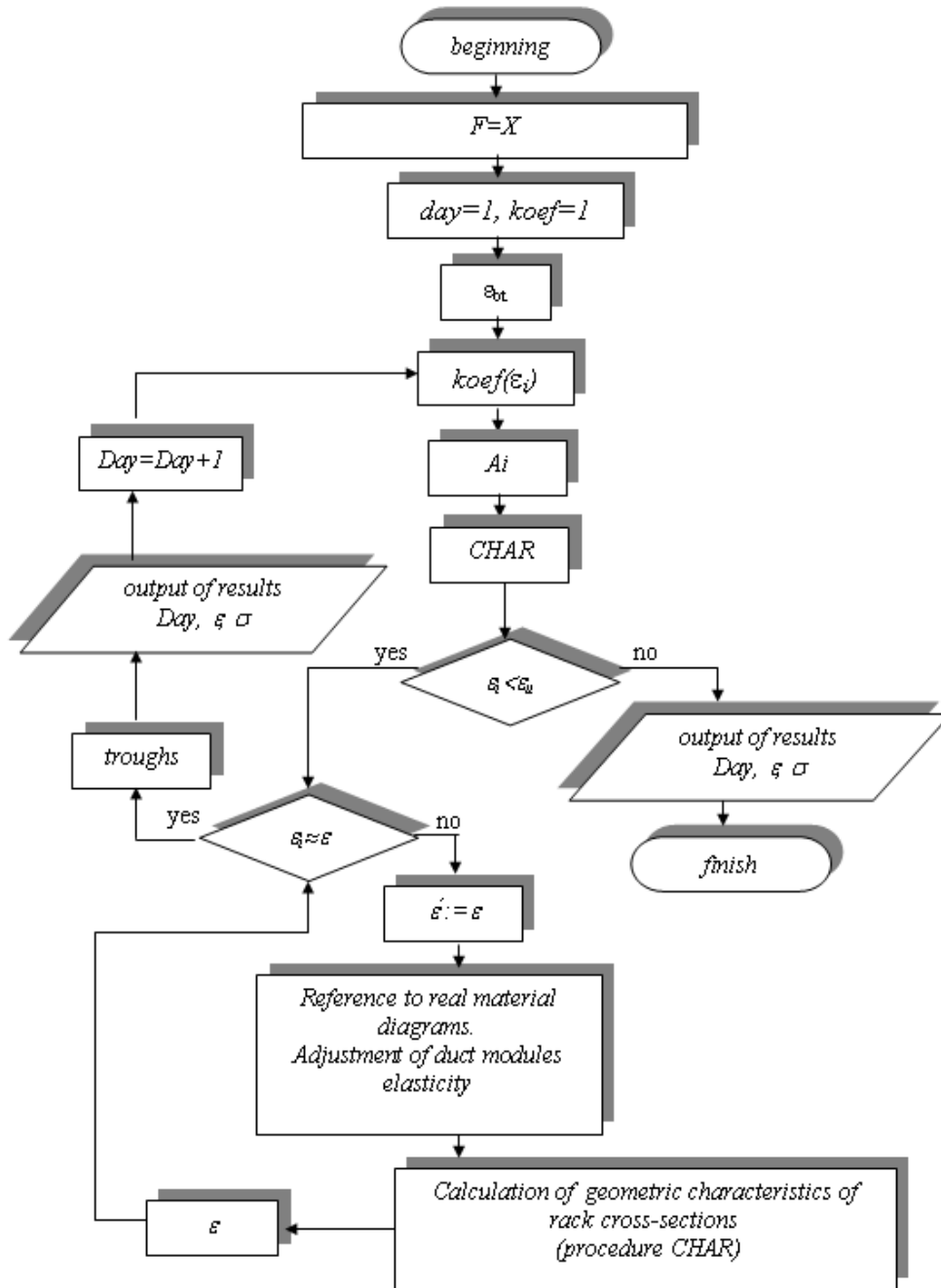**Fig.2.** Block diagram of the determination of the reinforced concrete beams stress-strain state

Fig.3. Block diagram of the determination of the reinforced concrete beams stress-strain state (due to intense corrosion of concrete).

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