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Piezoelectric Square Based Sensor-actuator Hybrid in Vibration Reduction

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Abstract This paper presents the results of comparison of vibration reduction levels between standard square based piezo actuators and piezoelectric sensor-actuator hybrids. Modelling was done using FEM method in ANSYS software. Model consisted of a steel plate with piezo elements attached. One of the elements was used as an actuator to excite plate's vibrations. The other was either a standard homogeneous square based actuator or a sensor actuator hybrid with 2 possible sizes of the sensor part of said hybrid. Harmonic analyses were performed for the 1st, 2nd, 4th and 5th mode shapes with the goal function being the minimalization of displacement vector sum of a number of nodes (there were 3 possible cases). Significant vibration reduction levels were obtained with no significant differences in said levels between standard actuators and sensor-actuator hybrids. Reducing the size of sensor part of sensor-actuator allowed for lower voltages needed to achieve vibration reduction levels.

Keywords: AVC, FEM, plates' vibration, sensors.

1. Introduction

The fundamental work of Fuller [1] and Hansen and Snyder [2] gave much of the necessary theoretical development for the static and dynamic behaviour of plates with piezoelectric actuators and sensors attached.

Developments in computing power allowed carrying advanced computer simulations [3, 4] as well as to control processes leading to vibration and noise reduction [5-9] Also an analytical approach is a subject of continuous development. New theories, mathematical and numerical models are widely used for problems of objects vibrations [10-12] and sound radiation [13-16].

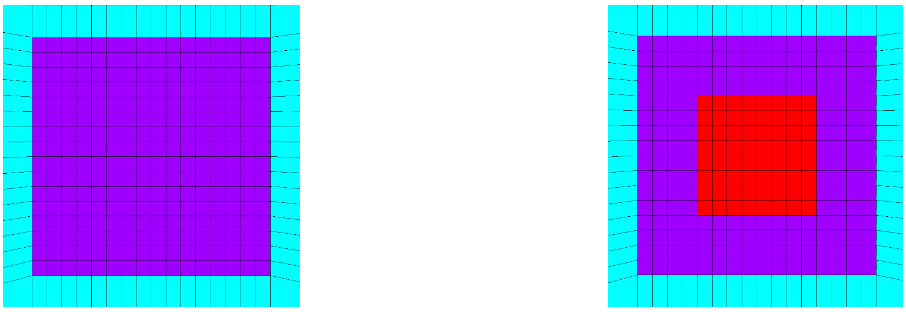
The authors previous works were based on functionally graded materials [17], which itself is a very dynamic field [18]. It concentrated on analysing the impact of a step change in material parameters on the effectiveness of piezoelectric actuators of various shapes (circular, rectangular, triangular) as well as the impact of a step change in material parameters of these actuators. These analyses showed slight changes in the achieved reduction of the vibration level with corresponding changes to the maximum sound pressure levels radiated by the plate. Changing the parameters of material parts of the certain part of the actuator results in an increase in voltage required to achieved these levels of reduction. [4, 16, 19, 20].

As a result, there was an idea to replace the inner part of the piezoelectric actuator with a sensor, which would simultaneously allow local measurement and reduction of vibrations. The sensor-actuator hybrid would be capable of achieving comparable vibration level reduction values with a homogeneous piezoelectric actuator (at the expense of the higher voltages required to achieve such levels). The sensor-actuator hybrid would also allow for an additional reduction in the weight of the control system [21].

This article presents results of numerical analyses of active vibration control said piezoelectric sensor-actuator hybrid. Numerical analyses were performed for a structure consisting of plate with 2 piezoelectric elements attached with different base- shapes (square and disc) and in case of sensor-actuator 2 possible ratios of sensor to actuator part sizes.

2. Materials and methods

The objects of interests were piezoelectric sensor-actuator hybrid. Based on the previous idea of an piezoelectric actuator with a step change in material properties [4, 16, 19, 20]. The idea for sensor-actuator hybrid was to replace inner part of standard piezoelectric actuator with a sensor is shown on Figure. 1. Such a hybrid allow for both functions of sensor and actuator, and possibly not impede either of them in significant manner.



(**a**) (**b**)

Figure 1. Idea of a sensor-actuator: a) standard square based piezo-actuator; b) square based sensor-actuator.

The first iteration of numerical models (Figure 1) presented in [21] was based on models used for actuators with a step change in material properties with the inner part of the actuator disabled. The second iteration was changed by adding empty space between the inner and outer part of piezoelectric element (Figure 2). This helps bring the model closer to a possible physical prototype by introducing an electric separation of 2 parts of piezo element. Introducing this gap should also simplify the creation of a prototype.

(**a**) (**b**)

Figure 2. Second iteration of sensor-actuator a) square based sensor-actuator with a larger sensor part; b) square based sensor-actuator with a smaller sensor part.

It should be noted, that to keep continuity with the previous analyses the total size of the actuator sensor hybrid, that is the sum of sizes of the sensor part, gap and actuator part is kept the same as in previous works.

Models were made using ANSYS software Each model consist of steel plate of 400 x 400 x 2 mm with 2 piezo elements attached (Figure 3a) and a half sphere of surrounding air (Figure 3b). Modelling of surrounding air allows for analyses of sound pressure radiated from the plate, which will not be shown in this article, but the air itself is left for continuity and consistency of results with previous and future works.

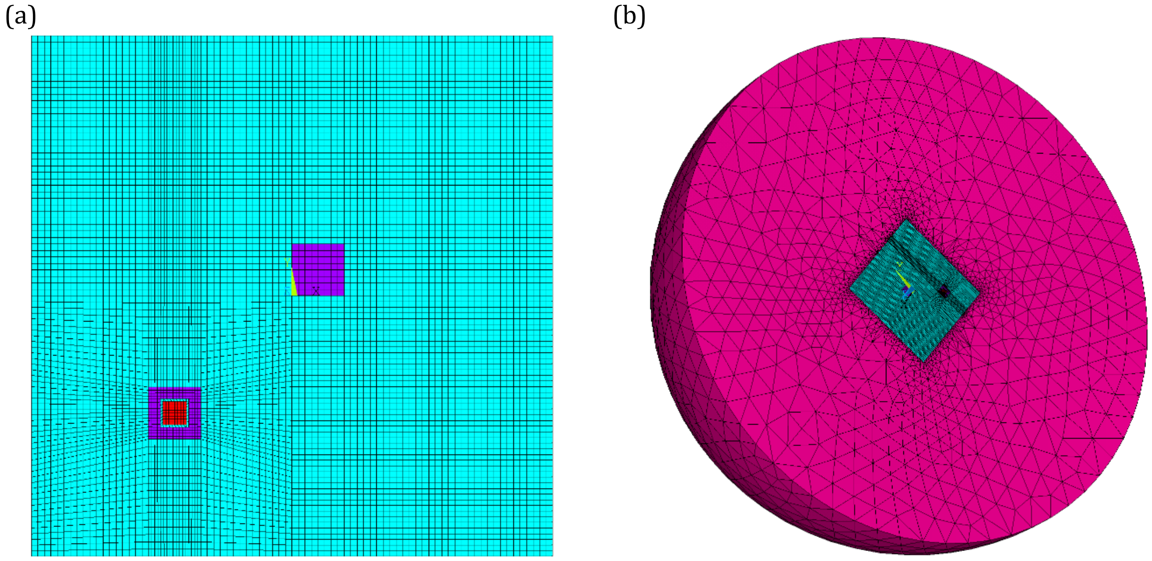


Figure 3. a) Modelled plate; b) modelled plate with surrounding air.

The size of modelled piezo elements is also kept the same as in previous works, so the thickness is 1 mm and the base area of a full actuators is 1600 mm2. For second iteration of sensor-actuator the outer dimensions are kept the same, the inner side with a larger sensor of the actuator is 0.55 of side or radius of the outer side and the outer side of the sensor is at 0.45. For hybrid with a smaller sensor the inner part of actuator is 0.35 and the outer part of sensor is 0.25 of full side or radius. This of course means that some of the base area of such hybrid is lost to the introduced gap.

Table 1. Parameters used in models.

Structural element	Element in ANSYS library	Properties
Plate	SOLSH190	$E = 1.93{\times}10^{11}$ Pa, $\nu = 0.29$, $\rho = 7800$ kg/m^3
Piezo elements	SOLID226	Properties of PZ28
Air	FLUID30	$\rho = 1.2$ kg/m^3 $c = 343$ m/s

Based on previous works a series of harmonic analyses was performed for the 1st, 2nd, 4th and 5th mode. The 3rd mode has been omitted as both the 2nd and 3rd mode for the square plate occur for the same frequency so only one of them is usually excited. For each of the analysed modes the plate was excited by applying a voltage with amplitude of 100 V and the phase angle 0° to the piezo actuator near the centre of the plate. Then an optimization procedure was performed using internal ANSYS functions to find the amplitude of the voltage to be applied to the second actuator to reduce plates' vibrations. One run of said procedure consisted of no more than 30 steps. For the first run the maximum amplitude of voltage that would be applied to the actuator was 600 V. After completion of the first run of the procedure another run was started with the value from the previous run as a starting point the narrower range of voltages. This was repeated until the amplitude range of voltage was ±2,5 V of the staring value.

The phase angle of the voltage applied to the actuator was not a variable for the analyses as it was shown in previous works that depending on the mode it would be either 0° or 180°.

If any of obtained vibration reduction values differed from other results significantly manual check and tuning was performed.

The goal function used in the optimization procedure is given by:

$$J_1 = \min \sum_{i=1}^{n} |\boldsymbol{X}_{\mathrm{sum}}(i)|, \tag{1}$$

where min is the smallest value of sum;

- $\mathbf{X}_{\mathrm{sum}}(i)$ is the displacement vector sum of the i-th node, n is the number of nodes used for calculations.

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There were 3 cases for the sensor placement analysed, therefore possible values for n are given as:

a) n is equal to every node making the back of the plate (here the back of the plate is the side to which the piezoelectric elements are not attached). The actual number is at least 7223 and the size of sensor part of sensor-actuator. This is considered a best case scenario;
b) n is equal to 25 nodes forming a square "virtual" sensor with a quarter of the size of the square based actuator placed on the same diagonal as piezo actuators but in the upper level side of the plate (near ¼th of its length);
c) n is equal to 65 (or more) nodes forming a "virtual" sensor the size of the sensor part of sensor-actuator placed directly under it (the number changes depending on the size of the sensor part).

3. Results

This section presents the results of carried out numerical analyses. First an equation for calculating vibration reduction levels will be presented. This is followed introduction of voltage efficiency parameter as a simplified way to somewhat compare energy needed to achieve said reduction.

$$L_{\text{red}} = 20\log\frac{\sum_{i=1}^{n}|\boldsymbol{X}_{1\text{sum}}(i)|}{\sum_{i=1}^{n}|\boldsymbol{X}_{2\text{sum}}(i)|}, \tag{2}$$

where $\mathbf{X}1_{\text{sum}}(i)$ is the displacement vector sum in i-th node before the reduction, $\mathbf{X}2_{\text{sum}}(i)$ is the displacement vector in the i-th node after the reduction, n is the number of nodes used (as per 3 cases mentioned before).

Introduced voltage efficiency is given by Equation 3:

$$UL = \frac{U}{L_{\text{red}}}, \tag{3}$$

where: U is the amplitude of voltage applied to actuator; L_{red} is the level of vibration reduction.

This parameter should help to compare the efficiency of piezo elements used in terms of voltages applied in relevance to obtained vibration reduction levels since all are modelled using the same material properties.

Table 2 presents the results for vibration reduction when using square based actuators and sensor-actuators and using the whole back area of the plate as a sensor.

Overall the obtained vibration reduction levels ranged from around 26 dB to more than 43 dB depending on the mode of the plate. It can be seen that for the 1st mode the sensor-actuator results are about 0.3 dB lower than for the full actuator. There seem to be no differences for the 2nd mode. For the 4th and 5th mode it can be seen that results obtained when using sensor-actuators are slightly higher than for the full actuator (up to around 0.4 dB). All of the above mentioned differences are small (less than 0.5 dB), but they are consistent and at the same time there are almost no differences between sensor-actuators with different ratio of sensor to actuator part.

As for the voltages that were needed to achieve said vibration reduction as expected the lowest values for voltage efficiency parameter can be seen when using a full actuator so they'll be treated as base. When using sensor-actuator with a larger sensor part the overall voltage efficiency values are around 1,55 times higher than for the full actuator. Which translates to almost 200 V higher voltage for the 1st mode. The values improve when using smaller sensor part as the actuator area is larger and are about 1,18 times higher than for full actuators (less than 60 V higher for the 1st mode).

Table 2. Results obtained for square based actuators, when using whole back area of the plate as sensor; mode - number of mode; type - full actuator, actuator-sensor; U_a - amplitude of voltage applied to actuator; φ_a - phase of the voltage applied to the actuator; L_{red} - vibration reduction; UL – voltage efficiency.

mode	type	U_a [V]	φ_a [°]	L_{red} [dB]	UL [V/dB]
1	actuator	365.29	180.00	41.3	8.8
2		57.77	360.00	43.1	1.3
4		12.05	180.00	25.8	0.5
5		159.95	360.00	35.0	4.6
1	actuator-sensor (larger)	558.69	180.00	41.0	13.6
2		88.40	360.00	43.1	2.1
4		18.38	180.00	26.1	0.7
5		247.37	360.00	35.4	7.0
1	actuator-sensor (smaller)	424.45	180.00	41.0	10.4
2		67.14	360.00	43.1	1.6
4		14.04	180.00	26.2	0.5
5		186.95	360.00	35.4	5.3

Table 3 presents the results of vibration levels reduction for the square based actuators and sensor-actuators for the scenario where a sensor would be placed on the same diagonal as actuator used for reduction but in the upper right quarter of the plate. Additional column introduced L_{redf} represents results of vibration level reduction recalculated using all nodes of the back of the plate but with the voltage obtained from smaller sensor. This will help with comparison of different sizes and placement of sensors.

Vibration reduction levels obtained in this scenario range from almost 24 dB for the 4th mode to almost 54 dB for the 2nd mode. There vibration reduction levels obtained when using sensor-actuators are slightly higher (up to 0.3 dB) for the 2nd mode and slightly lower for the 4th and 5th (up to 0.2 dB) when compared to full actuators. Again there is almost no difference in results obtained for sensor-actuators when changing the ratios of sensor and actuator parts.

Similarly to previous table higher values of UL were obtained when using sensor-actuators than full actuators. This should be expected as voltages applied to actuators are almost the same and although obtained vibration reduction levels differ from the previous scenario they are similar for different actuators used, therefore the ratios of UL values for different actuators should also stay similar.

Table 3. Results obtained for square based actuators when using a "virtual" sensor on the diagonal in the upper side of the plate; mode - number of mode; type - full actuator, actuator-sensor; U_a - amplitude of voltage applied to actuator; φ_a - phase of the voltage applied to the actuator; L_{red} - vibration reduction; L_{redf} - vibration reduction calculated for all nodes making the back of the plate; UL – voltage efficiency.

mode	type	U_a [V]	φ_a [°]	L_{red} [dB]	L_{redf} [dB]	UL [V/dB]
1	actuator	365.42	180.00	44.6	41.3	8.2
2		57.80	360.00	53.6	43.0	1.1
4		12.20	180.00	23.9	25.9	0.5
5		158.66	360.00	31.6	33.7	5.0
1	actuator-sensor (larger)	558.81	180.00	44.6	41.0	12.5
2		88.40	360.00	53.8	43.1	1.6
4		18.50	180.00	23.8	25.9	0.8
5		247.29	360.00	31.4	35.4	7.9
1	actuator-sensor (smaller)	424.48	180.00	44.6	41.0	9.5
2		67.14	360.00	53.9	43.1	1.2
4		14.04	180.00	23.8	26.2	0.6
5		186.81	360.00	31.5	35.4	5.9

Table 4 presents results for vibration reduction levels when using square based actuators in a scenario where a sensor is placed under the centre of the actuator (or sensor-actuator) and is the size of the sensor part of sensor-actuator.

For this scenario obtained results range from more than 25 dB for the 1st mode up to almost 43 dB for the 2nd mode. The differences in vibration reduction levels obtained for actuators and sensor actuators become more pronounced. For the 2nd mode values obtained when using sensor-actuator are up to 0.9 dB higher. Similar for the 4th (up to 0.8 dB) and 5th mode (up to 1.6 dB). So it would appear that sensor-actuator might be slightly better for vibration reduction. That isn't exactly true as will be shown below.

As for UL values the situation is similar as before. The values are different because of the change in vibration reduction levels (which come from changing the sensor), but the ratios are similar.

Table 4. Results obtained for square based actuators when using a "virtual" sensor placed under the centre of square based actuator; mode - number of mode; type - full actuator, actuator-sensor; U_a - amplitude of voltage applied to actuator; φ_a - phase of the voltage applied to the actuator; L_{red} - vibration reduction; L_{redf} - vibration reduction calculated for all nodes making the back of the plate; UL – voltage efficiency.

mode	type	U_a [V]	φ_a [°]	L_{red} [dB]	L_{redf} [dB]	UL [V/dB]
1	actuator	363.40	180.00	25.4	39.2	14.3
2		57.76	360.00	41.9	43.1	1.4
4		12.27	180.00	31.4	25.7	0.4
5		161.03	360.00	33.4	34.4	4.8
1	actuator-sensor (larger)	557.24	180.00	26.0	40.2	21.4
2		88.45	360.00	42.8	43.1	2.1
4		18.36	180.00	32.2	26.1	0.6
5		247.48	360.00	35.1	35.4	7.1
1	actuator-sensor (smaller)	423.19	180.00	25.5	40.0	16.6
2		67.14	360.00	42.2	43.1	1.6
4		14.05	180.00	32.0	26.1	0.4
5		186.85	360.00	34.0	35.4	5.5

As for the differences in vibration reduction levels they look differently when we compare them after recalculating these values for best case scenario – that is using the whole back of the plate as sensor (L_{redf} column for tables 3 and 4, and L_{red} column for table 2). These results are shown on Figure 4.

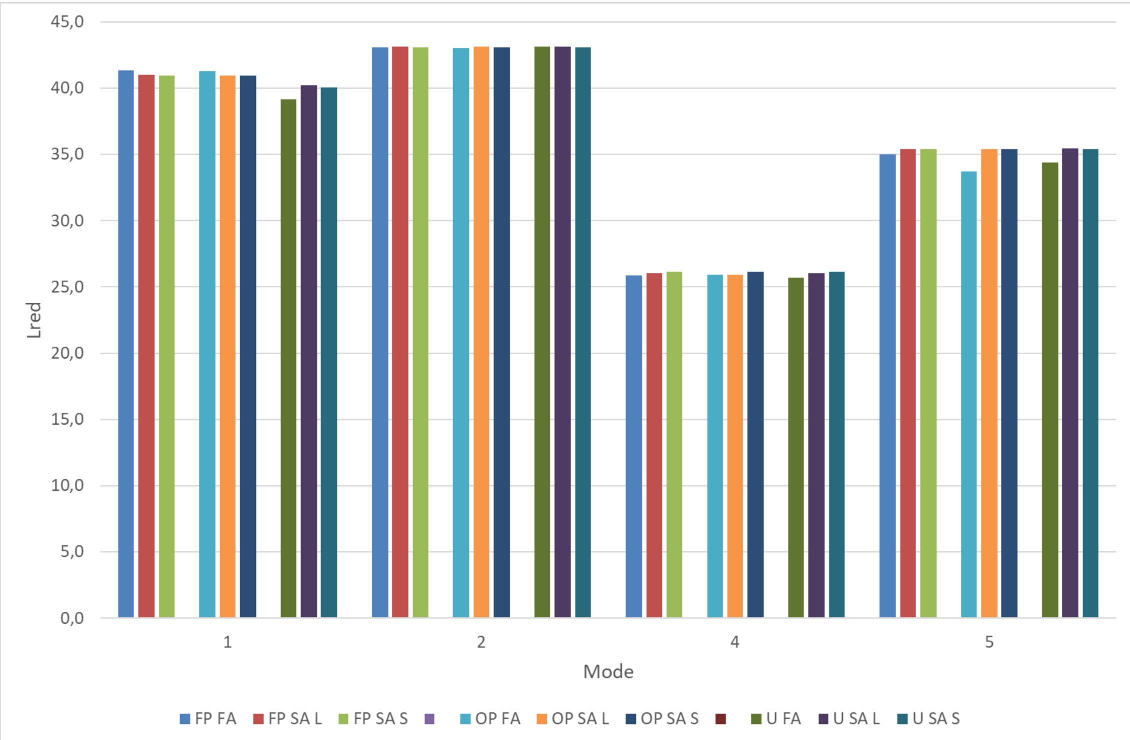

Figure 4. Comparison of vibration reduction levels obtained for square based actuator recalculated for the full plate; FP – using scenario with a whole back of the plate as a sensor; OP – using sensor placed on the diagonal in the upper right quadrant of the plate; Using sensor placed under actuator (or sensor-actuator); FA – full actuator; SA L – sensor-actuator with larger sensor part; SA S – sensor actuator with smaller sensor part.

The first 3 bars for each mode are the results when using every node on the back of the plate for obtaining the vibration level reduction values. These were discussed above.

The next 3 bars for each node are the results from the scenario when the sensor was placed on the diagonal in the upper right side of the plate recalculated using all nodes of the back of the plate. This mostly show us if there is a change when moving from global vibration reduction to more of a local approach. It can be seen that for the 1st mode when using full plate or the opposite sensor the results obtained using sensor actuator are slightly lower (about 0.3 dB) than when using full actuator. So that would be similar to the global scenario. There are basically no differences for the 2nd mode (at most 0.1 dB). For the 4th mode we also don't see any significant changes comparing to global reduction. For the 5th mode it can be seen that the normal actuator had the lowest reduction level at about 1.7 dB lower than for the sensor-actuator hybrids. Differences for the results from the sensor were at most 0.2 dB, which might indicate, that for some situations sensor-actuator of "frame" type actuator (with cut out middle part) might be better.

The last 3 bars represent the most interesting part of this comparison – the recalculated results for the scenario with a sensor corresponding in size and placement to the sensor part of sensor-actuator. For the 1st mode it can be seen that the vibration reduction levels are lower than for other scenarios. The difference for sensor-actuator is about 0.8-1.0 dB and for the full actuator 2.1 dB. Additionally in this scenario the full actuator reduction levels are lower than those for sensor-actuators by at least 0.8 dB. This last difference is probably due to there being a working part of actuator in the same place as sensor (so the minimum values of displacement vector of these nodes are slightly higher). Results for the 2nd and 4th modes are similar to those for the other scenarios. And as for the 5th mode it can be observed that again the results for sensor-actuators are higher than those for full actuator (by 1.0 dB).

From this we can assume that for modelled square based piezo elements the sensor part of sensor-actuator does not have a detrimental effect to the vibration reduction levels that can be obtained. And that reducing the size of said sensor part did not have any significant impact on the results. There is of course the matter of suboptimal placement of sensor as observed for the 1st mode.

One should remember, that numerical simulations show somewhat idealised results. It can be assumed that for physical experiments obtained values of reduction can be somewhat lower. For example, there was no intrinsic noise for modelled set-up. Still similar set-ups for physical experiment yielded vibration reduction values of up to 30-36 dB for one side clamped plate [22] and up to 34-40 dB for a 500x500x2mm plate clamped on all sides [23] and for more complicated set-ups using circular plate [24] up to 25-28 dB.

4. Conclusions

As was in the results the proposed sensor-actuator hybrid appear to work fairly well. There do not appear to be any significant differences between the vibration reduction levels obtained using sensor-actuator hybrids and full actuators. In some cases the hybrids shows very slight improvement in obtained reduction levels (although very slight). Also the sensor part seem to be serving its purpose. The results obtained when using a sensor placed directly as the sensor part and with its size are quite comparable to other scenarios. Of course as with normal sensor and actuators the issue of proper (optimal) placement is very important.

Of course there is a price to pay for introduced changes. Cutting out part of an actuator results in fair increase in voltages needed to obtain afore mentioned reduction levels. This can be partially negated by a smaller sensor part of sensor-actuator hybrid, but that will have limits. Although in presented results reducing the size of the sensor part did not introduce any detriments, the question remains what would be the optimal size and ratio. Of course we could also increase the overall size of the hybrid, but that also has its limits.

The next step of research would be creating a prototype of sensor-actuator hybrid on the basis of second iteration models. This would be done by cutting out the centre of piezoelectric actuator and then gluing the left outer part to beam or plate and gluing a smaller piezo electric in the centre.

The next steps in modelling would be introducing a dielectric between sensor and actuator parts of sensor-actuator hybrid instead of a gap and changing the shape of the sensor part. Assuming that a model of square based actuator part with a disc based sensor part would shows similar behaviour to current models, that would allow for additional prototypes to create and test.

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Additional information

The author(s) declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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