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Investigating the Strain State of Fibres Located on the Helical Line in Extended Yarn

Abstract
In this paper, the strain state of fibres located along the helical line in an extended yarn has been investigated. The slippage and mutual displacement of fibres relative to the yarn for analysis of the strain state of fibres in extended yarn are investigated. It is proposed for expression compressive transverse stress G, in our notation, to use the equation supposed in this work. The stress strain of fibres in extended yarn is examined and a comparison of the stresses between the cross-sectional and longitudinal directions is carried out. It is found that an increase in the twist angle leads to an increase in the compressive transverse stress of fibres in the centre of the yarn. It is also noticed that the axial stress strain depends on the twist angle of the yarn. The results obtained using this relationship are similar to those presented in previous studies.

Key words: *fibre strain, fibre slippage, mutual displacement, compressive stress, axial stress, twist angle.*

Symbols used

a – *various values of the parameter*
b – *length of slippage region, mm*
c = $\cos\alpha$
E_f – *Young's modulus for fibre (axial modulus of fibres), N/m²*
E_{fr} – *frictional force, N*
E_p – *pulling force, N*
g – *function of differential equation equilibrium of fibre in the matrix*
G – *specific stress, perpendicular to fibre axis, N/m²*
h – *yarn length, mm*
r_0 – *radii of fibre investigated, mm*
r – *distance of yarn element from center, mm*
$r^* = r/R$
R – *yarn radius, mm*
l – *fibre length, mm;*
$l = h/\cos\theta$
$L_b = 2\pi r_0$
$u = c/\cos\theta$
X – *tensile specific stress of fibres in yarn, N/m²*
α – *yarn twist angle, deg*
θ – *helix angle, deg*
ε_y – *yarn deformation*
ε_f – *fibre extension*
σ_l – *Poisson's ration for longitudinal deformation of fibre (axial Poisson's ratio)*
σ_y – *Poisson's ratio for yarn (lateral contraction ratio of yarn)*
μ – *coefficient of friction between fibres*

Introduction

It is well known that the structural composition of staple yarn is entirely different from that of filament yarn. The location of short fibres in the helical line is not similar with continuous filaments. Because of the limited length of fibres in staple yarn, migration, slippage and other phenomena occur. Kinematical and geometrical modifications made to the spinning process lead to changes within the structure of the yarn. These modifications primarily refer to the speed and diameter of the rotor and spindle. Except the geometry of the delta and tension, such changes in the system essentially influence the structure and mechanical properties of the textile product. It is necessary to consider that yarn formation conditions for prediction of the mechanical properties depends on the method of spinning and speed of machines. It is well known that as the fibres get straighter as the structure of the yarn undergoes stress and converge, depends on the level of its twisting. The tensile properties of yarn and the effect of the twist amount, twisting tension and stress distribution on the yarn structure have been discussed by many researchers [1-22]. Hence the radial stress in the cross section of yarn is increasing gradually, which leads to an increase in the stiffness of the product, thus affecting stretching. Applying twist in the yarn causes the fibres to follow a helical path along its axis, as illustrated in ***Figure 1***, which is given by Hearle et.al [13]. The stiffness of fibres located in the helical line generally increases to a certain level and then begins to decrease by increasing the inclination of the twist angle. As a result, the irregularity of the microstructure of the yarn and range of characteristics for yarn deformation (such as breaking load) can provide real yarn. In this case the heterogeneity of the microstructure of yarn, the presence of the relative displacement of fibres relative to each other and a large range of variation in some variables specific to the strain of the yarn allows to present the real yarn, in contrast to well-known works [14, 15, 17], as a combination of a

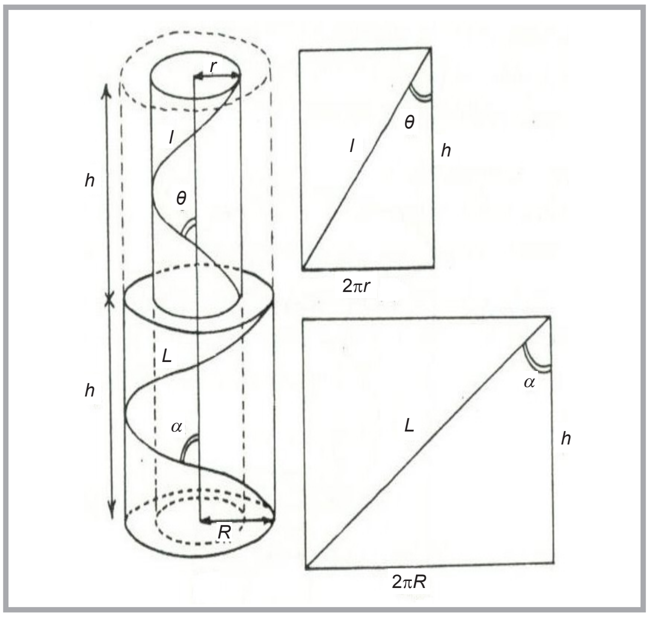

Figure 1. *Idealised helical yarn geometry: a) Idealized geometry, b) "opened-out" diagram of cylinder at radius r, and c) "opened-out" yarn surface [13].*

large number of elements with the simple laws of deformation, and the emergence and development of zones of slip in the yarn cross-section. This representation allows the yarn to be considered as ac-ontinuum model with a structural framework that enables to apply approaches for the study of the deformed state of the yarn. We have studied the deformation of the cylindrical filament form in the presence of yarn cross-sectional areas of stretching and slippage. Conditions under which all of the fibres will be able to slip were determined.

Theoretical approach

The strain properties of yarn are more obvious at an early stage of loading, and then by increasing the stress these properties are transferred from a single system to a compact system of fibres with high module elasticity and low parameters. In previous studies it is noticed that during twisting and stretching, the cross section of yarn depends on the fibres located in two areas, such as the slippage and non-slippage regions [14, 15]. In the absence of mutual displacement of fibres, and passing any point of cross section the deformation of it can be found by Hearle's equation (4.22) [16]

$$\varepsilon_f = \varepsilon_y (\cos^2\theta - \sigma_y \sin^2\theta) \quad (1)$$

Here $\cos\theta = h / \sqrt{h^2 + 4\pi^2 r^2}$.

If fibre is located on the surface of the yarn then it is assumed that $\theta = \alpha$ (where α - yarn twist angle). Let's suppose that stress acting on the fibre in the axial direction - X and compressive transverse stress - G. According to Hooke's law, the linear deformation of fibre under these forces can be determined by Hearle's equation (4.24) [16].

$$\varepsilon_f = \frac{X}{E_f} - \frac{2\sigma_1}{E_f}(-G) \quad (2)$$

A connection is established between the stress $X = x/E_f\varepsilon_y$ and compression $G = g/E_f\varepsilon_y$ [16] by comparing ***Equations 1*** and ***2***, as:

$$x = \cos^2\theta - \sigma_y \sin^2\theta - 2\sigma_1 g \quad (3)$$

Function g is the solution of the differential equation equilibrium of fibre in the matrix, and according to Hearle's equation (equation 4.45 [16]) it has the form:

$$g = \frac{1+\sigma_y}{(1+2\sigma_1)} \frac{c^2}{u^2}(1-u^{1+2\sigma_1}) - \sigma_y \frac{1-u^{2\sigma_1 - 1}}{(2\sigma_1 - 1)}$$
$$0 < \theta < \alpha \quad (4)$$

In the current work, it is proposed to compress the transverse stress G, in our notation using the equation derived by Chistoborodov et al [17].

$$G = E_f \varepsilon_f \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{2(r^2 \sin^2\alpha + R^2\cos^2\alpha)}$$
$$0 < r < R \quad (5)$$

or taking into account expression ε_f from ***Equation 1***, we have ***Equation 6***

Considering that $g = G/E_f\varepsilon_y$, we have ***Equation 7***

Substituting expression g from ***Equation 6*** into ***Equation 2***, we have

$$\varepsilon_f = \frac{X}{E_f} + \sigma_1 \varepsilon_f \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{(r^2 \sin^2\alpha + R^2\cos^2\alpha)} \quad (8)$$

Substituting expression ε_f from ***Equation 1*** in ***Equation 8***, we obtain function X, *see* ***Equation 9,*** and considering $x = X/E_f\varepsilon_y$, we have ***Equation 10.***

From ***Equation 10*** it is obvious that function $x(r)$ will take the form as in ***Equation 3***, where it is necessary to choose $g(r)$ by means of ***Equation 7***.

Figure 2 presents typical curves of the function of ratio g to $r^* = r/R$ for different values of twist angle α. It is seen that the compressive force (pressure) in the centre of the yarn takes the maximum value, which increase with an increase in the twist angle α significantly. A increas in the twist angle leads to the distribution of pressure along the radii of yarn irregularity. This pattern is clearly visible at angles $\alpha > 10°$.

Figure 3 (see page 22) shows distribution curves of axial stretch stress $x = X/E_f\varepsilon_y$ (according to ***Equations 10***) along the radii of yarn at various values of twist angles α and ratios σ_1 and σ_y.

It is observed from analyses that fibres located in the centre of the yarn exhibit higher tension as compared with those at larger values of the twist angle away from the centre. Consequently axial stress along the radii of the yarn is mainly influenced by the longitudinal Poisson's ratio for yarn σ_y.

Calculations by proposed scheme

Comparing the results of calculations carried out by the proposed scheme with dates in [16], we can conclude that they are both qualitatively and quantitatively similar to each other. This is due to the different expressions for compressive stresses G obtained by the two approaches based on equilibrium equations for yarn under the action of tensile stresses.

We define the value of the pulling force of single fibre and friction by means of equations derived by Chistoborodov et al [17], and Jumaniyazov et al [18].

$$F_p = \pi r_0^2 X,\ F_{fr} = \mu G l \cdot L_b \quad (11)$$

Substituting ***Equation 11*** into ***Equation 9*** for expression G and X from ***Equation 6*** and ***Equation 9***, repectively, we have ***Equation 12*** and ***Equation 13.***

As the analysis shows, for expression F_f the axial stress reaches its maximum value in the central fibre, and the greater the distance from the center of the yarn, the more it is reduced. For some values of

$$G = E_f \varepsilon_y (\cos^2\theta - \sigma_y \sin^2\theta)\ \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{2(r^2 \sin^2\alpha + R^2\cos^2\alpha)} \quad (6)$$

$$g = (\cos^2\theta - \sigma_y \sin^2\theta)\ \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{2(r^2 \sin^2\alpha + R^2\cos^2\alpha)} \quad (7)$$

$$X = E_f \varepsilon_y (\cos^2\theta - \sigma_y \sin^2\theta)\left[1 - \sigma_1 \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{r^2 \sin^2\alpha + R^2\cos^2\alpha}\right] \quad (9)$$

$$x = (\cos^2\theta - \sigma_y \sin^2\theta)\left[1 - \sigma_1 \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{r^2 \sin^2\alpha + R^2\cos^2\alpha}\right] \quad (10)$$

$$F_p = \pi r_0^2 E_f \varepsilon_y (\cos^2\theta - \sigma_y \sin^2\theta)\left[1 - \sigma_1 \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{r^2 \sin^2\alpha + R^2\cos^2\alpha}\right] \quad (12)$$

$$F_{fr} = 2\pi\mu r_0 h E_f \varepsilon_y \frac{\cos^2\theta - \sigma_y \sin^2\theta}{\cos\theta}\ \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{2(r^2 \sin^2\alpha + R^2\cos^2\alpha)} \quad (13)$$

Equations 6, 7, 9, 10, 12 and 13.

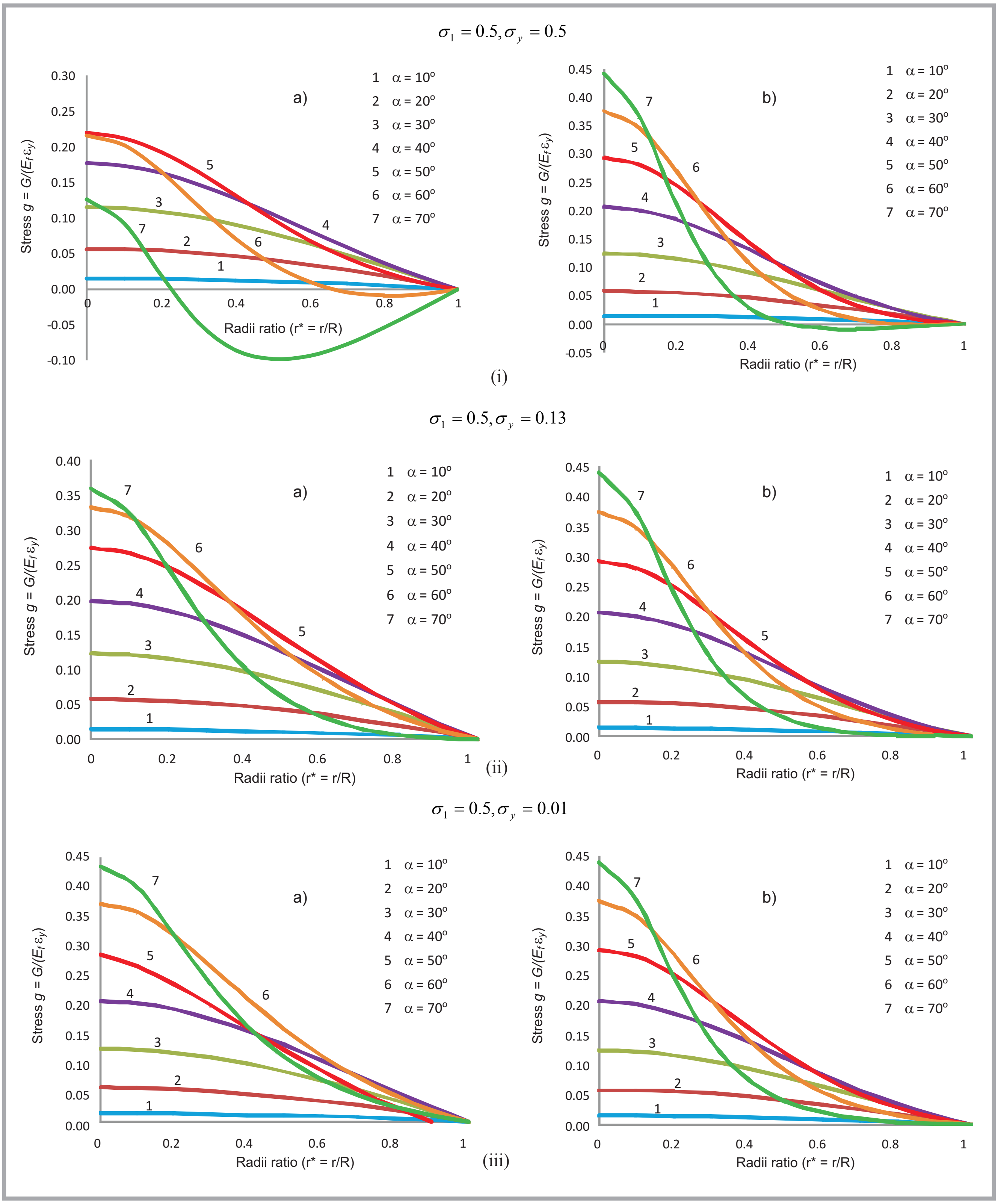

Figure 2. *Curves of compressed fibres with stress $g = G/E_f\varepsilon_y$ at radii (ratio $r^* = r/R$) for various values of twist angles α and ratios σ_1 and σ_y in (i), (ii), and (iii), a) – by means of Hearle's equation (equation 4.45 or 4.47 [16]), b) – by means of the equation proposed (10).*

this stress, a central layer may form in the yarn i.e. a boundary which interacts with the rest of it, where there is the slippage of fibres between each other. The force is determined according to Coulomb's low. If we denote the distance as $r = r_1$ from the centre of the yarn to the layer then the slip condition at its boundary is

$$F_p = F_{fr} \text{ at } r = r_1 \qquad (14)$$

Substituting expressions F_f and F_{fr} from ***Equations 12*** and ***13*** into ***Equation 14*** and assuming $(2\pi r_1/h)^2 \approx 0$ we can obtain

$$\frac{(R^2 - r_1^2)\cos^2\alpha\sin^2\alpha}{(r_1^2\sin^2\alpha + R^2\cos^2\alpha)} = \frac{r_0}{\mu h + r_0\sigma_1} \qquad (15)$$

Solving this equation with respect to r_1

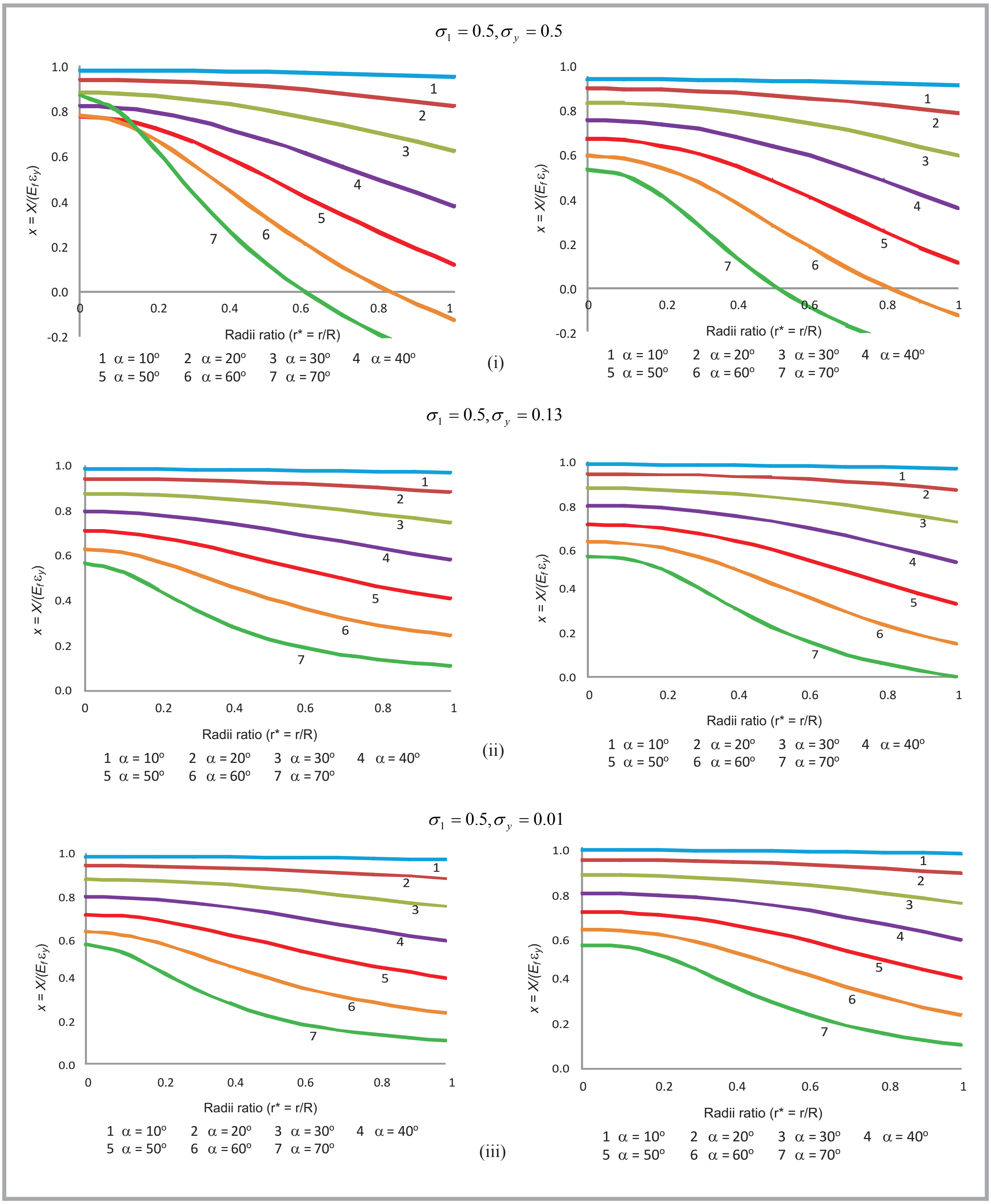

__Figure 3.__ Curves distribution of the tensile stress of fibers (given) $x = X/E_f\varepsilon_y$ at radii (ratio $r^* = r/R$) for various values of twist angles α and ratios σ_1 and σ_y in (i), (ii), and (iii), a) by means of Hearle's equation (equation 4.45 or 4.47 [16]), b) by means of equation proposed (10).

$$r_1 = R\frac{\cos\alpha}{\sin\alpha}\sqrt{\frac{\sin^2\alpha - a}{1-\sin^2\alpha + a}} \qquad (16)$$

$$0 \le r_1 \le R$$

where $\alpha = 1/(\mu_0 + \sigma_1)$, $\mu_0 = \mu h/r_0$. Further assuming $0 < \alpha < 50°$.

The length of the slip area is equal to $b = R - r_1$. Thus when the axial stress of fibre becomes equal to the frictional force of fibres two zones in the cross section of yarn are formed. In the first zone, where the condition is $r_1 < r < R$, all fibres starting from the boundary are in a slip condition relative to each other. In the second zone, where the condition is $0 < r < r_1$, slippage will be absent and the yarn structure in this zone is not distorted. When the condition is $a \ge 1$

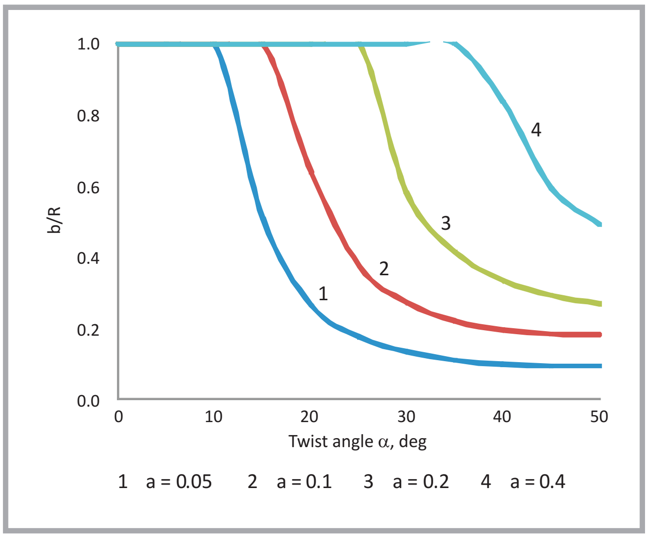

Figure 4. *The typical ratio curves of the length of slippage area b/R to twist angle α at various values of parameter a = 1/(μ_0 + σ_1).*

($\mu_0 \leq 1$ - σ_1), inequality (16) holds at any various values of r_1 ($0 \leq r_1 \leq R$), and all fibres will be in the slippage condition. Consider inequality (16) in a case where parameter is $a < 1$. From the condition of existence the under-root of $\sin^2\alpha$ - $a \geq 0$ and requirement of inequality $r_1 \leq R$, be $\alpha_{min} \leq \alpha \leq \alpha_{max}$, where $\alpha_{min} = \arcsin\sqrt{a}$, α_{max} - the result of equation at $r'_1(\alpha) = 0$.

If the inequality is $\alpha < \alpha_{min}$ all fibres in the yarn are in the slippage condition and twist angle values are $\alpha > \alpha_{max}$ in the yarn, which means there is no slippage area. ***Figure 4*** shows the curve ratio of b/R and twist angle α for parameters "a".

It is observed that at values of parameter $a = 0.05$ ($\mu = \frac{r_0}{h}(20$ - $\sigma_1)$) at $0 \leq \alpha \leq 13°$ all fibres remains in the slippage condition. Furthermore by increasing the twist angle, the length of the slip area decreases rapidly and reaches a limit value of $b = 0.0966\ R$. For a value of parameter $a = 0.4$ ($\mu = \frac{r_0}{h}(2.5 - \sigma_1)$) all fibres slip at $0 \leq \alpha \leq 39.25°$ and the length of the slippage area reaches at value of $b = 0.494\ R$.

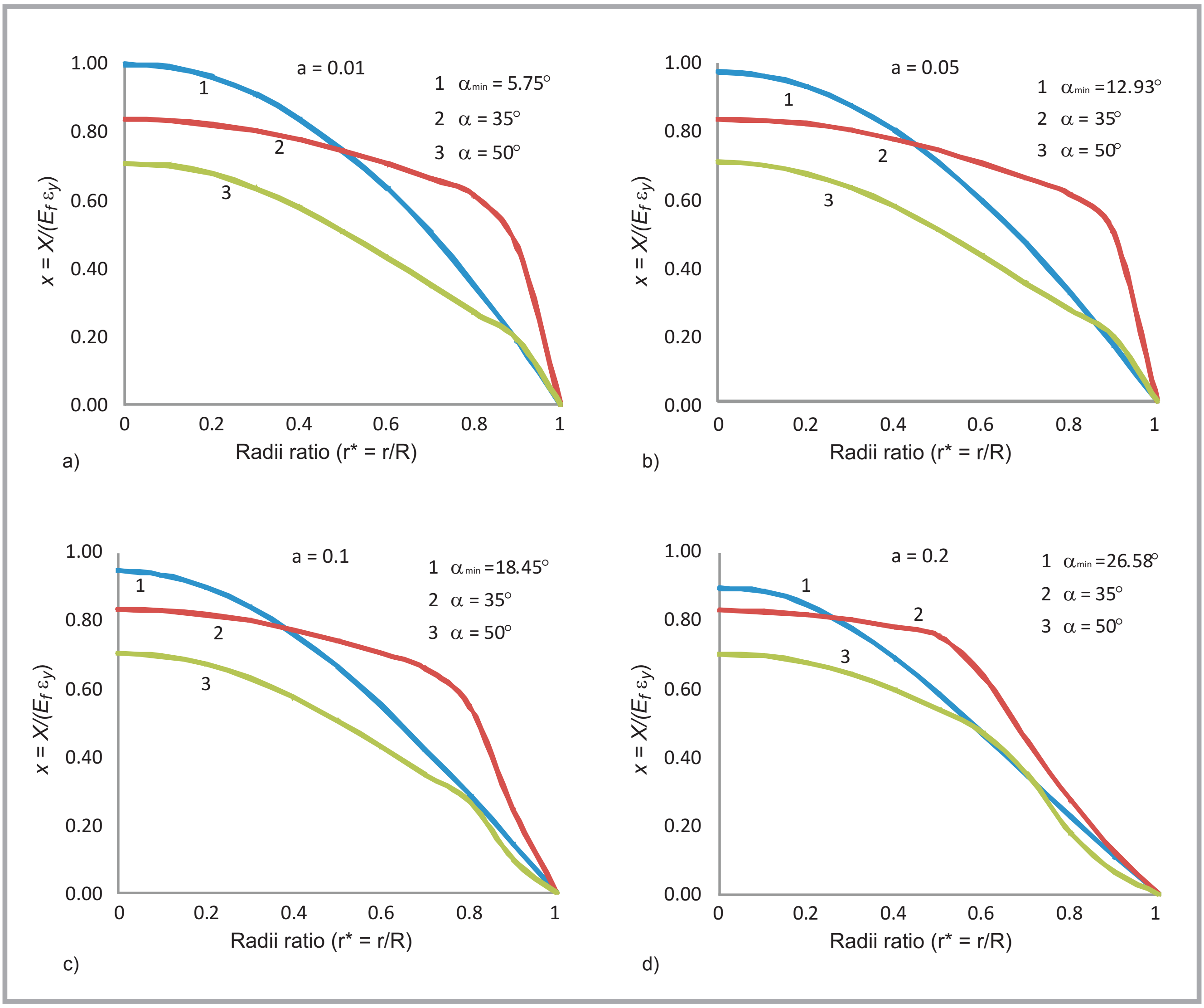

Figure 5. *Curve of the distribution of dimensionless axial stress x by the given fiber radii (ratio r* = r/R) for various values of parameter = a and twist angles = α.*

If there is a slippage area then the axial tensile $X(r)$ (referring to the value of $E_f \varepsilon_y$) can be found by the following equations:

$$x = \mu_0 \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{(r^2 \sin^2\alpha + R^2\cos^2\alpha)} \text{ at } 0 \le r \le R, \text{ at } a \ge 1, \quad (17)$$

$$x = (\cos^2\theta - \sigma_y \sin^2\theta) \times \left[1 - \sigma_1 \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{r^2 \sin^2\alpha + R^2\cos^2\alpha}\right] \quad (18)$$

at $0 \le r \le r_1$,

$$x = \mu_0 \frac{(R^2 - r^2)\cos^2\alpha \sin^2\alpha}{(r^2 \sin^2\alpha + R^2\cos^2\alpha)} \text{ at } r_1 \le r \le R, \text{ at } a \le 1. \quad (19)$$

Figure 5 presents typical curves of distribution tensile axial stresses $X(r)$ (referred to the value of $E_f \varepsilon_y$) from yarn radii (referred to the value of R) for various values of parameter $a = 1/(\mu_0 + \sigma_1)$ and twist angle α.

In ***Figure 5***, the blue curve corresponds to the minimum value of twist angle $\alpha = \alpha_{min} = \sqrt{a}$. For values of angles $\alpha \le \alpha_{min}$ all fibres in the matrix will be in the slippage condition, and they are not shown in ***Figures 5.a 5.b, 5.c*** and ***5.d***. It is observed that the stresses drop sharply in the slippage area. Thus at small values of the coefficient of friction μ and twist angle α there could be some fibres (stress free) which do not play any role during tensile stretching on the surface of the yarn.

Conclusion

Comparing the results of the calculations carried out by the proposed scheme with those obtained in [13], we can conclude that they both qualitatively and quantitatively give similar results to each other. Significant differences were found at a value of Poisson's ratio of $\sigma_1 = \sigma_y = 0.5$. Calculations based on two methods gave almost identical results at $\alpha < 30°$, differing with the largest relative deviation of about 9.5% for values $r = 0$ at $\alpha = 50°$. For small values σ_y of Poisson's ratio, the calculation results are almost identical which is due to the fact that the expression of compressive stresses G obtained by the two approaches is based on the use of equilibrium equations for the element of yarn under tension.

Particularly noteworthy results were obtained for large values of the torsion angle of yarn α. The magnitude of lateral tension G (***Figure 2***), calculated for $\sigma_1 = \sigma_y = 0.5$ at $\alpha \ge 60°$ at a certain distance from the centre of the yarn becomes zero and then negative, indicating the possibility of the occurrence of a migration zone of fibres (***Figure 3***). Furthermore the appearance of an area of fibre slippage relative to each other in the cross-section of yarn was studied. It was established that the length of this zone depends on the angle of torsion and parameter $a = 1/(\mu_0 + \sigma_1)$. When the condition is $a \ge 1$ all of the fibres will be able to slip at all values of the torsion angle. If it is $a < 1$, then the condition for the appearance of the slip zone is torsion angle inequality $\alpha_{min} \le \alpha \le \alpha_{max}$ where $\alpha_{min} = \arcsin\sqrt{a}$, α_{max} - the - root of equation $r'_1(\alpha) = 0$. When the inequality is $\alpha < \alpha_{min}$ all the fibres in the yarn will be in the condition of slip, and for values $\alpha > \alpha_{max}$ of the angles of twist in the yarn, there will be no slip zone. From the analysis of the curves shown in ***Figure 4***, it is seen that for parameter value $a = 0.05$ when $0 \le \alpha \le 13°$, all fibres are able to slip. With a further increase in the angle of twist the slip zone length decreases rapidly, reaching a limit value of about $b = 0.0966R$. For parameter value $a = 0.4$ (curve 4) all fibres slip at $0 \le \alpha \le 39.25°$ and the length of the sliding zone reaches the value $b = 0.494R$. In the slip area the tension drop sharply on the free surface of the yarn, and then vanishes. Thus for small values of the coefficient of friction μ and angle of twist α, on the surface of yarn fibres, there can be no stress.

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