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MATRIX REPRESENTATION AND THE ANALYSIS OF SHAPES OF ROOFS

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Abstract. The paper is the supplement of a series of articles devoted to geometry of roofs. Regular roofs generated by *k*-connected generalized polygon can treated as geometrical configurations in the form $((2v+2(k–2))_3, (3v+3(k–2))_2)$ and described by means incidence or adjacence matrices. After all, such representation results from the natural graph-theoretical characterization of roofs described in previous sections. So, a regular roof can be described as an incidence matrix mutually related to vertices ↔ edges, and as adjacency matrix mutually related to vertices ↔ hipped roof ends (in graph-theoretical interpretation for planar graphs: vertices ↔ regions). In order to built all topological types of roofs every case of the adjacency matrix satisfying the condition (10) has to be studied. Adjacency matrices are already rare matrices for v=6. Therefore such a combinatorical way should be too complicated to be used here. The way leading through algebraic-geometrical analysis proposed in papers [5,6] seems to be more familiar and simple. In paper [6] the analysis of the existence of topological types of roofs only for v=8 has been made. Here we complete the analysis for the remaining numbers v=3,4,5,6,7 of sides of the base of investigated roofs.

Key Words: geometry of roofs, planar graph, Euler theorem for roofs, equations of roof, straight skeleton

1. Introduction

This paper is the supplement of cycle of articles: published [5,6,7] and prepared, devoted to geometry of roofs. Geometry of roofs remains in the area of the research concerned with the so-called straight skeletons (cf. [1, 2]).

Roofs considered in this article are defined as geometric polyhedral surfaces on the basis of two assumptions:

1° all eaves of a roof form a planar (simple connected or *k*-connected) polygon called *a base of a roof*,

2° every hipped roof end makes the same angle with the horizontal plane which contains the base (cf. [4,9]).

Then every roof, and equivalently the orthographic projection of this roof onto a plane, is uniquely generated by its base. Namely, each ridge of a roof can be obtained as a line segment of the bisectrix of the angle formed by two appropriate edges of the base; if these edges are parallel, then a ridge is the axis of the symmetry (mirror line) of such edges. Disregarding the metric properties of a roof, we can treat them as planar graphs. Usually, i.e. if the vertices of the base of a given roof are in the general position, there are 3-regular graphs. For such graphs (with a simply-connected or *k*-connected base of the roof) we formulated and proved new Euler's formula (Euler's formula for regular roofs), and so-called the equations of a roof (cf. [5]) and other important properties (cf. [6]).

The results of the presented articles may be interesting from many points of view:
- a physically plausible engineering examples of 3-regular graphs,
- a new example of Euler's formula for special polyhedral surfaces,
- computational geometry of special polyhedral surfaces,
- computer graphics in design and a visualization of roofs of buildings
- and consequently, computer aided architectural design.

The geometry of roofs is also an interesting example of an application of Descriptive Geometry methods in the complete geometric design of a roof (shape of a roof, dihedral angle between two adjacent hipped roof ends, a true size of a hipped roof end).

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2. Geometric determination of a roof from the view point of Graph Theory

In this article we use the notation and terminology from [5,6]. In [5] we considered the generalized polygon $\mathbf{P}(\mathbf{C}_1,\mathbf{C}_2,...,\mathbf{C}_k)$ as the plan projection of a certain *base of a roof* R($\mathbf{P}$) (see Fig. 1. with k=1, v=8). We denoted by $V_1,V_2,...,V_v$ the *corner vertices* of the roof R($\mathbf{P}$), by $e_1,e_2, ...,e_v$ *the edges* (*eaves*) ($e_i=V_{i-1}V_i$, $e_1=V_vV_1$, i=2,3,...,v) of the roof R($\mathbf{P}$). The base $\mathbf{P}$ induce the set T=$\{T_1,T_2,...,T_t\}$ of *top points*. We additionally distinguished four sets:

- V=$\{V_1,V_2,...,V_v\}$ is the set of *vertices* of the base of the roof,
- E=$\{e_1,e_2, ...,e_v\}$ is the set of *edges* of the base of the roof,
- R=$\{r_1,r_2,...,r_r,r_{v+1},r_{v+2},...,r_{r+d}\}$ is the set of *ridges* of the roof $r_i=V_iT_j$ (for i=1,2,...,v and appropriate j),
- R'=$\{r_{v+1},r_{v+2},...,r_{r+d}\}$ is the set of *disappearing ridges* ($r_{v+i}=T_lT_j$ (for i=1,2,...,d and appropriate l,j).

The generalized polygon $\mathbf{P}(\mathbf{C}_1,\mathbf{C}_2,...,\mathbf{C}_k)$ determines the *basic graph* BG=(V,E) of the *contour of the roof.* It follows from the geometric properties of a roof that the graph BG=(V,E) induces the graph RG=(V∪T,E∪R) of the *projection of the skeleton* . We also distinguished the graph of *disappearing ridges* DRG=(T,R').

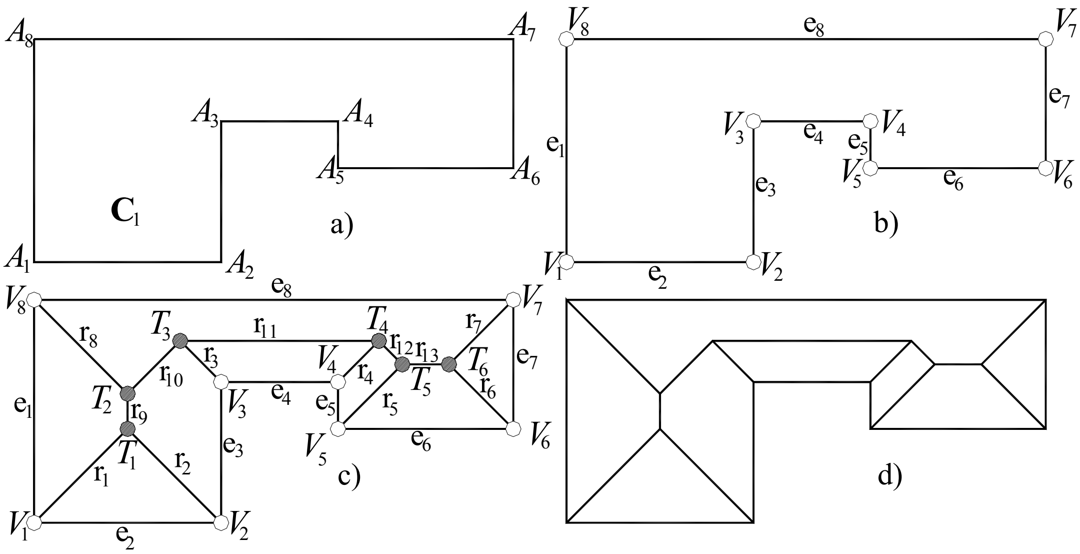

Fig.1: An illustration of the scheme of the description of a roof from Graph Theory view point: a) the generalized polygon $P(C_1)$ – the base of the roof R($\mathbf{P}$), b) the basic graph BG=(V,E) of the contour of this roof , c) the graph RG=(V∪T,E∪R) of the skeleton of the roof: $r_i= V_iT_j$ – corner roof ridges, $T_1T_2...T_6$ – a line of disappearing ridges, $r_9 =T_1T_2$, $r_{10} =T_1T_2$, $r_{11} =T_1T_2$, $r_{12} =T_1T_2$, $r_{13} =T_1T_2$ – the disappearing ridges, d) the projection of the skeleton of the roof R($\mathbf{P}$)

If we suppose that

1° at every top point of the skeleton of the roof there meet exactly three ridges of the roof (one or two corner ridges and one or two ridges of the roof) ,

2° v edges of the base of the roof induce exactly v polygons (hipped roof ends),

3° each hipped roof end contains exactly one eaves.

The above supposition is equivalent to the general position of the vertices of the roof in geometrical sense and the induced by $\mathbf{P}(\mathbf{C}_1,\mathbf{C}_2,...,\mathbf{C}_k)$ roof is called *regular.*

Now, let us recall the main results obtained in [5,6].

Theorem 1 (Euler formula for regular roofs**).** *If the base of a regular roof is k-connected generalized v-polygon, then the number r of the ridges of this roof is* $2v+3(k–2)$, *the number t of the top points of this roof is* $v+2(k–2)$ *and the number d of the disappearing ridges of this roof is* $v+3(k–2)$.

The equations of roofs which belong to the class of regular roofs are:

$$m_3 + m_4 + ... + m_v = v, \tag{1}$$

$$3m_3 + 4m_4 + ... + vm_v = 5v+6(k-2) \tag{2}$$

where m_i denotes the number of i-gonal hipped roof ends for i=3,4,...,v.

Proposition 1. *If a roof generated by k-connected generalized v-polygon is regular, then the graph* (T,R') *of the line of disappearing ridges induced by this roof is connected.*

Statement 1. *If two triangles as hipped roof ends adjoin in a regular, simply connected roof, then the base of this roof must be a triangle* (v=3).

Statement 2. *If in any regular, simply connected roof there exists a quadrangle as hipped roof ends which adjoin with two triangles as hipped roof ends, then the base of this roof must be a quadrangle* (v=4).

Corollary 1. *The graph* (T, R') *of the line of disappearing ridges of a regular simply onnected roof is connected.*

Corollary 2. *The graph of the line disappearing ridges* (T, R') *of arbitrary regular simply connected roof is a tree and has at least two leaves.*

Corollary 3. *Every regular simply connected roof contains at least two triangular hipped roof ends.*

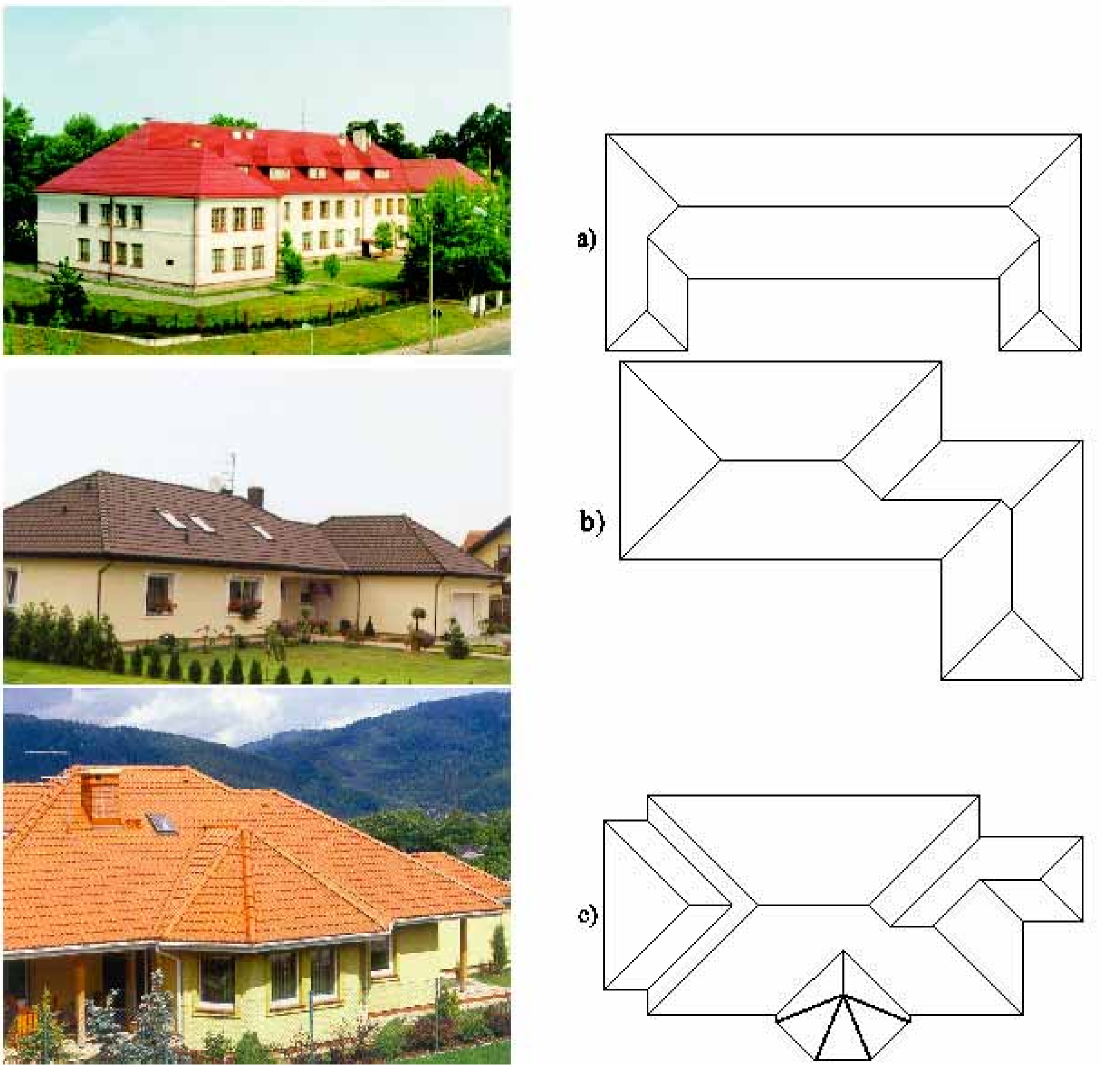

Fig. 2: Different shapes of roofs together with the roof skeleton: a) the building of the secondary school in Augustow in Poland (built in 1922) with a regular roof , b) a detached house with a regular roof, c) a detached house with an irregular roof

Theorem 2. *If a regular simply connected roof generated by a v-gon has* s_i $v-n-i+1$ *– gonal hipped roof ends for* $i=1,2,...,p$, *with* $n\geq 0$, $s_i>0$, $n+p+2\leq v$, $1<\sum_{i=1}^{p} s_i \leq v$, *then under the assumption that certain i-gonal hipped roof ends may have a c disappearing ridge*

$$v \leq n+3+\frac{n+\sum_{j=3}^{p}\sum_{i=2}^{j-1} s_j s_i+(s_1-1)\sum_{i=2}^{p} s_i+\sum_{i=2}^{p}\frac{s_i+1}{2}s_i+\frac{s_1(s_1-1)}{2}+\sum_{i=1}^{p} s_i(i-1)}{\sum_{i=1}^{p} s_i-1}, \tag{3}$$

and, under the assumption that every two i-gonal hipped roof ends not share any disappearing ridge

$$v \leq n+3+\frac{n+\sum_{i=1}^{p} s_i(i-1)}{\sum_{i=1}^{p} s_i-1}. \tag{4}$$

And as an immediate consequence of Theorem 2 the following corollaries.

Corollary 4. *If a regular simply connected roof generated by a v-gon has s v-n-gonal hipped roof ends with* $n+3\leq v$ *and* $1<s\leq v$, *then under the assumption that these hipped roof ends may have a common disappearing ridge, the inequality*

$$v \leq n+3+\frac{s}{2}+\frac{n}{s-1}, \tag{5}$$

holds, and under the assumption that every two of these hipped roof ends not share any disappearing ridge, the following inequality holds

$$v \leq n+3+\frac{n}{s-1}. \tag{6}$$

Corollary 5. *If a regular simply connected generated by a v-gon roof has two v-gonal hipped roof ends, then* $v\leq 4$.

Corollary 6. *If a regular simply connected generated by a v-gon roof has three v-gonal hipped roof ends, then* $v\leq 3$.

And the property in which the strong sequence of the number of sides of polygons is omited:

Theorem 3. *If in a regular simply connected roof generated by a v-gon there are s* m_i *-gonal hipped roof ends* ($m_i>3$ *for* $i=1,2,...,s$) *such that no two of which have a common disappearing ridge, then*

$$v \geq \sum_{i=1}^{s} m_i - 3s + 3. \tag{7}$$

Corollary 7. *If a regular simply connected roof generated by a v-gon has s m-gonal hipped roof ends* ($m>3$) *no two of which have a common disappearing ridge, then*

$$v \geq s(m-3)+3. \tag{8}$$

3. Matrix representation of regular roofs

We can treat the regular roofs generated by k-connected generalized polygon as geometrical configurations in the form $((2v+2(k-2))_3, (3v+3(k-2))_2)$ and describe it by means of incidence or adjacence matrices (cf. [8]). After all, such represention results from the natural graph-theoretical characterization of roofs described in previous sections. So, a regular roof can be described as an incidence matrix mutually related to vertices ↔ edges (Fig. 3a”), and as adjacence matrix mutually related to vertices ↔ hipped roof ends (in graph-theoretical interpretation for planar graphs: vertices ↔ regions) (Fig. 3a’’’).

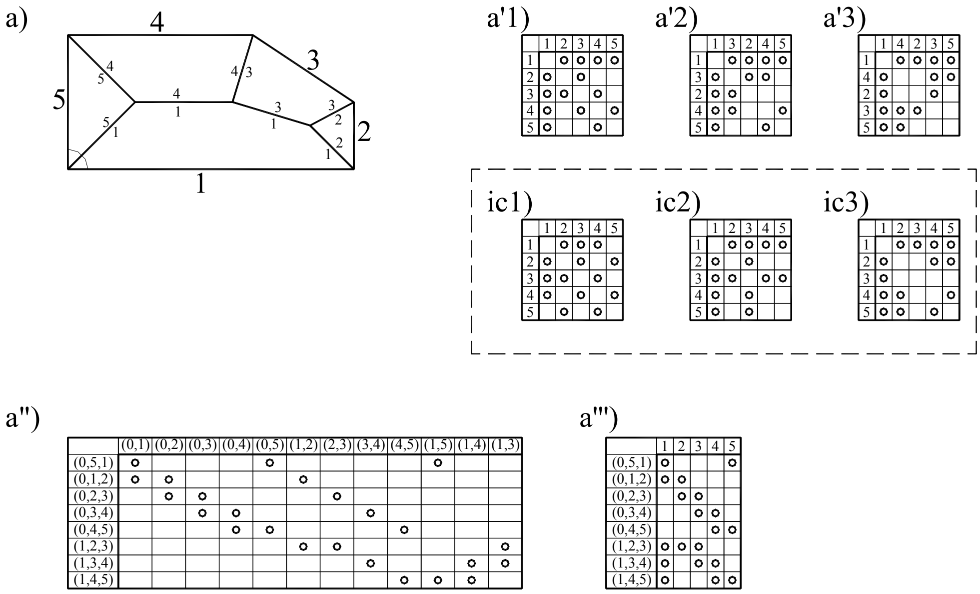

Fig. 3: The roof skeleton generated by pentagon: a’1) its adjacency matrix; a’2÷a’3) adiacency matrices others roofs over pentagons, mutually izomorphic; ic1÷ic3) matrices of no existing roofs (impossible cases); a”) incidence matrix of the roof as the image of the geometric configuartion (in relation edges ↔ vertices); a’’’) incidence matrix of the roof in relation: hipped roof ends ↔ vertices

Let’s mark rows and columns of a matrix $v\times v$ with numbers $1,2,...,v$ and treat these numbers as the indices of hipped roof ends, then we can define an adjacency matrix of the roof generated by any v-gon. We define the adjacency matrix $[a_{ij}]_{0\le i\le v,\ 0\le j\le v}$ in the following simple manner:

$a_{ij}=1$ if and only if the hipped roof ends i and j have a common ridge (only one) and $a_{ij}=0$ if and only if the hipped roof ends i and j have not any common ridge. We assume that $a_{ii}=0$. Obviously

$$[a_{ij}] = [a_{ji}] \text{ for } 0\le i\le v,\ 0\le j\le v. \tag{9}$$

Then, due to (1), (2) the matrix $[a_{ij}]_{0\le i\le v,\ 0\le j\le v}$ has $2r = 4v + 6(k-2))$ of ones and $(v^2-v)-2(2v+3(k-2)) = v^2-5v-6(k-2)$ zeros.

Therefore we have

$$\sum_{i=1}^{v}\sum_{j=1}^{v} a_{ij} = 4v+6(k-2)). \tag{10}$$

The number $\sum a_{ij} = w_i$ denotes that i-th hipped roof end has w_i+1 sides. Similarly, the number $\sum a_{ij} = u_j$ denotes that j-th hipped roof end has u_j+1 sides. By (9) we have $w_i = u_j$ for $i=j$, $0\le i\le v$, $0\le j\le v$.

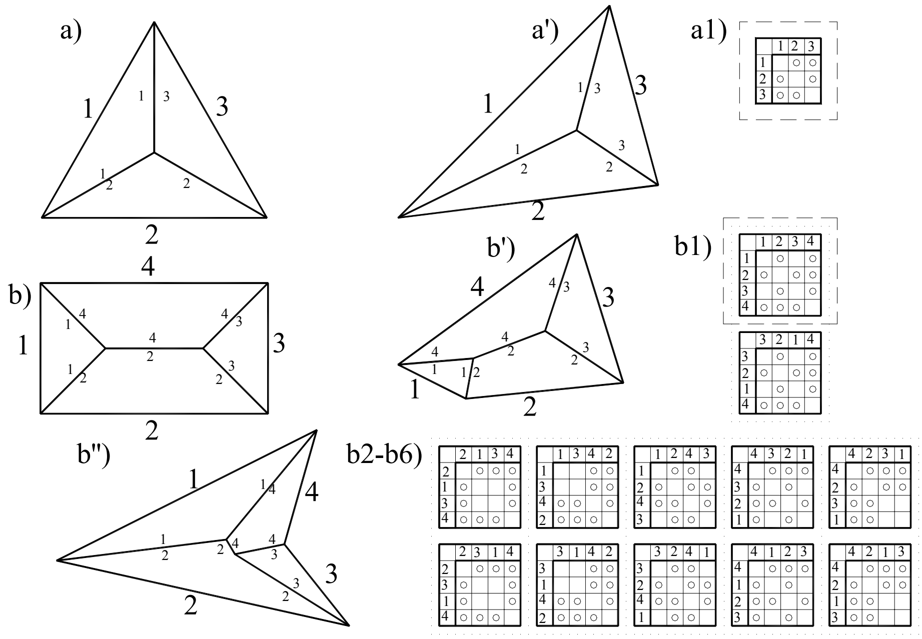

Fig. 4: The roof skeleton generated by triangle and quadrangle and its adjacency matrix: a1) a unique adjacency matrix of a roof over a triangle; b1÷b6) all possible adiacency matrices of a roof over quadrangle

Hence for k-connected v-gon there exist $\binom{\frac{v(v-1)}{2}}{2v+3(k-2)}$ systems, of which some only induce matrices describing roofs. In order to build all topological types of roofs every case of the adjacency matrix satisfying the condition (10) should be studied. Notice that adjacency matrices are already rare matrices for v=6. Therefore such a combinatorical way should be too complicated to be used here. Moreover, the drawings of investigated roofs have to be made. The way leading through algebraic-geometrical analysis proposed in papers [5,6] seems to be more familiar and simple. In paper [6] the analysis of the existence of topological types of roofs only for v=8 has been made. Here we complete the analysis for the remaining numbers v=3,4,5,6,7 of sides of the base of investigated roofs.

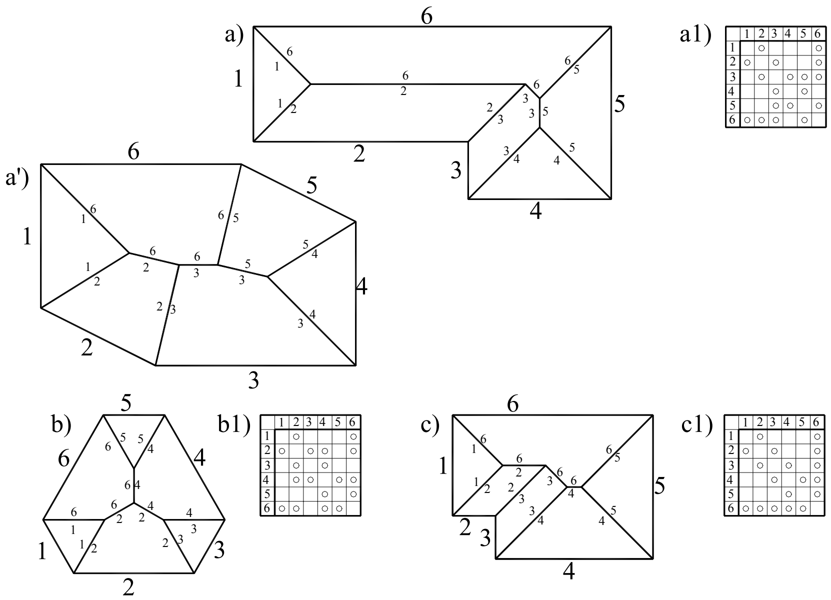

Fig. 5: The different roof skeletons over hexagons and its adjacency matrices

4. Algebraic and geometrical analysis of the equations of roof

In this section we will consider roofs generated by simply connected polygons in cases which the analysis has been omited in [6]. So we put k=1. Then the equations (2), (1) assume the following form

$$3m_3 + 4m_4 + \ldots + vm_v = 5v - 6. \tag{11}$$

$$m_3 + m_4 + \ldots + m_v = v. \tag{12}$$

4.1. Regular roofs for *v*=3,4,5,6,7

We will use now the properties presented above of roofs in an examination of shapes of roofs. We adopt the following convention: if a roof has n m-gonal hipped roof ends then we will write $m(n)$. In order to study the shapes of roofs we will examine the equations of a roof (1), (2) for k=1, i.e. the equations (11), (12). Let us consider the following cases:

C3) Let v=3.

For v=3 the equations (11), (12) assume the form

$$m_3 = 3,$$
$$3\,m_3 = 5\cdot 3 - 6.$$

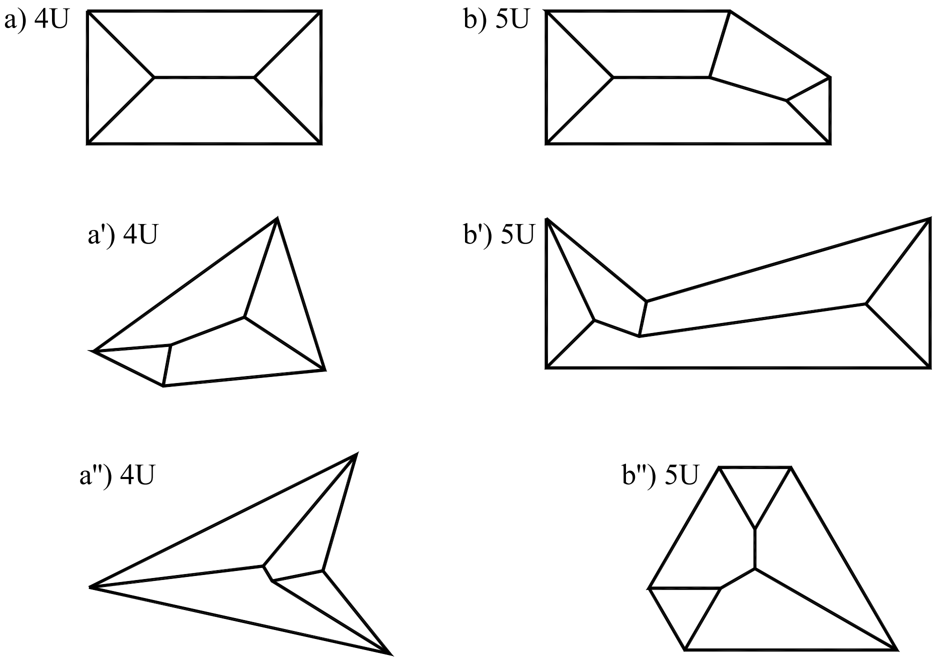

Fig. 6: Roofs generated by a quadrangle and a pentagon: a), a'), a") seemingly different realizations of roofs determined by a quadrangle; b), b'), b") seemingly different realizations of roofs determined by a pentagon

We have one solution m_3=3. Then there exists only one topological type (3U) of a roof generated by a triangle.

C4) Let v=4.

For v=4 the equations (11), (12) assume the form

$$m_3 + m_4 = 4,$$
$$3m_3 + 4m_4 = 14.$$

We have one solution m_3=2, m_4=2. Then there exists only one topological type (4U) of a roof generated by a quadrangle. Seemingly different realizations of such roofs are displayed in Fig. 6a,a',a".

C5) Let v=5.

For v=5 the equations (11), (12) assume the form

$$m_3 + m_4 + m_5 = 5,$$
$$3m_3 + 4m_4 + 5m_5 = 19.$$

Consider the subcases:

C51) Let us assume m_5=2. We have the solution 5(2), 4(0), 3(3) , i.e. there are two pentagons, zero quadrangles, three triangles. Due to Corollary 5 this case is geometrically impossible, however it is algebraically correct.

C52) For m_5=1 we have one solution 5(1), 4(2), 3(2). Notice that it is a solution of the type 5U (cf. [6]). Then we have again one solution only. Then there exists only one topological type of a roof generated by a pentagon. Seemingly different realizations of such roofs are displayed in Fig. 3b,b',b''. In reality it is one universal type 5U.

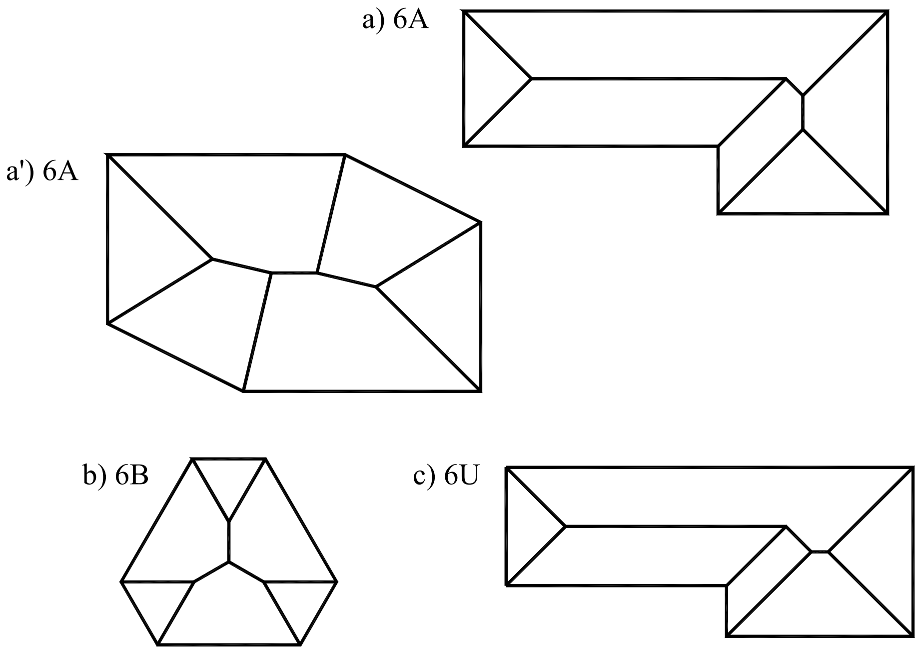

Fig. 7: Different topological kinds of roofs generated by hexagon: a), a') seemingly different realizations of roofs determined by a hexagon

C6) Let v=6.
For v=6 the equations (11), (12) assume the form

$$m_3 + m_4 + m_5 + m_6 = 6,$$
$$3m_3 + 4m_4 + 5m_5 + 6m_6 = 24.$$

Consider the subcases:

C61) For m_6=2 we have the solution 6(2), 5(0), 4(0), 3(4) which due to Corollary 5 is geometrically impossible.

C62) Let m_6==1, then must be

C621) 6(1), 5(1), 4(1), 3(3). This case is geometrically impossible, because due to Statement 1 and Statement 2 a quadrangle must be adjacent with two triangles, which leads to the inequality v < 5. Notice that due to uniqueness of the solution (2}) this case could be omitted.

C622) 6(1), 5(0), 4(3), 3(2). This solution is geometrically correct. The geometric realization of such a topological universal type is displayed in Fig. 6c. Notice that it is a solution of the type 6U.

C63) Let m_6=0, then must be

C631) 6(0), 5(2), 4(2), 3(2). This solution is geometrically correct. In Fig. 6b the geometric realization of such a type of a roof is displayed.

C632) 6(0), 5(3), 4(0), 3(3). We obtain again the geometrically correct solution. In Fig. 6a,a' the geometric realization of such a type of a roof is displayed.

C7) Let v=7.

For v=7 the equations (11), (12) assume the form

$$m_3 + m_4 + m_5 + m_6 + m_7 = 7,$$
$$3m_3 + 4m_4 + 5m_5 + 6m_6 + 7m_7 = 29.$$

Consider the subcases:

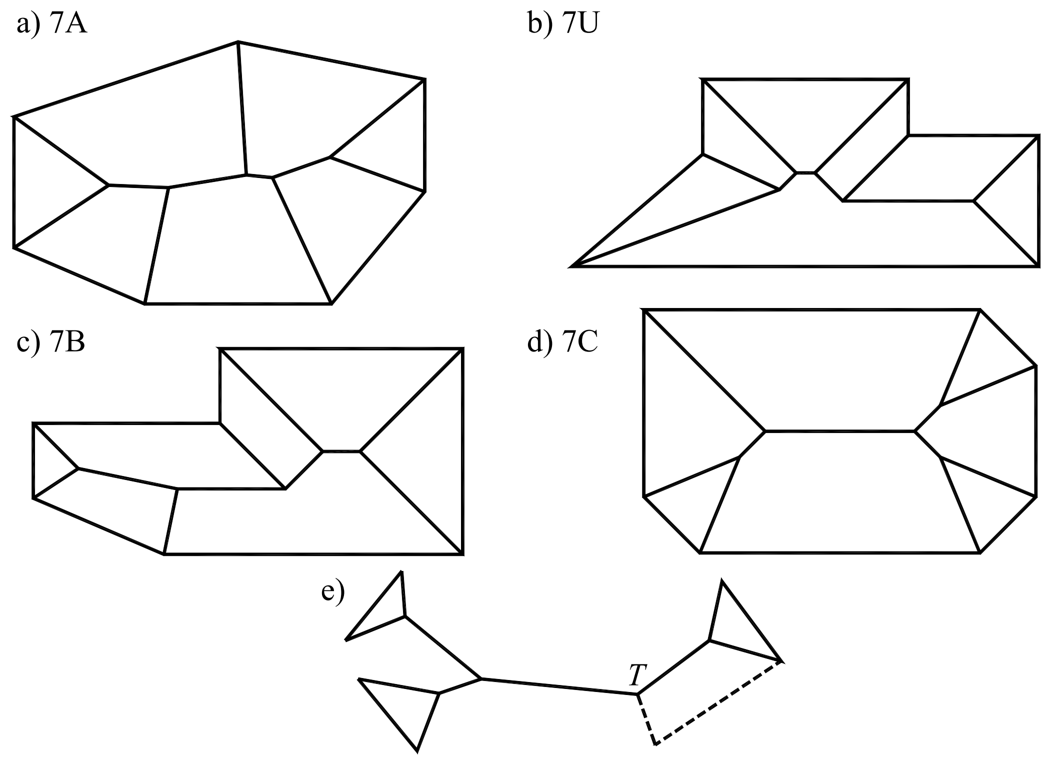

Fig. 8. Different topological kinds of roofs determined by 7-gon and: e) a sketch of the analysis of some roofs for the case C725

C71) For m_7=1 due to (2) we have the only one geometrically correct solution 7(1), 6(0), 5(0), 4(4), 3(2). It is an universal type 7U of a roof (see Fig. 8b).

C72) Let us assume m_7=0.

Consider the subcases:

C721) m_6=1. Notice that the case m_6=s with s>1 due to Corollary 7 is impossible. It suffices to assume v=7, m=6, s>1 in (8). However e.g. the system 7(0), 6(2), 5(1), 4(0), 3(4) is a solution of (11), (12) for v=7.

We have subsubcases:

C7211) Let m_6=2. Then we have the geometrically correct solution 7(0), 6(1), 5(2), 4(1), 3(3). Due to Statement 2 two triangles may not be adjacent with a quadrangle. Similarly, nor quadrangle nor pentagon may not be adjacent with hexagon. Such a case leads to an unicoursality of a line of disappearing ridges which is impossible, because the roof contains three triangles. Therefore it is necessary such a sequence of hipped roof ends: 6,3,5,3,5,4,3.

In Fig. 7d the geometric realization of such a type of a roof is displayed.

C7212) m_5=1. Then we have 7(0), 6(1), 5(1), 4(3), 3(2). The roof has exactly two triangles. Then the line of disappearing ridges must be unicoursal and due to Theorem 1 its length p must be v-3. Then a hexagon and a quadrangle must be adjacent. Notice that a subsequence of hipped roof ends: a quadrangle, a triangle, a quadrangle is impossible. Such a configuration leads to a roof generated by a quadrangle (v=4). Therefore the roof in this case must have a form displayed in Fig. 7c.

C722) m_6=0.

Consider subsubcases:

C7221) m_5=3. Then we have a solution 7(0), 6(0), 5(3), 4(2), 3(2) (see Fig. 7a).

C7222) m_5=s for s>3. The cases s=5,6,7 are immediate impossible.

The remaining case is s=4. Then we have a solution 7(0), 6(0), 5(4), 4(0), 3(3). Hence due to Corollary 3 the line of disappearing ridges must have three leaves and four components (see Fig. 8a). From the point T may not start a corner ridge (see Fig. 8a). Indeed, if from point T started a corner ridge, then the point T would be a vertex of any quadrangle, contrary to the considered solution. Then such a roof do not exist. All cases have been exhausted.

The detailed analysis of shapes of roofs generated by octagon has been made in [6]. The description of the shapes of roofs for arbitrary number v (v≥9) will be possible in the following papers devoted to roofs, where the analysis by means of decomposition is realized.

In Table 1 we list the above results.

Table 1.

Code	Base of the roof	Number of v-gons					Kind of solution	
		3	4	5	6	7	algebraic	geometrical
T01	triangle	3					+	+ (uniwersal roof)
Q01	quadrangle	2	2				+	+ (uniwersal roof 4U, Figs. 6aa'a")
P01[n)]	pentagon	3	0	2			+	– (Statement1)
P02[n)]		1	4	0			+	– (Corollary 3)
P03		2	2	1			+	+(uniwersal roof 5U, Figs. 6bb'b")
H01[n)]	hexagon	4	0	0	2		+	– (Statement 1)
H02[n)]		3	1	1	1		+	– (Statement 2)
H03		2	3	0	1		+	+(uniwersal roof 6U, Fig. 7c)
H04		2	2	2	0		+	+(6A, Fig. 7aa')
H05		3	0	3	0		+	+(6B, Fig. 7b)
S01	7-gon	2	4	0	0	1	+	+ (uniwersal roof 7U, Fig. 8b)
S02		3	1	2	1	0	+	+ (7C, Fig. 8d)
S03		2	3	1	1	0	+	+ (7B, Fig. 8c)
S04		2	2	3	0	0	+	+ (7A, Fig. 8a)
S05[n)]		3	2	0	2	0	+	– (Corollary 7)
S06[n)]		4	0	1	2	0	+	– (Corollary 1)
S07[n)]		3	0	4	0	0	+	– (Theorem 1, Fig. 8e)

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MACIERZOWA REPREZENTACJA A ANALIZA KSZTAŁTÓW DACHÓW

Praca stanowi uzupełnienie cyklu artykułów poświęconych geometrii dachów. Dachy regularne, generowane przez k-spójne wielokąty uogólnione mogą być traktowane jako konfiguracje geometryczne postaci $((2v+2(k-2))_3, (3v+3(k-2))_2)$ i opisywane za pomocą macierzy incydencji lub adjacencji. Reprezentacja taka wynika z naturalnej, grafowej, charakteryzacji dachów. Ale wówczas, w celu opisania wszystkich topologicznych typów, każdy przypadek macierzy adjacencji, spełniający opisany w pracy warunek, musiałby być rozpatrzony. Ponieważ macierze adjacencji są rzadkie (już dla v=6), ich analiza kombinatoryczna, w przypadkach v=6, 7, 8, wymagałaby rozpatrzenia bardzo dużej liczby przypadków. Pozostaje więc zdecydowanie prostsza droga algebraiczno-geometryczna oparta na własnościach grafów dachów. W artykule przeprowadzono analizę kształtów dachów dla v=3, 4, 5, 6, 7 uzupełniając tym samym treść cyklu pierwszych prac na ten temat.