Scientific Journals
of the Maritime University of Szczecin

Zeszyty Naukowe
Akademii Morskiej w Szczecinie

2016, 45 (117), 37–43
ISSN 1733-8670 (Printed)
ISSN 2392-0378 (Online)
DOI: 10.17402/083

Received: 31.08.2015
Accepted: 17.02.2016
Published: 25.03.2016

Using elements of integral calculus in the process of optimising the configuration of navigation positioning systems

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Key words: marine navigation, integral calculus, aids to navigation, land radio-navigation system, navigation – hydrographic support, accuracy of navigation, criteria for evaluating navigation systems

Abstract

This paper presents the authors' reflections on using integral calculus when preparing the process of locating single ground-based chains of positioning systems in marine navigation. These reflections are purely theoretical and presented research results apply to a hypothetical navigation system that operates based on the ratio of distances, which is regarded as a navigation parameter.

Introduction

The problems associated with positioning single chains of navigation systems always come down to ensuring that a vessel will be able to carry out specific tasks at sea (Czaplewski, 1998).

Navigation systems may be used by vessels which can be divided into three basic groups based on how they "move" and in what sea areas they operate:

- vessels that have to maintain their position;
- vessels that have to maintain a given course;
- vessels that move in restricted waters (sea areas).

Among vessels that have to maintain their position are, for example:

- vessels that are used by marine administration services which position navigation marks in coastal areas;
- floating dive bases for scuba-divers and skin-divers; these bases must maintain their position during underwater works;
- warships that have to maintain their position in order to carry out a given combat mission properly.

A vessel's position fix must be determined with a given accuracy and on a continuous basis with a particularly high degree of precision when tasks of this kind are carried out because this ensures proper execution of an operation.

The group of vessels that move along a predetermined route or fairway is the largest one. Each vessel that leaves or enters a port moves along a specific fairway. Additionally, this group can also include hydrographic survey vessels that conduct survey work and move along a given route.

There are two types of warships. Landing craft constitute the first type. Vessels of this kind require that a passage route through coastal minefields be accurately determined when approaching the land in order to put the landing force ashore. Minesweepers, which maintain a given tack when carrying out minesweeping operations, constitute the second type of warships. These vessels can also be used to lay naval mines in the water and it is also necessary to accurately determine specific positions in this case. Moreover, merchant ships proceeding along coastal fairways also belong to the group of vessels that move along a predetermined route.

Among the vessels that are used to carry out various tasks in restricted waters the main ones are the following:

- rescue vessels that operate within a designated search area during a rescue operation;
- warships that are used when carrying out military exercises in specific maritime training areas;
- fishing vessels that carry out fishing in specific sea areas which are inaccessible to non-fishing vessels.

It is particularly important to obtain continuous and highly accurate information about the position of one's vessel near the boundaries of these areas. If a civilian vessel entered a military area, it could, for example, come under fire from battleships during an artillery operation. Fishing boats pulling fishing nets could, in turn, pose a threat to the safety of merchant ships, etc. The accuracy with which a vessel's position is determined can also differ depending on the location of restricted waters. The requirements concerning the determination of a position fix are much stricter in the coastal zone and near fairways, as well as in sailing routes, than they are for sea areas that are located far from the shore and fairways.

From the perspective of navigation system designers, it is important to select proper criteria which would, on the one hand, provide a basis for assessing these systems in general, and on the other hand, constitute comparative criteria when optimising the process of positioning navigation stations. The set of criteria for evaluating navigation systems, in particular radio-navigation systems, are presented in the documents: (IALA, 2010) and (AFSC, 1997), as well as in the papers written by (Kopacz et al., 1994; 1996; Kopacz, 1995; Felski & Specht, 1996a; 1996b; Specht, 1997). Therefore, this study will not describe in detail all of the criteria that are used to assess navigation systems. However, those that are recommended by the IALA are presented below for the sake of order.

According to the IMO's regulations and the data that are presented in the literature on the subject (Kopacz, 1995; Kopacz et al., 1996; AFSC, 1997; Specht, 1997), the following definitions of **position accuracy** are employed, depending on geographic coordinate systems that are used:

- absolute accuracy or geodetic accuracy – this is the accuracy of a position that is expressed in geodetic coordinates;
- predictable accuracy – this is the accuracy with which a position will be determined if predictable errors occur; this type of accuracy depends on the state of knowledge about the sources of error;
- relative accuracy – this is the accuracy with which one user can determine his or her position relative to another user, within the same navigation system and at the same time;
- repeatable accuracy – this is the accuracy with which a given system allows the user to return to a previously determined position; it is expressed in coordinates that are specific to a given navigation system.

The concept of a navigation **system's availability** refers to the likelihood that the services provided by a given system will be available to the navigator of a vessel within that system's coverage zone (IALA, 2010). This is one of the most important comparative criteria because it gives a real picture of the services that are provided by a given system.

A **system's reliability** describes its ability to inform the users about its operational status, or more precisely (and according to the definition), a radio-navigation system's reliability defines its capability to issue periodic warnings about when not to use it for navigation.

The **frequency with which positions are determined** refers to the number of positions (or position parameters) that are established by a given system within a time unit. As is to be expected, the frequency with which positions are determined depends on many factors, such as the principles of operation and design of a given system's station, the receiver's technical capabilities and the method of presenting output data.

The concept of a **system's capacity** refers to the number of users who can utilise a given system at the same time.

Ambiguity occurs when a navigation system determines two or more position fixes during the same measurement, and it is not possible to establish which of them is the closest to the actual position.

The level of **technological advancement represented by a navigation system** defines its technical properties and technological capabilities as well as the operational features of devices that are part of this system and their compliance with applicable standards.

Purchase and operating costs describe the financial side of a project involving the implementation and maintenance of a functioning navigation system. This is one of the key criteria although it is not connected with navigational matters.

Using integral calculus

The simplest solutions are always the most effective, and this is true in this case. The process of adapting integral calculus for the purposes of the analyses that are presented in this paper involves using definite integrals to carry out specific tasks. Analysis of the properties of definite integrals shows that they are most applicable to two of the three above-mentioned groups of vessels, i.e.:

- vessels that have to maintain a given course;
- vessels that move in restricted waters.

If one wants to use a definite integral to analyse a given navigation system with respect to a specific criterion, for example, when performing optimisation work, one should be able to describe that criterion by using a continuous function. In (Czaplewski, 1999), an analysis of a ratio-based navigation system in terms of two criteria is presented. Those criteria are:

- The angle at which position lines intersect, which is described by the formula:

$$\cos\Theta = \frac{d_{1,2}^2 - r_1^2 - r_2^2}{2r_1r_2} \qquad (1)$$

where:

Θ – angle of intersection between position lines;
$d_{1,2}$ – distance between the centres of position circles;
r_1, r_2 – radii of position circles.

- The mean error of a position fix, which is given by the formula:

$$M = \operatorname{cosec}\Theta\sqrt{m_{l_1}^2 + m_{l_2}^2 - 2r_{l_{1,2}}m_{l_1}m_{l_2}\cos\Theta} \qquad (2)$$

where:

M – mean error of a position fix;
m_{l1}, m_{l2} – mean error of a position line;
$r_{l1,2}$ – correlation coefficient.

Coordinates of a single ground-based chain for a vessel maintaining its position

For a vessel that is following a predetermined route, it is important that the value of the optimisation parameter, in relation to which a given system's stations are to be positioned, fall within a range of values that are useful in navigation along the entire route to be followed (the entire fairway).

The formulas that describe the angle at which position lines intersect (1) and the mean error of a position fix (2) make it possible to obtain the distribution of these values for the entire predetermined route. Example distributions of these parameters are shown in the Figures 1 and 2.

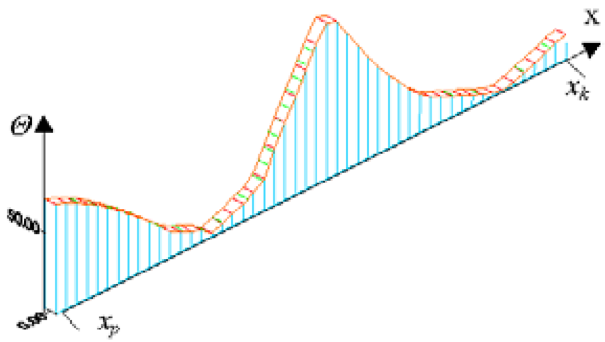

Figure 1. Example distribution of the angle of intersection between position lines (θ) along a fairway with the coordinates (x_p = –110 km, y_p = 090 km), (x_k = 170 km, y_k = 090 km) for a base system described in Table 1

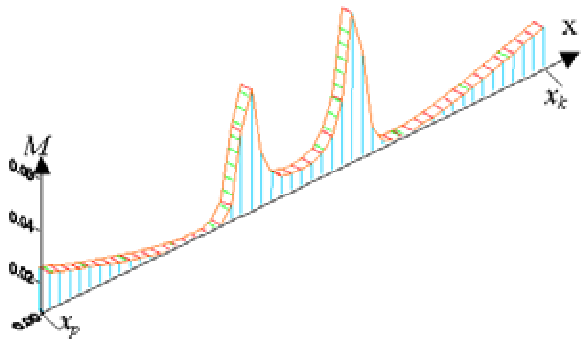

Figure 2. Example distribution of the mean error of a position fix (M) along a fairway with the coordinates (x_p = –110 km, y_p = 090 km), (x_k = 170 km, y_k = 090 km) for a base system which is described in Table 1

Table 1. Coordinates of a based quotient navigation system's stations for the examples presented in Figures 1, 2 and 5

Base angle θ [°]	Station A		Station B		Station C	
	x [km]	y [km]	x [km]	y [km]	x [km]	y [km]
090°	–57	35	0	–20	55	35

In order to find the optimal coordinates of the locations at which these stations are to be positioned with respect to one of the above-mentioned criteria, one must introduce an auxiliary parameter, i.e. the area under the curve representing the function that describes a given criterion. This area is calculated by using a definite integral:

$$P = \int_{x_p}^{x_k} f \, \mathrm{d}x \qquad (3)$$

where:

P – area under the curve of function f;
f – function describing a given optimisation criterion;
x_p, x_k – rectangular coordinates of the beginning and ending points of a vessel's passage route for y = const.

The area under the curve representing the function that has been obtained by using formula (3) can be evaluated in the process of optimising the positions of the system's stations, for example, by searching for the minimum or maximum values of this area, depending on the criterion used.

As for the angle of intersection between position lines, one should search for the maximum values of the area under the curve representing the distribution of angle θ. This will allow one to find coordinates of the stations for which angle θ would approach 090° along the entire route to be followed. Those pairs of coordinates of the stations' positions that generate angle $\theta < 030°$ at any point of the given route should be excluded from further analysis (Figure 3).

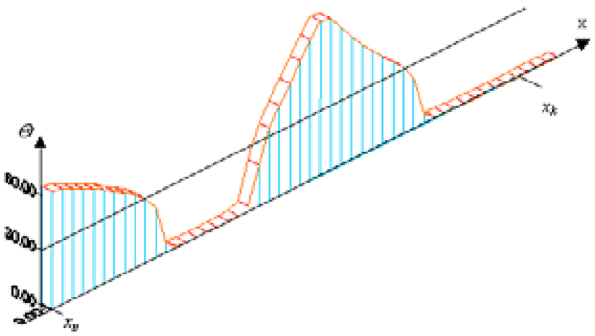

Figure 3. Example distribution of cosines of the angle at which position lines intersect for the system's configuration that does not ensure that the angle's values will fall within the required range along the entire route

This will allow one to avoid a situation in which the area under the curve would have the maximum value, but it would not be possible to use the system along the entire route. Only a pair of coordinates that meets the following conditions should be selected for further optimisation analysis (Figure 4).

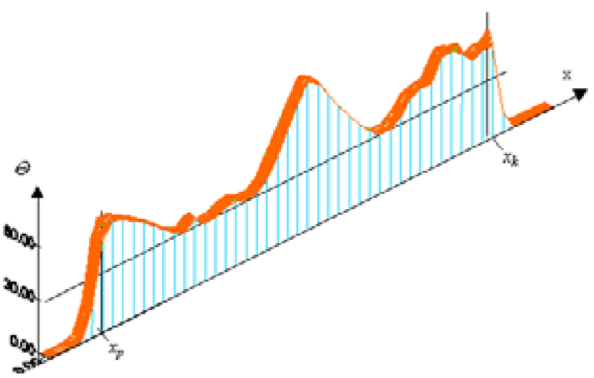

Figure 4. Example distribution of cosines of the angle at which position lines intersect for the system's configuration that can be used in further analysis

a) The angle's value is 030° ≤ Θ ≤ 150° along the entire route;
b) The value of the area under the curve is the largest of the areas for all pairs of coordinates that satisfy condition a).

As for the mean error of a position fix, one should search for the minimum value of the area under the curve representing the function along the given route without restrictions (Figure 5).

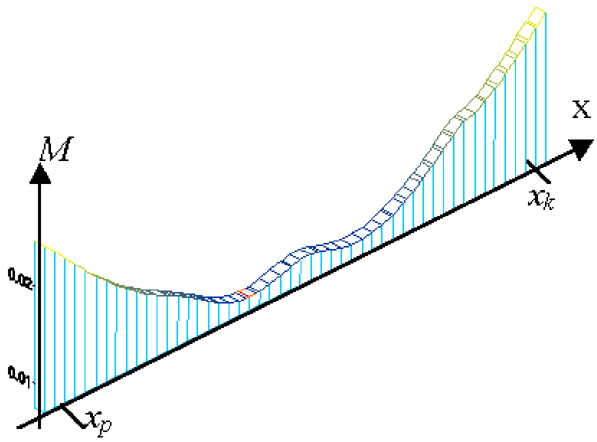

Figure 5. Example distribution of the mean error of a position fix on the fairway. The coordinates of the stations' positions are presented in Table 1, whereas the coordinates of the fairways are described in the captions of Figures 1 and 2

This ensures that the smallest possible value of the mean error of a position fix will be obtained along the entire route followed by a vessel.

Optimal coordinates of a single ground-based chain for a vessel moving in restricted waters

Just as for a fairway, it is also important that the values of the criteria for the entire sea area (i.e. in a two-dimensional system) be within acceptable ranges that have been adopted for the purpose of carrying out a given navigational task.

In order to find the optimal coordinates of the locations at which the stations are to be positioned with respect to the mean error of a position fix, or the angle at which position lines intersect, one should also introduce an auxiliary parameter. For a sea area that is limited by coordinates, the volume of the solid that has formed under the curve of the function that describes a given optimisation criterion will be an auxiliary parameter (Figures 6 and 7).

The solid that has formed (the sections in the middle of Figures 6 and 7) is limited by the water surface, the coordinates of the sea area and the area

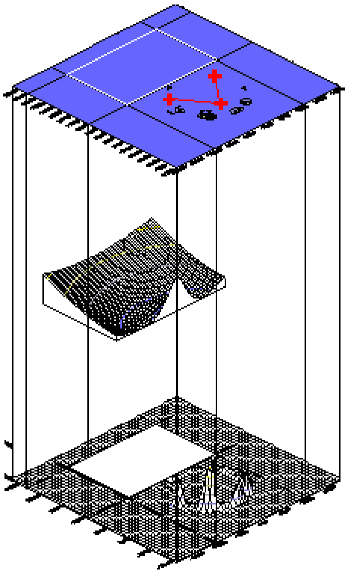

Figure 6. Example distribution of the mean error for the given sea area

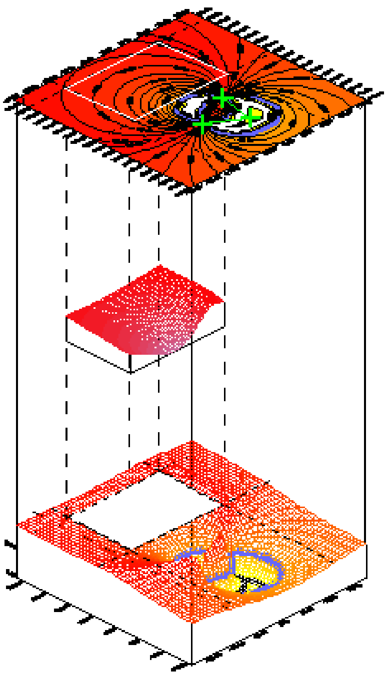

Figure 7. Example distribution of the angle of intersection between position lines for the given sea area

of the optimisation parameter's distribution. This solid's volume can also be expressed by using a definite integral, but this time it will be a double integral:

$$V = \int_{x_p}^{x_k} \int_{y_p}^{y_k} \Psi \,\mathrm{d}\,x\,\mathrm{d}\,y \qquad (4)$$

where:

V – volume of the solid under the curve representing function f;

Ψ – function describing a given optimisation criterion;

(x_p, y_p), (x_k, y_k) – boundary coordinates of the area in which a given vessel is moving.

The volumes that have been obtained based on formula (4) should be subjected to optimisation analysis, as is the case when considering the areas under the curve representing the function for the fairway.

Using the integral criterion when solving optimisation problems

Optimisation analyses were carried out in accordance with the assumptions made in this paper. Sets of acceptable solutions for each of the system's stations were determined by establishing the acceptable areas in which these stations could be positioned, depending on the region in which vessels would operate, as well as with respect to the system's base angle. The boundaries of the areas in which the stations are to be positioned were determined based on Polish nautical charts Nos. 251 and 252, and these boundaries were transformed into a rectangular coordinate system in order to simplify the calculations.

After determining the acceptable areas within which the stations can be positioned, calculations were performed by using auxiliary parameters for a three-element ratio-based navigation system, as well as by employing computer programs that facilitated these calculations. The location of each of the system's stations, for each of the system's analysed chains, was optimised for a vessel to improve the accuracy of its position in system's zone of action. In order to standardise the intervals between the positions of these stations on land and those of the vessel at sea, it was determined that:

- positions had been established at a 1000-metre interval while optimising the stations' coordinates for a given sea area;
- vessel's positions at sea had been established at a 500-metre interval while optimising the stations' coordinates for a given section of a fairway;
- subsequent positions of the stations had been established at a 500-metre interval.

The Tables 2–5 and Figures 8–11 present selected results of optimisation analyses which were carried out with respect to:

- angle of intersection between position lines;
- mean error of a position fix.

These tables contain the coordinates of the locations at which a ratio-based navigation system's stations are to be positioned for two types of coordinate systems:

- rectangular coordinates (calculations);
- geographic coordinates (the graphical representation of the results).

Table 2. Optimal coordinates of the positions of a quotient navigation system's stations for a base angle of 180° with respect to the angle of intersection between position lines for sea area No. 6A

Item No.	Station	Geographic coordinates		Rectangular coordinates	
		φ	λ	X	Y
1	Station A	54°27.7' N	016°25.7' E	6037438.0	462963.0
2	Station B	54°32.0' N	016°47.0' E	6045283.4	485996.7
3	Station C	54°40.8' N	017°03.9' E	6061560.3	504259.5

Table 3. Optimal coordinates of the positions of a ratio-based navigation system's stations for a base angle of 120° with respect to the angle of intersection between position lines for the coastal section of the Świnoujście–Gdynia fairway

Item No.	Station	Geographic coordinates		Rectangular coordinates	
		φ	λ	X	Y
1	Station A	54°14.3' N	015°56.6' E	6013044.2	431126.3
2	Station B	54°16.0' N	016°21.1' E	6015755.8	457803.3
3	Station C	54°30.3' N	016°27.9' E	6042202.0	465403.7

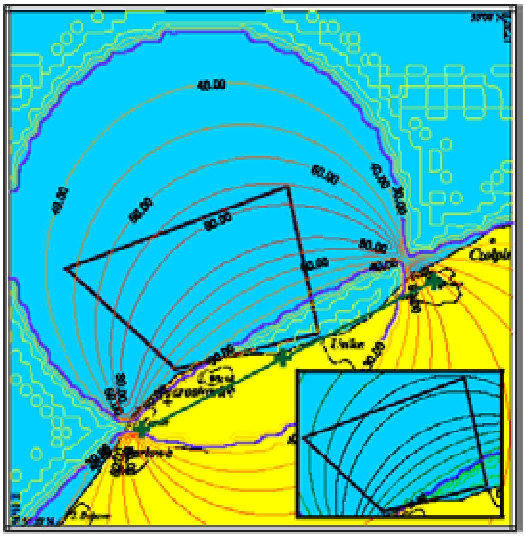

Figure 8. Distribution of the values of the angle at which position lines intersect for a ratio-based navigation system with a base angle of 180° in the southern part of the Baltic Sea for sea area No. 6A

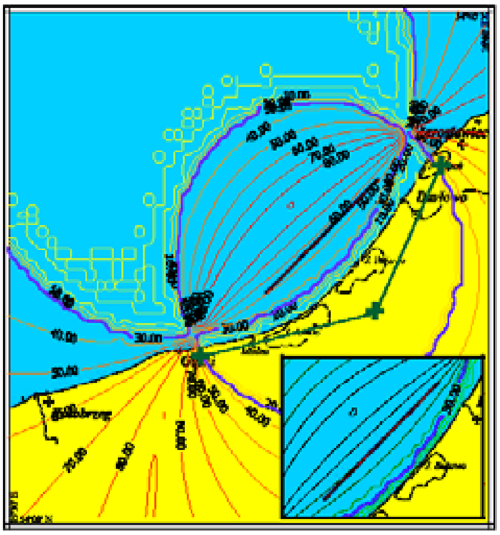

Figure 9. Distribution of the values of the angle at which position lines intersect for a ratio-based navigation system with a base angle of 120° in the southern part of the Baltic Sea for the coastal section of the Świnoujście–Gdynia fairway

Table 4. Optimal coordinates of the positions of a quotient navigation system's stations for a base angle of 180° with respect to the mean error of a position fix for sea area No. 6A

Item No.	Station	Geographic coordinates		Rectangular coordinates		Auxiliary parameter [m^3]
		φ	λ	X	Y	
1	Station A	54°27.7' N	016°26.2' E	6037447.0	463468.0	7068885.7
2	Station B	54°31.9' N	016°47.1' E	6045283.4	486096.3	3651521.9
3	Station C	54°39.7' N	017°04.0' E	6059660.6	504259.5	3524149.9

Table 5. Optimal coordinates of the positions of a ratio-based navigation system's stations for a base angle of 120° with respect to the mean error of a position fix for the coastal section of the Świnoujście–Gdynia fairway

Item No.	Station	Geographic coordinates		Rectangular coordinates		Auxiliary parameter [m^3]
		φ	λ	X	Y	
1	Station A	54°14.8' N	015°56.6' E	6014044.0	431126.2	128180.5
2	Station B	54°16.0' N	016°21.1' E	6015755.8	457803.3	103809.3
3	Station C	54°30.3' N	016°27.9' E	6042201.9	465403.7	887315.2

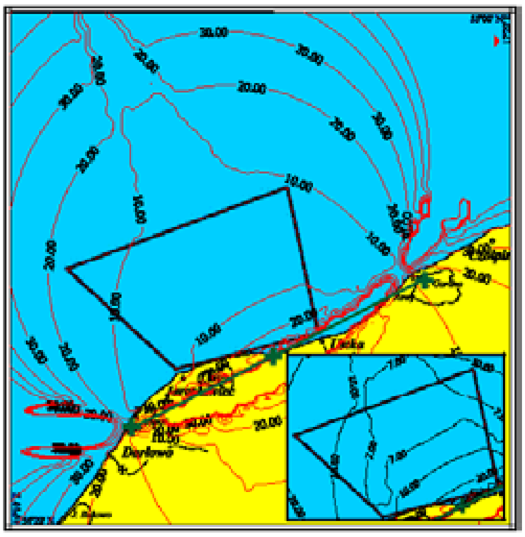

Figure 10. Distribution of the values of the mean error of a position fix for a ratio-based navigation system with a base angle of 180° in the southern part of the Baltic Sea for sea area No. 6A

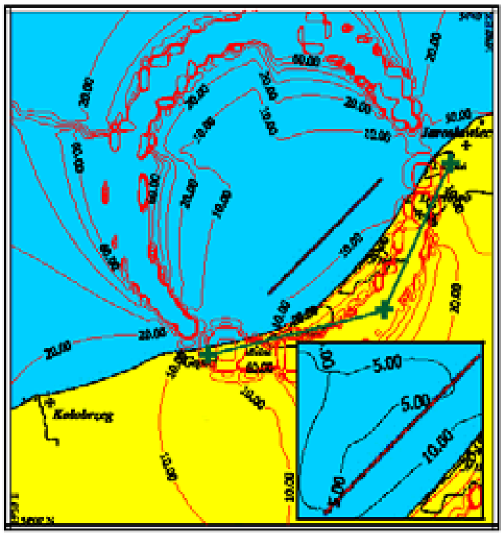

Figure 11. Distribution of the mean error of a position fix for a ratio-based navigation system with a base angle of 120° in the southern part of the Baltic Sea for the coastal section of the Świnoujście–Gdynia fairway

The sea areas for which the optimisation was performed are presented in the bottom corners of the images. In the figures, the angle at which position lines intersect is presented at an interval of 10°, and the mean error of a position fix at an interval of 10 m.

Conclusions

Introducing auxiliary parameters when optimising the coordinates of the positions of a navigation system's stations (the area under the curve representing a function or the volume of the solid under the curve of a function) allows one to determine the optimal coordinates of the locations, at which these stations are to be positioned, for a given set of acceptable solutions in an unambiguous manner and based on the interval that has been adopted for calculations.

The method for determining the location of a ratio-based navigation system's chain can be successfully applied to navigation systems that use different position lines. Then, however, one should carry out the appropriate theoretical, adaptive analyses that would be relevant to the specific observations under discussion.

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