OFFSETS IN GEOMETRIC CREATION OF ROOF SKELETONS WITH VARYING SLOPE AND CUT-AND-FILL PROBLEMS IN TOPOGRAPHIC PROJECTION

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Abstract: The article describes the construction and some properties of roof skeletons with varying slope. In particular, the author shows that for an arbitrarily fixed polygon we can obtain different shapes of roofs of varying slope depending on the sequence of real positive numbers (distances).

Keywords: offset construction, roof, varying slope, cut-and-fill problem, topographical projection

1 Offset planar curves and offset surfaces

For a smooth planar curve c, we define an *offset* curve c_d ($c'_d \cup c''_d$) at distance d (for simplicity often just called an *offset*) in the following way: on each curve normal, we mark the two points that are at distance d from the curve c. The set of all of these points forms the offset c_d ([3], 335). The curves c'_d, c''_d will be called half offsets (see Fig.1a). By varying the distance d, we can generate a family of offset curves (see Fig. 1b).

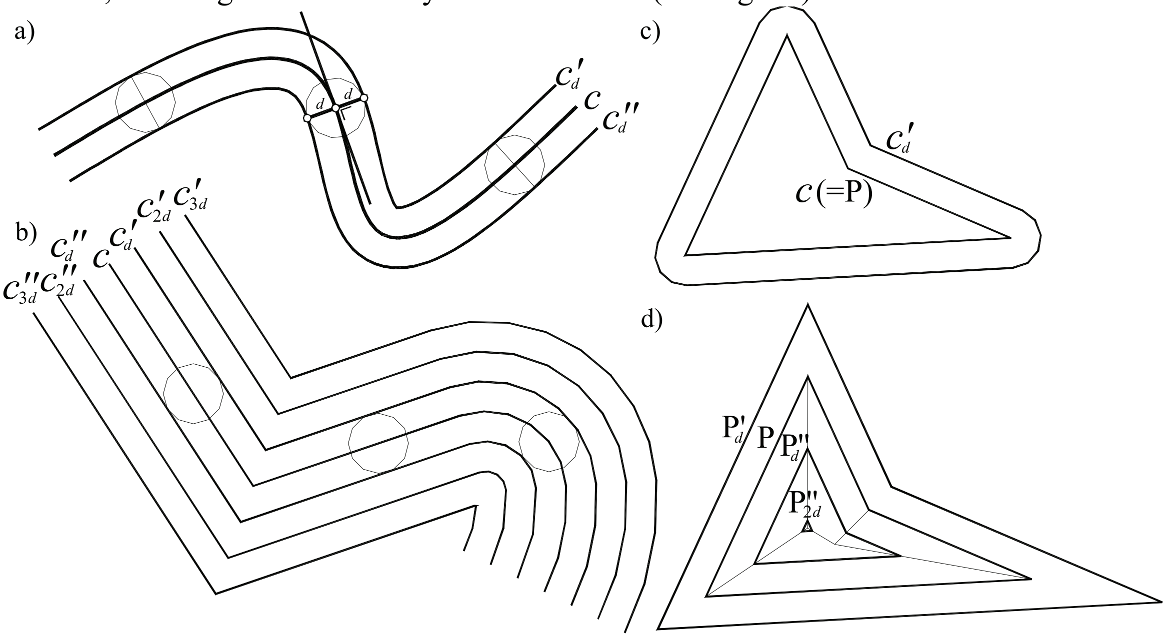

Figure 1 Offsets of: a) a spline; b) a pline; c) a half offset of a polygon (obtained in AutoCAD with variable value *Offsetgaptype*=1) d) a sequence of "half" discrete offsets of a polygon (obtained in AutoCAD with *Offsetgaptype*=0)

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AutoCAD system provides alternatives for creating the offset of a planar polygon. Instead of replacing a vertex of a polygon **P** with a circular arc (Fig. 1c), the offset will have a sharp corner (Fig. 1d). Thus, the offset $\mathbf{P}_d$, the so called *discrete offset of a polygon* **P** is again a polygon. Figure 1d shows the sequence of "half" discrete offsets $\mathbf{P'}_d$, $\mathbf{P''}_d$. It is obvious that the offsets of a straight line are parallel straight lines. Note that the offsets of a circle are concentric circles but the offsets of an ellipse are of a more general nature and are no longer ellipses (cf. Fig.2).

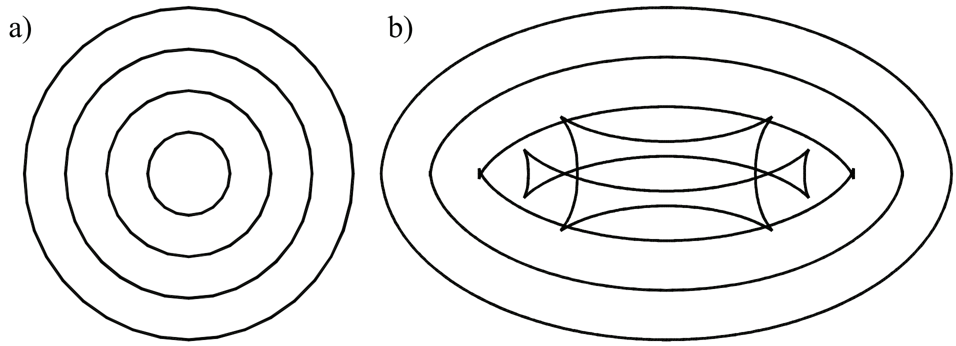

Figure 2: The offsets of (elementary figures): a) a circle are concentric circles but b) the offsets of an ellipse are of a more general nature and are no longer ellipses ([3])

Figure 3: Offsets of special surfaces used in architecture - the Jubilee Church by Richard Meier [5]

The definition of an *offset surface* is analogous to the definition of an offset curve. The offset surfaces of a sphere with radius r at distance d are concentric spheres of radius r+d (r-d) and consist of two concentric spheres of radius r+d and r-d. The offset surfaces of a rotational cylinder of radius r consist of two rotational cylinders of radius r+d and r-d with the same axis. The offsets of a torus are tori with the same axis and the same middle circle. In general, the offsets of rotational surfaces are again rotational surfaces.

Offsets of special surfaces are used in architectural design. Figure 3 shows parts of concentric spheres - the Jubilee Church by Richard Meier.

2 Designing roofs of varying slope

Let us return to "half" discrete offsets of a polygon. Let us first consider a simple geometric object - an angle. For an angle we can construct a discrete offset and define the so called (d_1,d_2)-bisectrix (cf. Fig.4e) in the following way. Let us explode any given angle into two half lines 1 and 2 and construct for the half line 1 the sequence of offset half lines with distance d_1, and for the half line 2 the sequence of offset lines with distance d_2 (cf. Fig.4e). The intersection points of pairs of appropriate half lines define the line which we will call (d_1,d_2)-*bisectrix*.

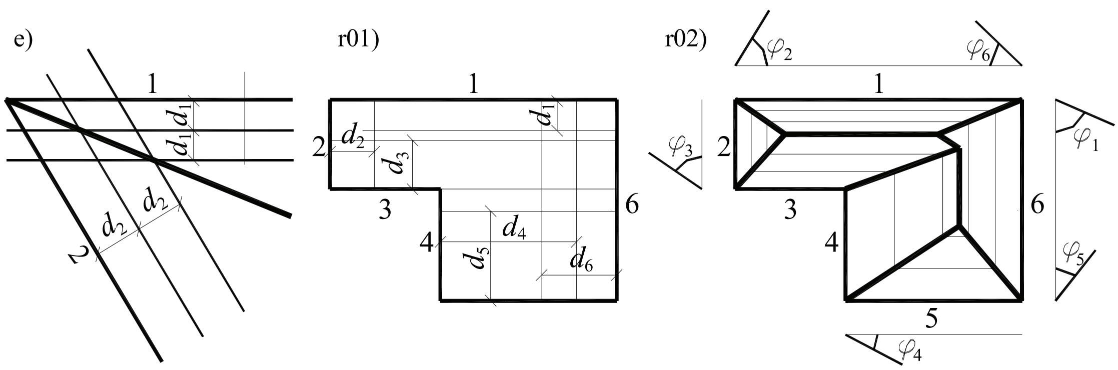

Figure 4: Generalized discrete offsets: e) of an angle with d_1,d_2 parameters; data d_1,d_2,d_3,d_4,d_5,d_6 for a discrete offset of the polygon; r02) a sequence of "half" discrete offsets of a polygon (1,d_1; 2,d_2; 3,d_3; 4,d_4; 5,d_5; 6,d_6)

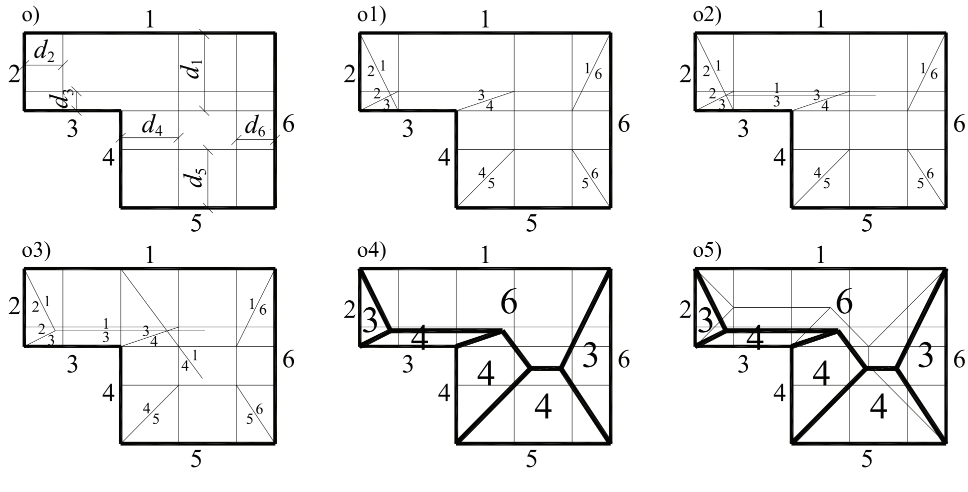

Figure 5: Roof design for roof planes of different slope generated by a hexagon (1,d_1; 2,d_2; 3,d_3; 4,d_4; 5,d_5; 6,d_6)

Similarly for a given n-gonal polygon $\mathbf{P}_n$ and a sequence of real positive numbers (distances) $d_1,d_2,...,d_n$ (satisfying certain constraints) we can number (i=1,2,…,n) the sides of n-gon and create the sequence of pairs (i,d_i). Each pair (i,i+1) (i=1,2,…,n-1) and (n,1) determines (d_i,d_{i+1})-bisectrices for i=1,2,…,n-1 and (d_n,d_1)-bisectrix. Let us construct these bisectrices and let us continue using the construction of roof skeleton described in [1,2] to obtain the roof skeleton of roof of varying slope (Fig.5). Such a construction allows to design roofs of varying slope (Fig.5).

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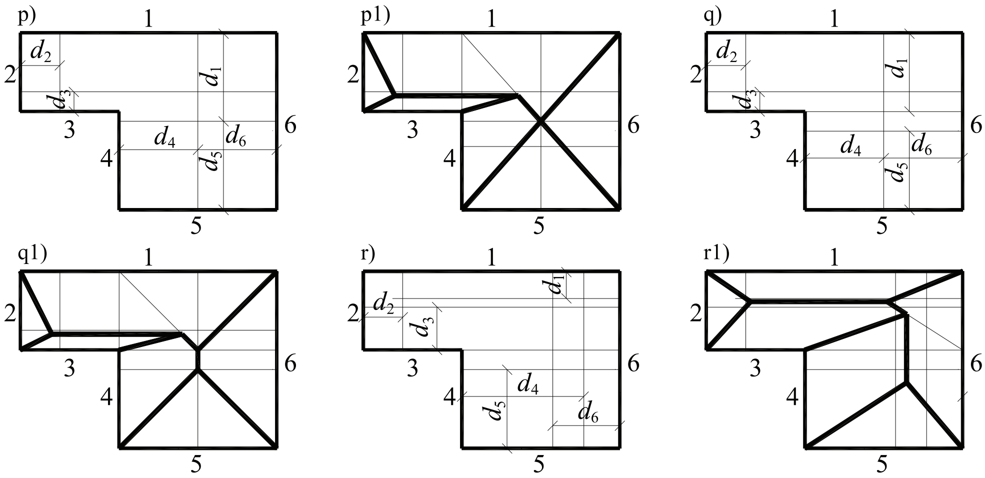

Figure 6: Different topological types of roofs of varying slope generated by a hexagon ($1,d_1$; $2,d_2$; $3,d_3$; $4,d_4$; $5,d_5$; $6,d_6$)

In comparison to roof skeletons of constant slope we obtain some interesting properties of skeletons of roofs with varying slope. Let us remember that within the class of roof skeletons with constant slope the basic polygon uniquely determine the topological type of a roof skeleton [1,2]. We have one base polygon – one topological type of a skeleton. It is easy to see that for an arbitrarily fixed polygon we can obtain different shapes of roofs of varying slope depending on the sequence of real positive numbers (distances) $d_1,d_2,\ldots,d_n$ (cf. Fig.6). In addition, analyzing elementary roofs of a given arbitrary polygon, we obtain all possible topological types of roof skeletons.

3 Cut-and-fill problems solved by using topographic projection

The second important application of offset curves of a line and a circle are solutions of cut-and-fill problems in earth works. Namely, to establish the boundaries of cut-and-fill areas in highway construction or other operations involving earthwork, it is simply necessary to superimpose the proposed contours on the existing contours, mark the points where the two contours systems are in agreement, and connect the marked points to outline the extent of the cut and fill ([4]). To solve this problem we can use the offset construction curves discussed above (implemented in AutoCAD).

The line of intersection between the horizontal construction site at the elevation of 120m and the 120 contour line will be the boundary line between the cut and the fill areas (cf. Fig. 7). Simply, the intersections of the 120m contour with the edges of the construction site will mark the limiting points of change from cut to fill. The determined points divide the polygon of a construction site into two parts (cf. Fig. 7). On the left-hand sides of the area (cf. Fig. 8, the “bottom” part of the polygon) the proposed contours, obtained by offset construction by using another distance, are laid off 1m apart to give a 1 horizontal to 1 vertical slope for the cut. On the right-hand sides of the area (cf. Fig. 9, the “top” part of the polygon) the proposed contours, obtained by offset construction, are laid off 1 ½ m apart to give a 1 ½ horizontal to 1 vertical slope for the fill.

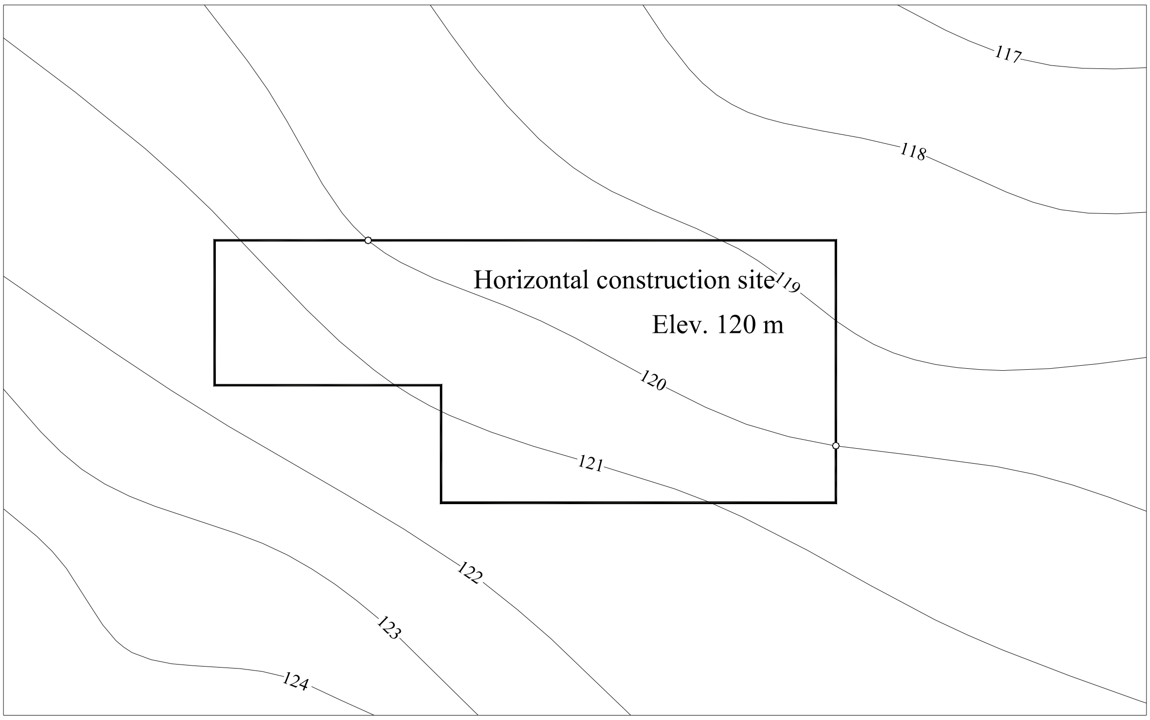

Figure 7: The intersections of the 120m contour with the edges of the building site will mark the points of change from cut to fill

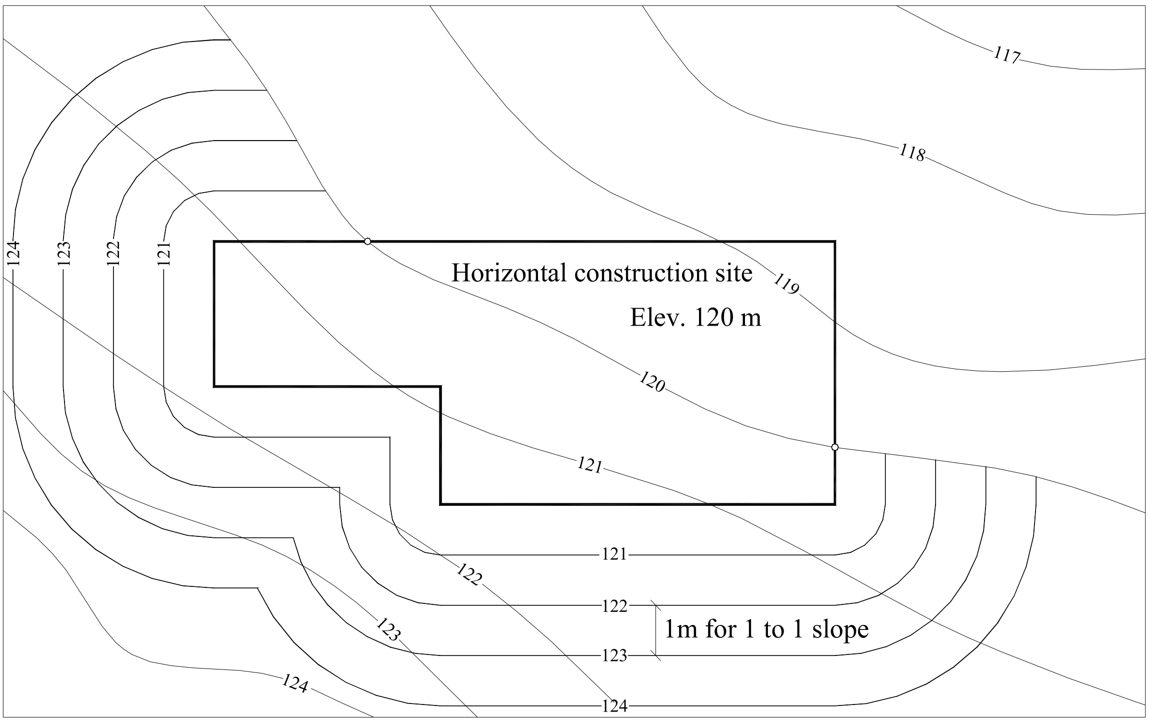

Figure 8: On the left-hand side of the area the proposed contours, obtained by offset construction with another distance, are laid off 1 m apart to give a 1 horizontal to 1 vertical slope for the cut

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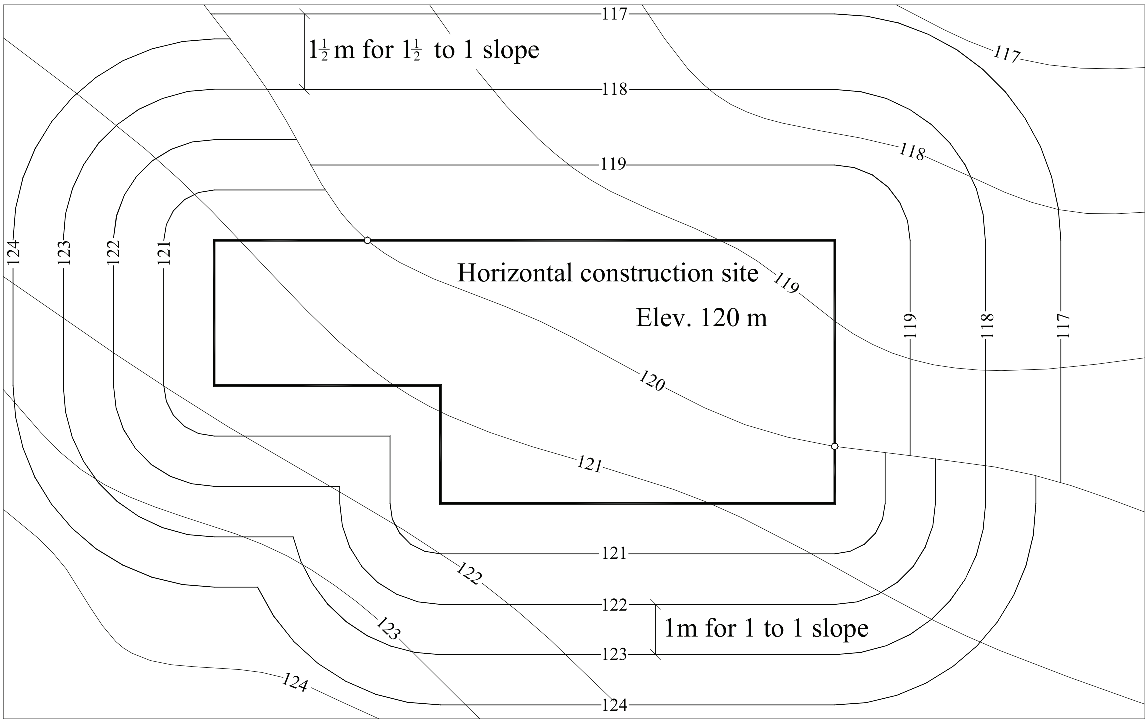

Figure 9: On the right-hand side of the area the proposed contours, obtained by offset construction, are laid off 1 ½ m apart to give a 1 ½ horizontal to 1 vertical slope for the fill

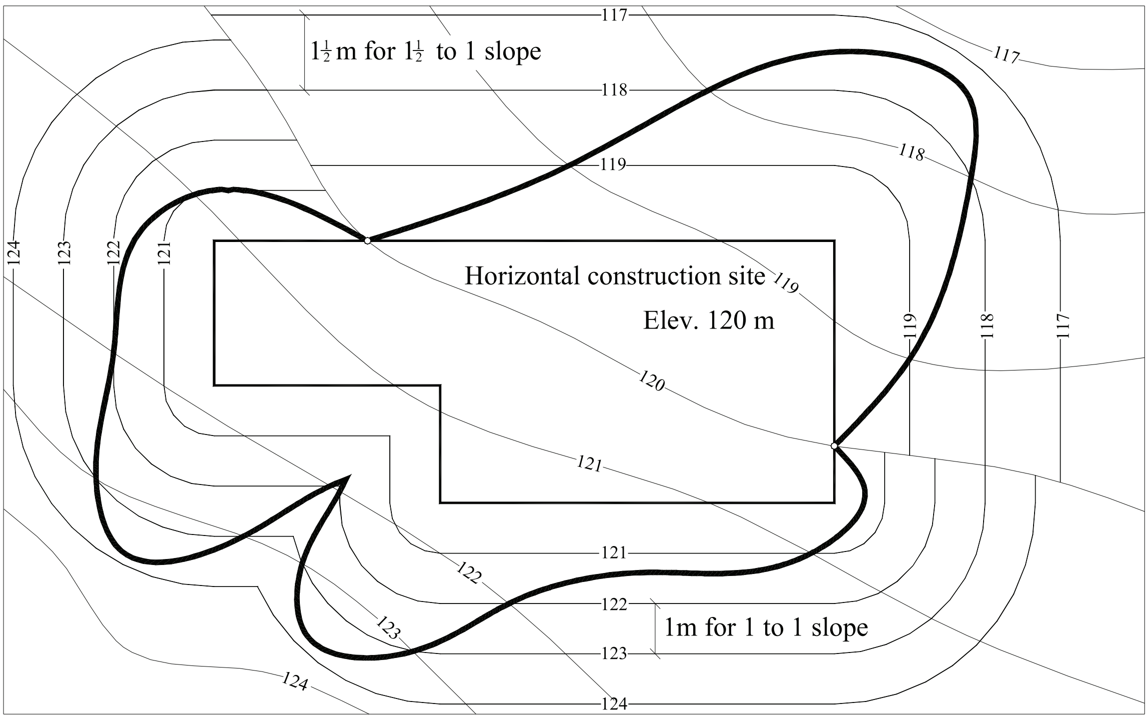

Figure 10: Cut-and-fill boundaries

4 Conclusions

1. To construct the roof skeleton of varying slope generated by a polygon ($1,d_1$; $2,d_2$; …; n,d_n) we successively construct

a) the (d_i,d_j)- bisectrices of all angles,
b) two of them close the polygon – hipped roof end,
c) the third ridge (as the (d_i,d_j)- bisectrix) starts at the obtained vertex (in general),
d) meets the fourth (d_i,d_j) - bisectrix and closes the second hipped roof end, etc.

2. The classical methods of designing roofs of constant slope and roofs of varying slope are the same.
3. In topographical projection cut-and-fill problem can be solved by using offset curve construction.
4. The proposed methods can also be used in didactics of descriptive geometry.

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OPERACJE OFFSET W GEOMETRYCZNYM ROZWIĄZYWANIU DACHÓW O RÓŻNYM NACHYLENIU POŁACI I GEOMETRYCZNYM OKREŚLANIU BILANSU ROBÓT ZIEMNYCH

W pracy zaproponowano, opartą na koncepcji konstrukcji krzywych offsetowych, zręczną konstrukcję dachu o różnym nachyleniu połaci. Pokazano, że metoda konstrukcji szkieletów dachów o różnym nachyleniu niczym nie różni się od klasycznej metody rozwiązywania dachów. Zauważono w szczególności, że kształt wielokąta podstawy dachu nie determinuje, jak ma to miejsce w geometrii dachów o jednakowym nachyleniu, jednoznacznego topologicznego kształtu dachu. Dokładniej dla danych dachów elementarnych kształt topologiczny zależy od kątów nachylenia poszczególnych połaci. Krzywe offsetowe wielokątów wykorzystano do topograficznego wyznaczania geometrii obszarów transportu urobku w robotach ziemnych.

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