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Bogalecka Magda

Kołowrocki Krzysztof
Maritime University, Gdynia, Poland

Modeling, identification and prediction of environment degradation initiating events process generated by critical infrastructure accidents

Keywords

critical infrastructure, accident consequences, initial events, environment degradation

Abstract

The paper is the first part of the probabilistic general model of critical infrastructure accident consequences including the process of initiating events, the process of environment threats and the process of environment degradation models. Basic notions concerned with the events initiating dangerous for the environment after the critical infrastructure accident or its loss of safety critical level are introduced. The methods and procedures of estimating the process of initiating events unknown basic parameters and identifying the distributions of its conditional sojourn times at its states are proposed. Under these all assumptions from the constructed model and after its unknown parameters identification, the main characteristics of the process of the initiating events are predicted. Finally, the proposed model and methods are applied to modelling, identification and prediction of the process of initial events generated by the critical infrastructure defined as a ship operating in the Baltic Sea basin.

1. Introduction

Some kinds of critical infrastructure accidents concerned with its safety level decrease may occur during its operation [3], [5], [7], [9], [10]-[12], [15]-[20], [24]. Those accidents may bring some dangerous consequences for the environment and have disastrous influence on the human health and activity [7]-[8]. Such accidents can generate the initiating event causing dangerous situations in the critical infrastructures operation surroundings. The process of those initiating events can result in the environment treats and lead to the environment dangerous degradations [1]-[3]. There is the need to join those three interacting processes, i.e. the process of initiating events, the process of environment threats and the process of environment degradation into one general process of the critical infrastructure accident consequences model. The paper is concerned with the first one of the three processes modelling, identification and prediction and its preliminary application to the critical infrastructure a ship operating at the Baltic Sea waters.

The process of initiating events and its states are defined. The vectors of initial probabilities of these process staying at its particular states, the matrix of probabilities of this process transitions between its particular states, the matrix of conditional distribution functions and the matrix of conditional density functions of this process conditional sojourn times at its particular states are defined.

The formulae estimating the probabilities of this process staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states are proposed. The mean values of the process conditional sojourn times at its particular states are determined. Moreover, the distribution functions of this process unconditional sojourn times at its particular states, the mean values of these processes unconditional sojourn times at their particular states, the limit values of the transient probabilities of this process at its particular operation states and the approximate mean values of this process sojourn times at its particular states for the fixed sufficiently large time are determined.

The proposed model and methods are applied to modelling, identification and prediction of the process of initial events generated by the critical infrastructure defined as a ship operating at the sea waters.

2. Process of initial events

2.1. Process of initial events modelling

We fix the time interval $\langle 0,T\rangle$, $T>0$, where T is the time of a critical infrastructure operation and we distinguish n_1, $n_1 \in N$, events initiating the dangerous situation of the critical infrastructure operation environment and mark them by $E_1, E_2, ..., E_{n_1}$. Moreover, we introduce the set

$$E = \{e: \quad e = [e_1, e_2, ..., e_{n_1}],\ e_i \in \{0,1\}\},$$

where

$$e_i = \begin{cases} 1, \text{if initiating event } E_i \text{ occurs,} \\ 0, \text{if initiating event } E_i \text{ does not occur} \end{cases}$$

for $i = 1,2,...,n_1$.

Definition 1. A function $E(t)$ defined on the time interval $\langle 0,T\rangle$ and having values from the set E i.e.

$$E: \langle 0,T\rangle \to E$$

is called a process of initiating events.
The vectors of the set *E* are called the states of the process $E(t)$ while the set *E* is called the set of states of the process $E(t)$.
We number the states of the process of initiating events $E(t)$ and we assume that this process has *v* different states from the set, i.e., we assume that

$$E = \{e^1, e^2, ..., e^v\},$$

where

$$e^k = [e_1^k, e_2^k, ..., e_v^k],\ k = 1,2,...,v, \tag{1}$$

and

$$e_j^k \in \{0,1\},\ j = 1,2,...,n_1.$$

Further, we assume a semi-Markov model [4], [8], [13]-[14], [22]-[23] of the process $E(t)$ and denote by θ^{kl} its random conditional sojourn time in the state e^k while its next transition will be done to the state e^l, $k,l = 1,2,...,v$, $k \neq l$. Then, the process is described by the vector of probabilities of its initial states at the moment $t = 0$

$$[p(0)]_{1xv} = [p^1(0), p^2(0), ..., p^v(0)], \tag{2}$$

and by the matrix of probabilities of transitions between the states

$$[p^{kl}]_{vxv} = \begin{bmatrix} p^{11} & p^{12} & . & . & . & p^{1v} \\ p^{21} & p^{22} & . & . & . & p^{2v} \\ . & . & . & . & . & . \\ p^{v1} & p^{v2} & . & . & . & p^{vv} \end{bmatrix}, \tag{3}$$

where

$$\forall k = 1,2,...,v \ \ p^{kk} = 0.$$

Moreover this process is defined by the matrix of conditional distribution functions of sojourn times θ^{kl} of the process *E*(*t*) in the state e^k while its next transition will be done to the state e^l, $k,l = 1,2,...,v$, $k \neq l$,

$$[H^{kl}(t)]_{vxv} = \begin{bmatrix} H^{11}(t) & H^{12}(t) & . & . & . & H^{1v}(t) \\ H^{21}(t) & H^{22}(t) & . & . & . & H^{2v}(t) \\ . & . & . & . & . & . \\ H^{v1}(t) & H^{v2}(t) & . & . & . & H^{vv}(t) \end{bmatrix}, \tag{4}$$

where $\forall k = 1,2,...,v \ \ H^{kk}(t) = 0.$

This matrix is complied with the matrix of conditional densities of sojourn times θ^{kl} of the process *E*(*t*) in the state e^k while its next transition will be done to the state e^l, $k,l = 1,2,...,v$, $k \neq l$,

$$[h^{kl}(t)]_{vxv} = \begin{bmatrix} h^{11}(t) & h^{12}(t) & . & . & . & h^{1v}(t) \\ h^{21}(t) & h^{22}(t) & . & . & . & h^{2v}(t) \\ . & . & . & . & . & . \\ h^{v1}(t) & h^{v2}(t) & . & . & . & h^{vv}(t) \end{bmatrix}, \tag{5}$$

where

$$\forall k = 1,2,...,v \ \ h^{kk}(t) = 0.$$

2.2. Process of initial events identification

In order to estimate parameters of the process of initiating events $E(t)$, we firstly fix the number of states v of the process $E(t)$ and define the states $e^1, e^2, ..., e^v$ of the set E. Further, we fix the vector of realisations $n^k(0),\ k = 1,2,...,v,$ of the numbers of the process $E(t)$ transients in the particular states e^k at the initial moment $t = 0$

$$[n^k(0)]_{1xv} = [n^1(0), n^2(0), ..., n^v(0)], \tag{6}$$

and we fix the matrix of realisations n^{kl} $k,l = 1,2,...,v,$ of the numbers of the process $E(t)$ transitions from the state e^k into the state e^l during the experimental time

$$[n^{kl}]_{vxv} = \begin{bmatrix} n^{11} & n^{12} & . & . & . & n^{1v} \\ n^{21} & n^{22} & . & . & . & n^{2v} \\ . & . & . & . & . & . \\ n^{v1} & n^{v2} & . & . & . & n^{vv} \end{bmatrix}. \tag{7}$$

Having these numbers, we estimate the vector of realisations $[p^k(0)]_{1xv},\ k = 1,2,...,v,$ of the initial probabilities of the process $E(t)$ transients in the particular states e^k at the moment $t = 0$ according to the formula

$$p^k(0) = \frac{n_k(0)}{n(0)},\ k = 1,2,...,v, \tag{8}$$

where

$$n(0) = \sum_{k=1}^{v} n_k(0), \tag{9}$$

is the total number of the process $E(t)$ realisations at $t = 0$.

Next, we evaluate the matrix of realisations $[p^{kl}]$ $k,l = 1,2,...v,$ of the transitions probabilities of the process $E(t)$ from the state e^k into the state e^l during the experimental time according to the formula

$$p^{kl} = \frac{n^{kl}}{n^k},\ k,l = 1,2,...,v,\ k \neq l, \tag{10}$$

and

$$p^{kk} = 0,\ k = 1,2,...,v,$$

where

$$n^k = \sum_{l \neq k}^{v} n^{kl},\ k = 1,2,...,v, \tag{11}$$

is the realisation of the total number of the process $E(t)$ transitions from the state e^k during the experimental time.

Further, we formulate and verify the hypotheses about the conditional distribution functions of the process $E(t)$ lifetime $\theta^{kl},\ k,l = 1,2,...,v,\ k \neq l,$ in the state e^k while the next transition is to the state e^l on the base of their realisations θ^{kl}_{γ}, $\gamma = 1,2,...,n^{kl}$.

Prior to estimating the parameters of the distributions of the conditional sojourn times of the system operation process at the particular operation states, we have to determine the following empirical characteristics of the realizations of the conditional sojourn time of the system operation process at the particular operation states:

- the realizations of the empirical mean values $\bar{\theta}^{kl}$ of the conditional sojourn times $\theta^{kl},\ k,l = 1,2,...,v,$ $k \neq l,$ in the state e^k while the next transition is to the state e^l on the base of their realizations θ^{kl}_{γ}, $\gamma = 1,2,...,n^{kl}$, according to the formula

$$\bar{\theta}^{kl} = \frac{1}{n^{kl}} \sum_{\gamma=1}^{n^{kl}} \theta^{kl}_{\gamma},\ k,l = 1,2,...,v,\ k \neq l; \tag{12}$$

- the number $\bar{r}^{kl}$ of the disjoint intervals $I_j = \langle a^{kl}_j, b^{kl}_j),\ j = 1,2,...,\bar{r}^{kl}$, that include the realizations θ^{kl}_{γ}, $\gamma = 1,2,...,n^{kl}$, of the conditional sojourn times $\theta^{kl},\ k,l = 1,2,...,v,\ k \neq l,$ in the state e^k while the next transition is to the state e^l, according to the formula

$$\bar{r}^{kl} \cong \sqrt{n^{kl}}; \tag{13}$$

- the length d^{kl} of the intervals $I_j = \langle a^{kl}_j, b^{kl}_j)$, $j = 1,2,...,\bar{r}^{kl}$, according to the formula

$$d^{kl} = \frac{\bar{R}^{kl}}{\bar{r}^{kl} - 1}, \tag{14}$$

where

$$\bar{R}^{kl} = \max_{1 \le \gamma \le n^{kl}} \theta_\gamma^{kl} - \min_{1 \le \gamma \le n^{kl}} \theta_\gamma^{kl}, \tag{15}$$

- the ends a_j^{kl}, b_j^{kl}, $j = 1,2,...,\bar{r}^{kl}$, of the intervals $I_j = < a_j^{kl}, b_j^{kl})$, $j = 1,2,...,\bar{r}^{kl}$, according to the formulae

$$a_1^{kl} = \max\{\min_{1 \le \gamma \le n^{kl}} \theta_\gamma^{kl} - \frac{d^{kl}}{2}, 0\},$$

$$b_j^{kl} = a_1^{kl} + jd^{kl}, \quad j = 1,2,...,\bar{r}^{kl}, ,$$

$$a_j^{kl} = b_{j-1}^{kl}, \; j = 2,3,...,\bar{r}^{kl}, \tag{16}$$

in such a way that

$$I_1 \cup I_2 \cup ... \cup I_{\bar{r}^{kl}} = < a_1^{kl}, b_{\bar{r}^{kl}}^{kl})$$

and

$$I_i \cap I_j = \emptyset \text{ for all } i \ne j, \; i, j = 1,2,...,\bar{r}^{kl},$$

- the numbers n_j^{kl} of the realizations θ_γ^{kl}, $\gamma = 1,2,...,n^{kl}$, in the intervals $I_j = < a_j^{kl}, b_j^{kl})$, $j = 1,2,...,\bar{r}^{kl}$, according to the formula

$$n_j^{kl} = \#\{\gamma : \theta_\gamma^{kl} \in I_j, \gamma \in \{1,2,...,n^{kl}\}\}, \; j = 1,2,...,\bar{r}^{kl}, \tag{17}$$

where

$$\sum_{\gamma=1}^{\bar{r}^{kl}} n_\gamma^{kl} = n^{kl},$$

whereas the symbol # means the number of elements of the set;

The way, how to estimate the parameters of the distributions of the conditional sojourn times θ^{kl}, $k, l = 1,2,...,v$, $k \ne l$, in the state e^k while the next transition is to the state e^l, is presented in Chapter 2 [14].

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution of the process of initial events conditional sojourn time θ^{kl}, at the state e^k while the next transition is to the state e^l, on the basis of its realizations θ_γ^{kl}, $\gamma = 1,2,...,n^{kl}$, it is due to proceed according to the following scheme:

- to construct and to plot the realization of the histogram of the process of initial events conditional sojourn time θ^{kl}, at the operation state e^k, defined by the following formula

$$\bar{h}_{n^{kl}}^{kl}(t) = \frac{n_j^{kl}}{n^{kl}} \text{ for } t \in I_j, \tag{18}$$

- to analyse the realization of the histogram $\bar{h}_{n^{kl}}(t)$, comparing it with the graphs of the density functions of the distinguished in Chapter 2 [14] distributions, to select one of them and to formulate the null hypothesis H_0, concerning the unknown form of the distribution of the conditional sojourn time θ^{kl} in the following form:

H_0: The initiating events process conditional the conditional sojourn times θ^{kl} in the state e^k while the next transition is to the state e^l has the distribution with the density function $h^{kl}(t)$;

- to join each of the intervals I_j that has the number n_j^{kl} of realizations less than 4 either with the neighbour interval I_{j+1} or with the neighbour interval I_{j-1} this way that the numbers of realizations in all intervals are not less than 4;

- to fix a new number of intervals $\bar{\bar{r}}^{kl}$;

- to determine new intervals

$$\bar{I}_j = < \bar{a}_j^{kl}, \bar{b}_j^{kl}), \quad j = 1,2,...,\bar{\bar{r}}^{kl};$$

- to fix the numbers $\bar{n}_j^{kl}$ of realizations in new intervals $\bar{I}_j$, $j = 1,2,...,\bar{\bar{r}}^{kl}$;

- to calculate the hypothetical probabilities that the variable θ^{kl} takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j = P(\theta^{kl} \in \bar{I}_j) = P(\bar{a}_j^{kl} \le \theta^{kl} < \bar{b}_j^{kl}) = H^{kl}(\bar{b}_j^{kl})$$

$$- H^{kl}(\bar{a}_j^{kl}), \; j = 1,2,...,\bar{\bar{r}}^{kl}; \tag{19}$$

where $H^{kl}(\bar{b}_j^{kl})$ and $H^{kl}(\bar{a}_j^{kl})$ are the values of the distribution function $H^{kl}(t)$ of the random variable θ_{bl} corresponding to the density function $h^{kl}(t)$; assumed in the null hypothesis H_0;

- to calculate the realization of the χ^2 (chi-square)-Pearson's statistics $U_{n^{kl}}$, according to the formula

$$u_{n^{kl}} = \sum_{j=1}^{\bar{\bar{r}}^{kl}} \frac{(\bar{n}_j^{kl} - n^{kl} p_j)^2}{n^{kl} p_j}; \tag{20}$$

- to assume the significance level α (for instance $\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test;
- to fix the number $\bar{\bar{r}}^{kl} - l - 1$ of degrees of freedom, substituting for l the number of unknown parameters of the distribution function $H^{kl}(t)$ estimated on the basis of the sojourn time θ^{kl} realizations;
- to read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}}^{kl} - l - 1$ such that the following equality holds

$$P(U_n^{kl} > u_\alpha) = \alpha, \tag{21}$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,
- to compare the obtained value $u_{n^{kl}}$ of the realization of the statistics $U_{n^{kl}}$ with the read from the Tables critical value u_α of the chi-square random variable and to decide on the previously formulated null hypothesis H_0 in the following way: if the value $u_{n^{kl}}$ does not belong to the critical domain, i.e. when $u_{n^{kl}} \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value $u_{n_{bl}}$ belongs to the critical domain, i.e. when $u_{n^{kl}} > u_\alpha$, then we reject the hypothesis H_0.

2.3. Process of initiating evens prediction

Under the previous assumptions about the process of the initiating events $E(t)$, after the identification of its model unknown parameters its selected characteristics parameters can be found.

Namely, the expected values $E[\theta^{kl}]$ and variances $D[\theta^{kl}]$ of variables θ^{kl} are respectively determined by

$$M^{kl} = E[\theta^{kl}] = \int_0^\infty t h^{kl}(t) dt, \tag{22}$$

and

$$D^{kl} = D[\theta^{kl}] = \int_0^\infty (t - E[\theta^{kl}])^2 h^{kl}(t) dt$$
$$= E[(\theta^{kl})^2] - (M^{kl})^2,$$

where

$$E[(\theta^{kl})^2] = \int_0^\infty t^2 h^{kl}(t) dt$$

and M^{kl} are determined by (22)

From the formula for total probability it follows that the unconditional distribution functions of sojourn times θ^k of the process of initiating events $E(t)$ in states e^k, $k = 1,2,...,v$, are determined by

$$H^k(t) = \sum_{l=1}^{v} p^{kl} H^{kl}(t), \ k = 1,2,\ldots,v, \tag{23}$$

and their corresponding density functions are given by

$$h^k(t) = \sum_{l=1}^{v} p^{kl} h^{kl}(t), \ k = 1,2,\ldots,v. \tag{24}$$

Hence, the expected values $E[\theta^k]$ and variances $D[\theta^k]$ of variables θ^k are given respectively by

$$M^k = E[\theta^k] = \sum_{l=1}^{v} p^{kl} M^{kl}, \ k = 1,2,\ldots,v, \tag{25}$$

$$D^k = D[\theta^k] = E[(\theta^k)^2] - (M^k)^2, \ k = 1,2,\ldots,v,$$

where

$$E[(\theta^k)^2] = \int_0^\infty t^2 h^k(t) dt = \sum_{l=1}^{v} p^{kl} (M^{kl})^2$$

and M^{kl} are defined by (22), p^{kl} are defined by (3) and M^k are determined by (25).

The limit values of the instantaneous probabilities (the transient probabilities) of the process of initiating events $E(t)$ in its particular states

$$p^k(t) = P(E(t) = e^k), \ k = 1,2,...,v,$$

are calculated from the formula

$$p^k = \lim_{t \to \infty} p^k(t) = \frac{\pi^k M^k}{\sum_{l=1}^{v} \pi^l M^l}, \ k = 1,2,...,v, \tag{26}$$

where probabilities π^k satisfy the system of the following equations

$$\begin{cases}[\pi^k] = [\pi^k][p^{kl}] \\ \sum_{l=1}^{v} \pi^l = 1,\end{cases} \quad (27)$$

where

$$[\pi^k] = [\pi^1, \pi^2, ..., \pi^v]$$

and $[p^{kl}]$ is given by (3).

The asymptotic distribution of the sojourn total time $\hat{\theta}^k$ of the process of initiating events $E(t)$ in the time interval $\langle 0, \theta \rangle$, $\theta > 0$, in the state e^k is normal with the expected value

$$\hat{M}^k = E[\hat{\theta}^k] \cong p^k \theta, \quad (28)$$

where p^k are given by (26).

3. Application – preliminary analysis of environment degradation initial events process generated by an accident of a ship operating at Baltic Sea

The Baltic Sea and nearby ecosystems are vulnerable to pollution and contamination as a result of sea accident during the dangerous goods transportation. These days, one major accident at the Baltic Sea happens every year approximately. There are more than 50,000 ships entering and leaving the Baltic Sea every year and about 2,000 vessels are at the Baltic Sea at any given moment as illustrated in *Figure 1* presenting the map of the region under discussion.

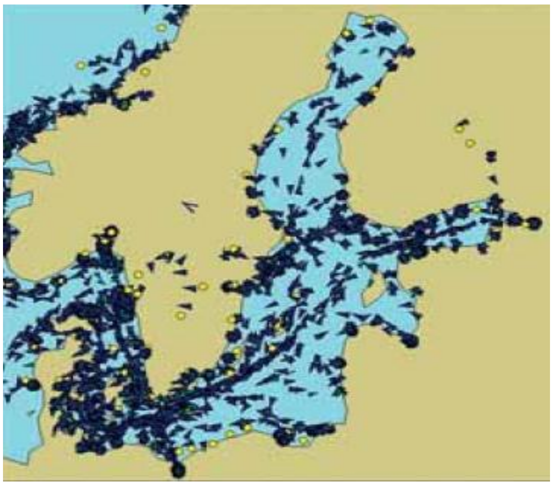

Figure 1. Snapshot from Automatic Identification System registered ships at the Baltic Sea (each spot on the map is a ship, the source: HELCOM)

This huge traffic across the Baltic Sea was observed. In the analysis, 11 events initiating dangerous situations was distinguished

3.1. Modelling process of initiating events generated by ship accidents

The initiating events that may be dangerous for the sea environment initial sea accidents are classified according to the International Maritime Organization document – MSC-MEPC.3/Circ.1 [6]. These events are marked by E_i, i = 1,2, ..., 11, and they are as follows:

- E_1 – collision,
- E_2 – stranding or grounding,
- E_3 – contact,
- E_4 – fire or explosion,
- E_5 – hull failure or failure of watertight doors, ports, etc.,
- E_6 – machinery damage,
- E_7 – damages to ship or equipment,
- E_8 – capsizing or listing,
- E_9 – missing (assumed lost),
- E_{10} – accidents with life-saving appliances,
- E_{11} – other events.

Considering *Definition 1*, we distinguish the following states of the process $E(t)$ of initiating events:

e^1=[0,0,0,0,0,0,0,0,0,0,0], e^2=[1,0,0,0,0,0,0,0,0,0,0],

e^3=[0,1,0,0,0,0,0,0,0,0,0], e^4=[0,0,1,0,0,0,0,0,0,0,0],

e^5=[0,0,0,1,0,0,0,0,0,0,0], e^6=[0,0,0,0,1,0,0,0,0,0,0],

e^7=[0,0,0,0,0,1,0,0,0,0,0], e^8=[0,0,0,0,0,0,1,0,0,0,0],

e^9=[0,0,0,0,0,0,0,1,0,0,0],
e^{10}=[0,0,0,0,0,0,0,0,1,0,0],

e^{11}=[0,0,0,0,0,0,0,0,0,1,0],
e^{12}=[0,0,0,0,0,0,0,0,0,0,1],

e^{13}=[0,0,0,0,0,0,1,1,0,0,0],
e^{14}=[0,1,0,0,1,0,0,0,0,0,0],

e^{15}=[0,1,0,0,0,0,1,0,0,0,0],
e^{16}=[0,0,0,0,1,1,0,1,0,0,0],

e^{17}=[0,1,0,0,0,0,0,1,0,0,0],
e^{18}=[0,1,0,0,0,1,1,0,0,0,0],

e^{19}=[0,0,0,0,1,0,1,0,0,0,0],
e^{20}=[0,1,0,0,0,1,0,0,0,0,0],

e^{21}=[0,0,0,1,0,0,1,0,0,0,0],
e^{22}=[0,0,1,0,0,1,0,0,0,0,0],

e^{23}=[0,0,0,1,1,1,1,0,0,0,0],
e^{24}=[0,0,0,1,1,1,1,1,0,0,0],

e^{25}=[0,1,0,1,1,1,1,1,0,0,0],
e^{26}=[0,0,0,0,1,1,0,0,0,0,0],

e^{27}=[0,0,0,1,0,1,0,0,0,0,0],
e^{28}=[0,0,0,0,1,0,0,1,0,0,0],

e^{29}=[0,1,0,0,0,0,0,0,0,0,1],
e^{30}=[0,1,0,0,1,0,1,0,0,0,0],

e^{31}=[0,0,0,0,1,1,1,0,0,0,0],
e^{32}=[0,0,0,0,0,1,0,1,0,0,0],

e^{33}=[0,1,0,1,0,0,0,0,0,0,0],
e^{34}=[0,0,0,1,0,1,1,0,0,0,0],

e^{35}=[0,0,0,0,0,1,1,0,0,0,0],
e^{36}=[1,0,0,0,1,0,0,0,0,0,0],

e^{37}=[1,0,0,0,0,0,1,0,0,0,0],
e^{38}=[0,1,0,1,0,1,1,0,0,0,0],

e^{39}=[1,1,0,0,1,0,0,0,0,0,0],
e^{40}=[0,0,1,0,1,0,0,0,0,0,0],

e^{41}=[1,0,0,0,0,1,0,0,0,0,0],
e^{42}=[0,0,0,0,0,1,0,0,1,0,0],

e^{43}=[0,1,0,0,0,1,0,0,1,0,0].

The state

e^{1}=[0,0,0,0,0,0,0,0,0,0,0]

means that no initial event dangerous for the environment takes place after the ship accident.
Then, according to (2)-(5), the process of initiating events is described by the vector of probabilities $[p(0)]_{1x43}$ of its initial states at the moment $t=0$, the matrix of probabilities of transitions between the states $\left[p^{kl}\right]_{vx43}$ and the matrix of conditional distribution functions $\left[H^{kl}(t)\right]_{vx43}$ of sojourn times of the process of initiating events at the particular states or equivalently by corresponding to this matrix the matrix of conditional density functions $\left[h^{kl}(t)\right]_{vx43}$.

3.2. Identification of process of initiating events generated by ship accidents

The experiment was performed in the region of the Baltic Sea basin in the years 2004-2007. The number of the observed ship accidents that generated the distinguished states of the process of initiating events was $n(0)=357$. The initial moment $t=0$ of the process of initiating event for each ship was fixed at the moment when the ship after an accident generated one of the distinguished states.
After this experiment it was possible to fix the vector of realisations $n^k(0)$, $k=1,2,...,11$, of the numbers of the process $E(t)$ of initiating events staying at the particular states e^k at the initial moment $t=0$. The fixed vector those realisations is

$[n_b(0)]_{1x11}$ = [0, 79, 99, 24, 35, 17, 29, 19, 16, 1, 1, 7, 1, 5, 2, 0, 1, 1, 2, 1. 5, 1, 1, 1, 0, 0, 1, 2, 0, 0, 0, 1,0, 3, 1, 1, 0, 0, 0, 0, 0, 1, 0].

Hence, by (9), the total number of the process $E(t)$ realisations at the moment $t=0$ is

$$n(0)=\sum_{k=1}^{11} n_k(0) = 0 + 79 + ... + 1 + 0 = 357$$

and further, according to (8), the evaluations of the initial probabilities are as follows:

$p^1(0) = 0$, $p^2(0) = 0.2213$, $p^3(0) = 0.2773$,

$p^4(0) = 0.0672$, $p^5(0) = 0.0980$, $p^6(0) = 0.0476$,

$p^7(0) = 0.0812$, $p^8(0) = 0.0532$, $p^9(0) = 0.0448$,
$p^{10}(0) = 0.0028$, $p^{11}(0) = 0,0028$, $p^{12}(0) = 0.0196$,

$p^{13}(0) = 0.0028$, $p^{14}(0) = 0.0140$, $p^{15}(0) = 0.0056$,

$p^{16}(0) = 0.0000$, $p^{17}(0) = 0.0028$, $p^{18}(0) = 0.0028$,

$p^{19}(0) = 0.056$, $p^{20}(0) = 0.0028$, $p^{21}(0) = 0.0140$,

$p^{22}(0) = 0.0028$, $p^{23}(0)= 0.028$, $p^{24}(0) = 0.0028$,

$p^{25}(0) = 0.0000$, $p^{26}(0) = 0.0000$, $p^{27}(0) = 0.0028$,

$p^{28}(0) = 0.0056$, $p^{29}(0) = 0.0000$, $p^{30}(0) = 0.0000$,

$p^{31}(0) = 0.000$, $p^{32}(0) = 0.0028$, $p^{33}(0) = 0.0000$,

$p^{34}(0) = 0.0084$, $p^{35}(0) = 0.0028$, $p^{36}(0) = 0.0028$,

$p^{37}(0) = 0.0000$, $p^{38}(0) = 0.0000$, $p^{39}(0) = 0.0000$,

p^{40} (0) = 0.0000, $p^{41}(0)$ = 0.0000, $p^{42}(0)$ = 0.0028,

$p^{43}(0)$ = 0.0000. (29)

At the experiment it also was possible to fix the realisations n^{kl} $k,l = 1,2,\ldots,43,$ of the numbers of the process $E(t)$ transitions from the state e^k into the state e^l during the experimental time:

$n^{1\,3}$ = 2, $n^{1\,4}$ = 1, $n^{1\,12}$ = 1; $n^{2\,3}$ = 14, $n^{2\,5}$ = 1, $n^{2\,6}$ = 26,

$n^{2\,8}$ = 17, $n^{2\,9}$ = 2; $n^{2\,19}$ = 7, $n^{2\,31}$ = 1; $n^{3\,1}$ = 23,

$n^{3\,6}$ = 3, $n^{3\,8}$ = 2, $n^{3\,14}$ = 19, $n^{3\,15}$ = 12, $n^{3\,17}$ = 1,

$n^{3\,19}$ = 2, $n^{3\,30}$ = 2; $n^{4\,3}$ = 2, $n^{4\,6}$ = 11, $n^{4\,8}$ = 6, $n^{4\,26}$ = 1,

$n^{4\,31}$ = 1, $n^{4\,35}$ = 1; $n^{5\,1}$ = 6, $n^{5\,21}$ = 16, $n^{5\,23}$ = 1,

$n^{5\,27}$ = 4, $n^{5\,33}$ = 4, $n^{5\,34}$ = 6; $n^{6\,14}$ =7, $n^{6\,19}$ = 1,
$n^{6\,28}$ = 1, $n^{6\,36}$ = 1, $n^{6\,40}$ = 1; $n^{7\,15}$ = 1, $n^{7\,20}$ = 4,

$n^{7\,22}$ = 2, $n^{7\,26}$ = 1, $n^{7\,27}$ = 4, $n^{7\,32}$ = 3, $n^{7\,35}$ = 1,

$n^{7\,41}$ = 1, $n^{7\,42}$ = 1; $n^{8\,1}$ = 1, $n^{8\,13}$ = 2, $n^{8\,15}$ = 2, $n^{8\,19}$ = 1,

$n^{8\,21}$ = 3, $n^{8\,37}$ = 1; $n^{9\,2}$ = 1, $n^{9\,3}$ = 6, $n^{9\,13}$ = 1, $n^{9\,31}$ = 1;

$n^{10\,3}$ = 1; $n^{11\,3}$ =1; $n^{12\,5}$ = 2, $n^{12\,7}$ = 2, $n^{12\,8}$ = 1,

$n^{12\,9}$ = 1, $n^{12\,29}$ = 1; $n^{13\,3}$ =1; $n^{14\,6}$ = 3, $n^{14\,8}$ = 1,

$n^{14\,39}$ = 1; $n^{15\,8}$ = 2, $n^{15\,30}$ = 2; $n^{16\,12}$ = 1; $n^{17\,8}$ = 1;

$n^{18\,15}$ = 1, $n^{18\,35}$ = 1; $n^{19\,2}$ = 1, $n^{19\,30}$ = 1; $n^{20\,18}$ = 1;

$n^{21\,8}$ = 5; $n^{22\,19}$ = 1; $n^{23\,24}$ = 1; $n^{24\,25}$ = 1; $n^{25\,8}$ = 1;

$n^{26\,12}$ = 1; $n^{27\,7}$ = 1; $n^{28\,14}$ = 2; $n^{29\,8}$ = 1; $n^{30\,8}$ = 1;

$n^{31\,8}$ = 1; $n^{32\,16}$ = 1; $n^{33\,8}$ = 1; $n^{34\,35}$ = 2, $n^{34\,38}$ = 1;

$n^{35\,8}$ = 5, $n^{35\,18}$ = 1; $n^{36\,28}$ = 1; $n^{37\,12}$ = 1; $n^{38\,19}$ = 1;

$n^{39\,8}$ = 1; $n^{40\,8}$ = 1; $n^{41\,19}$ = 1; $n^{42\,43}$ = 1; $n^{43\,8}$ = 1.

Hence, according to (11), the realisation of the total numbers of the process $E(t)$ transitions from the state e^k, $k = 1,2,\ldots,43,$ during the experimental time are:

n^1 = 4, n^2 = 68, n^3 = 64, n^4 = 22, n^5 = 37, n^6 = 11,

n^7 = 18, n^8 = 10, n^9 = 9, n^{10} = 1, n^{11} = 1, n^{12} = 7,

n^{13} = 1, n^{14} = 5, n^{15} = 4, n^{16} = 1, n^{17} = 1, n^{18} = 2,

n^{19} = 2, n^{20} = 1, n^{21} = 5, n^{22} = 1, n^{23} = 1, n^{24} = 1,

n^{25} = 1, n^{26} = 1, n^{27} = 1, n^{28} = 2, n^{29} = 1, n^{30} = 1,

n^{31} = 1, n^{32} = 1, n^{33} = 1, n^{34} = 3, n^{35} = 6, n^{36} = 1,

n^{37} = 1, n^{38} = 1, n^{39} = 1, n^{40} = 1, n^{41} = 1, n^{42} = 1,

n^{43} = 1.

Applying the formula (10), we can evaluate the matrix of realisations $[p^{kl}]$ $k,l = 1,2,\ldots 43,$ of the transitions probabilities of the process *E*(*t*) from the state e^k into the state e^l during the experimental time. The probabilities of transitions that are not equal to 0 are as follows:

$p^{1\,3}$ = 0.5000, $p^{1\,4}$ = 0.2500, $p^{1\,12}$ = 0.2500;

$p^{2\,3}$ = 0.2059, $p^{2\,5}$ = 0.0147, $p^{2\,6}$ = 0.3824,

$p^{2\,8}$ = 0.2500, $p^{2\,9}$ = 0.0294, $p^{2\,19}$ = 0.1029,

$p^{2\,31}$ = 0.0147;

$p^{3\,1}$ = 0.3594, $p^{3\,6}$ = 0.0469, $p^{3\,8}$ = 0.0313,

$p^{3\,14}$ = 0.2969, $p^{3\,15}$ = 0.1875, $p^{3\,17}$ = 0.0156,

$p^{3\,19}$ = 0.0312, $p^{3\,30}$ = 0.0312;

$p^{4\,3}$ = 0.0909, $p^{4\,6}$ = 0.5000, $p^{4\,8}$ = 0.2727,

$p^{4\,26}$ = 0.0455, $p^{4\,31}$ = 0.0455, $p^{4\,35}$ = 0.0454;

$p^{5\,1}$ = 0.1622, $p^{5\,21}$ = 0.4324, $p^{5\,23}$ = 0.0270,

$p^{5\,27}$ = 0.1081, $p^{5\,33}$ = 0.1081, $p^{5\,34}$ = 0.1622;

$p^{6\,14}$ =0.6364, $p^{6\,19}$ = 0.0909, $p^{6\,28}$ = 0.0909,

$p^{6\,36}$ = 0.0909, $p^{6\,40}$ = 0.0909;

$p^{7\,15}$ = 0.0555, $p^{7\,20}$ = 0.2222, $p^{7\,22}$ = 0.1111,

$p^{7\,26}$ = 0.0556, $p^{7\,27}$ = 0.2222, $p^{7\,32}$ = 0.1666,

$p^{7\,35}$ = 0.0556, $p^{7\,41}$ = 0.0556, $p^{7\,42}$ = 0.0556;

$p^{8\,1}$ = 0.1000, $p^{8\,13}$ = 0.2000, $p^{8\,15}$ = 0.2000,

$p^{8\,19} = 0.1000$, $p^{8\,21} = 0.3000$, $p^{8\,37} = 0.1000$;

$p^{9\,2} = 0.1111$, $p^{9\,3} = 0.6667$, $p^{9\,13} = 0.1111$,

$p^{9\,31} = 0.1111$; $p^{10\,3} = 1$; $p^{11\,3} = 1$;

$p^{12\,5} = 0.2857$, $p^{12\,7} = 0.2857$, $p^{12\,8} = 0.1429$,

$p^{12\,9} = 0.1429$, $p^{12\,29} = 0.1428$; $p^{13\,3} = 1$;

$p^{14\,6} = 0.6$, $p^{14\,8} = 0.2$, $p^{14\,39} = 0.2$,

$p^{15\,8} = 0.5$, $p^{15\,30} = 0.5$; $p^{16\,12} = 1$; $p^{17\,8} = 1$;

$p^{18\,15} = 0.5000$, $p^{18\,35} = 0.5000$;

$p^{19\,2} = 0.5000$, $p^{19\,30} = 0.5000$;

$p^{20\,18} = 1$; $p^{21\,8} = 1$; $p^{22\,19} = 1$; $p^{23\,24} = 1$; $p^{24\,25} = 1$;

$p^{25\,8} = 1$; $p^{26\,12} = 1$; $p^{27\,7} = 1$; $p^{28\,14} = 1$; $p^{29\,8} = 1$;

$p^{30\,8} = 1$; $p^{31\,8} = 1$; $p^{32\,16} = 1$; $p^{33\,8} = 1$;

$p^{34\,35} = 0.6666$, $p^{34\,38} = 0.3334$;

$p^{35\,8} = 0.8333$, $p^{35\,18} = 0.1667$;

$p^{36\,28} = 1$; $p^{37\,12} = 1$; $p^{38\,19} = 1$; $p^{39\,8} = 1$; $p^{40\,8} = 1$;

$p^{41\,19} = 1$; $p^{42\,43} = 1$; $p^{43\,8} = 1$. (30)

On the basis of the statistical data presented in Appendix 1, using the procedure and the formulae given in Section 2.2, it is possible to determine the empirical parameters of the conditional sojourn times θ^{kl}. To illustrate the application of this procedure, we perform it for θ^{31} that is one of the conditional sojourn times having most populous set of realizations.

The results for the conditional sojourn time θ^{31} are:

- the realization $\bar{\theta}^{31}$ of the defined by (12) mean value of the conditional sojourn time θ^{31} of the initial events process state e^3 when the next transition is to the initial events process state e^1

$$\bar{\theta}^{31} = \frac{1}{23}\sum_{\gamma=1}^{24}\theta_\gamma^{31} = 3513.2; \qquad (31)$$

- the number $\bar{r}^{31}$ of the disjoint intervals $I_j =< a_j^{31}, b_j^{31})$, $j = 1,2,\ldots,\bar{r}^{31}$, that include the realizations θ_γ^{31}, $\gamma = 1,2,\ldots,23$, of the conditional sojourn time θ^{31} of the initial events process state e^3 when the next transition is to the initial events process state e^1 defined by (13)

$$\bar{r}^{31} \cong \sqrt{23} \cong 5,$$

- the length d^{31} of the intervals $I_j =< a_j^{31}, b_j^{31})$, $j = 1,2,\ldots,5$, that after considering (16)

$$\bar{R}^{31} = \max_{1\le\gamma\le23}\theta_\gamma^{31} - \min_{1\le\gamma\le23}\theta_\gamma^{31} = 57600 – 10 = 57590,$$

is

$$d^{31} = \frac{\bar{R}^{31}}{\bar{r}^{31} - 1} = \frac{57590}{4} = 14397.5;$$

- the ends a_j^{31}, b_j^{31}, of the intervals $I_j =< a_j^{31}, b_j^{31})$, $j = 1,2,\ldots,5$, that after considering

$$\min_{1\le\gamma\le24}\theta_\gamma^{31} - \frac{d^{31}}{2} = 10 - \frac{14397.5}{2} = -7193.75,$$

are

$$a_1^{31} = \max\{–7193.75, 0\} = 0,$$

$$b_1^{31} = a_1^{31} + 14397.5 = 0 + 14397.5 = 14397.5,$$

$$a_2^{31} = b_1^{31} = 14397.5,$$

$$b_2^{31} = a_1^{31} + 2 \cdot 14397.5 = 0 + 28795 = 28795,$$

$$a_3^{31} = b_2^{31} = 28795,$$

$$b_3^{31} = a_1^{31} + 3 \cdot 14397.5 = 0 + 431925 = 431925,$$

$$a_4^{31} = b_3^{31} = 431925,$$

$$b_4^{31} = a_1^{31} + 4 \cdot 14397.5 = 0 + 57590 = 57590,$$

$$a_5^{31} = b_4^{31} = 57590,$$

$$b_5^{31} = a_1^{31} + 5 \cdot 14397.5 = 0 + 71987.5 = 71987.5; \qquad (32)$$

- the numbers n_j^{31} of the realizations θ_j^{31} in particular intervals $I_j =< a_j^{31}, b_j^{31})$, $j = 1,2,\ldots,5$, defined by (17)

$$n_1^{31} = 22,\ n_2^{31} = 0,\ n_3^{31} = 0,\ n_4^{31} =,\ n_5^{31} = 1. \qquad (33)$$

Using the procedure given in Section 2.2 and the statistical data from Appendix 1 and the above results, we may verify the hypotheses on the distributions of the process of initiating events conditional sojourn times θ^{kl}, $k,l = 1,2,\ldots,43$, $k \ne l$, at the particular states. To do this, we need a sufficient number of realizations of these variables, namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the statistical data we have in disposal and that are presented in *Appendix 1*. However, to make the procedure familiar to the reader, we perform it for the conditional sojourn time θ^{31}, the one of that having most numerous set of realizations and preliminarily analysed in this section.

Table 1. The realization of the histogram of the process initiating events conditional sojourn time θ^{31}

Histogram of the conditional sojourn time θ^{31}		
$I_j =< a_j^{31}, b_j^{31})$	n_j^{31}	$\bar{h}_{23}^{31}(t) = n_j^{31} / n^{31}$
0–14397.5	22	22/23
14397.5–28785	0	0/23
28795–43192.5	0	0/23
43192.5–57590	0	0/23
57590-71987.5	1	1/23

The realization $\bar{h}_{23}^{31}(t)$ of the histogram of the process of initiating events conditional sojourn time θ^{31}, defined by (18), is presented in *Table 1* and illustrated in *Figure 2.*

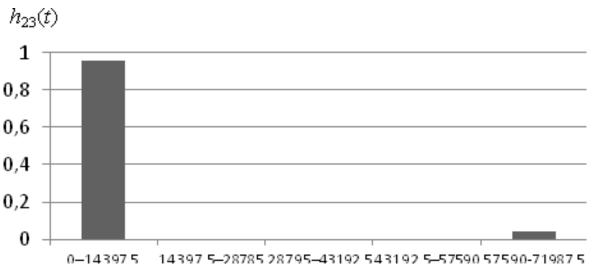

*Figure 2***.** The graph of the histogram of the process of initiating events conditional sojourn time θ^{31}

After analysing and comparing the realization $\bar{h}_{23}^{31}(t)$ of the histogram with the graphs of the density functions $h^{31}(t)$ of the previously distinguished in Chapter 2 [14] distributions, we formulate the null hypothesis H_0 in the following form:

H_0: The process of initiating events conditional sojourn time θ^{31} at the state e^3 when the next transition is to the operation state e^1, has the chimney distribution with the density function of the form

$$h^{31}(t) = \begin{cases} 0, & t < x_{31} \\ \dfrac{A_{31}}{z_{31}^1 - x_{31}}, & x_{31} \le t \le z_{31}^1 \\ \dfrac{C_{15}}{z_{31}^2 - z_{31}^1}, & z_{31}^1 \le t \le z_{31}^2 \\ \dfrac{D_{15}}{y_{31} - z_{31}^2}, & z_{31}^2 \le t \le y_{31} \\ 0, & t > y_{31}. \end{cases} \quad (34)$$

Since, according to (4.16)-(4.17) from [14], we have

$$\hat{n}^{31} = \max_{1 \le j \le 5}\{n_j^{31}\} = 22 \text{ and } n_1^{31} = \hat{n}_{31} = 22,$$

then $i = 1$. Moreover, by (4.22) in [14], we get

$$n_2^{31} = 0.$$

Therefore, we estimate the unknown parameters of the density function of the hypothetical chimney distribution using the formulae (4.15) and (4.18) from [14] and we obtain the following results

$$x_{31} = a_1^{31} = 0,$$

$$y_{31} = x_{31} + \bar{r}^{31} d^{31} = 0 + 5 \cdot 14397.5 = 71987.5,$$

$$z_{31}^1 = x_{31} + (1-1) d_{31} = 0,$$

$$z_{31}^2 = x_{31} + 1 d_{31} = 0 + 1 \cdot 14397.5 = 14397.5,$$

$$A_{31} = 0, \ C_{31} = \frac{n_1^{31}}{n^{31}} = \frac{22}{23},$$

$$D_{31} = \frac{n_2^{31} + n_3^{31} + n_4^{31} + n_5^{31}}{n^{31}} = \frac{0+0+0+1}{23} = \frac{1}{23}. \quad (35)$$

Substituting the above results into (34), we get completely defined the hypothetical density function in the form

$$h^{31}(t) =$$

$$\begin{cases} 0, & t < 0 \\ \dfrac{22/23}{14397.5 - 0}, & 0 \le t < 14397.5 \\ \dfrac{1/23}{71987.5 - 14397.5}, & 14397.5 \le t < 71987.5 \\ 0, & t \ge 71987.5 \end{cases}$$

$$= \begin{cases} 0, & t < 0 \\ 0.0000664367, & 0 \le t < 14397.5 \\ 0.0000000755, & 14397.5 \le t < 71987.5 \\ 0, & t \ge 71987.5. \end{cases} \quad (36)$$

Hence the hypothetical distribution function $H^{31}(t)$ of the conditional sojourn time θ^{31}, after taking the integral of the hypothetical density function $h^{31}(t)$ given by (36), takes the following form

$$H^{31}(t) = \int_0^t h^{31}(t)dt$$

$$= \begin{cases} 0, & t < 0 \\ 0.0000664367t, & 0 \le t < 14397.5 \\ 0.0000000755t + 0.9945649438, & 14397.5 \le t < 71987.5 \\ 1, & t \ge 71987.5. \end{cases}$$

(37)

Next, we join the intervals defined in the realization of the histogram $h^{31}(t)$ that have the numbers n_j^{31}, of realizations less than 4 into new intervals and we perform the following steps:
- we fix the new number of intervals

$$\bar{\bar{r}}^{31} = 2;$$

- we determine the new intervals

$$\bar{I}_1 = < 0, 14397.5), \ \bar{I}_2 = < 14397.5, 71987.5);$$

- we fix the numbers of realizations in the new intervals

$$\bar{n}_1^{31} = 13, \ \bar{n}_2^{31} = 1;$$

- we calculate, using (19), the hypothetical probabilities that the variable θ^{31} takes values from the new intervals

$$p_1 = P(\theta^{31} \in \bar{I}_1) = P(0 \le \theta^{31} < 14397.5)$$

$$= H^{31}(14397.5) - H^{31}(0)$$

$$\cong 0.9946 - 0 = 0.9946,$$

$$p_2 = P(\theta^{31} \in \bar{I}_2) = P(14397.5 \le \theta^{31} < 71987.5)$$

$$= H^{31}(71987.5) - H^{31}(14397.5)$$

$$\cong 1 - 0.9946 = 0.0054;$$

- we calculate, using (20) the realization of the χ^2 (chi-square)-Pearson's statistics

$$u_{31} = \sum_{j=1}^{2} \frac{(\bar{n}_j^{31} - n^{31} p_j)^2}{n^{31} p_j}$$

$$= \frac{(22 - 23 \cdot 0.9946)^2}{23 \cdot 0.9946} + \frac{(1 - 23 \cdot 0.0054)^2}{23 \cdot 0.0054}$$

$$\cong 0.034 + 6.176 = 6.21,$$

- we assume the significance level $\alpha = 0.01$,
- we fix the number of degrees of freedom

$$\bar{\bar{r}}^{31} - l - 1 = 2 - 0 - 1 = 1;$$

- we read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level $\alpha = 0.01$ and the number of degrees of freedom $\bar{\bar{r}}^{31} = 1$, such that, according to (21), the following equality holds

$$P(U_{31} > u_\alpha) = \alpha = 0.01$$

that amounts $u_\alpha = 6.63$ and we determine the critical domain in the form of the interval $(6.63, +\infty)$ and the acceptance domain in the form of the interval $< 0, 6.63 >$;
- we compare the obtained value $u_{31} = 6.21$ of the realization of the statistics U_{31} with the read from the Tables critical value $u_\alpha = 6.63$ of the chi-square random variable and since the value $u_{31} = 6.61$ does not belong to the critical domain, i.e.

$$u_{31} = 6.621 \le u_\alpha = 6.63,$$

then we do not reject the hypothesis H_0.
After getting such a result, in the case we have enough statistical data, we may assume that the sojourn time θ^{31} has the chimney distribution with the density function given by (36). Otherwise, if the

null hypothesis H_0 is rejected, we should select other density function from the distinguished distributions and repeat the procedure of testing.
In the case when as a result of the experiment, coming from experts, we only have the number of realizations of the process of initiating events and its all realizations are equal to an approximate value, we assume that this time has the uniform distribution in the interval from this value minus its half to this value plus its half. For instance, the process initiating events conditional time θ^{26} assumed $n^{26} = 26$ values equal to 1, we assume that it has the density function given by

$$h^{26}(t) = \begin{cases} 0, & t < 0.5 \\ 1, & 0.5 \le t < 1.5 \\ 0, & t \ge 1.5. \end{cases} \qquad (38)$$

and the distribution function given by

$$H^{26}(t) = \int_0^t h^{26}(t)dt = \begin{cases} 0, & t < 0.5 \\ t, & 0.5 \le t < 1.5 \\ 1, & t \ge 1.5. \end{cases} \qquad (39)$$

We can proceeding with the remaining conditional times in the states of the process of initiating events in the same way and approximately fix they distribution

3.3. Prediction of process of initiating events generated by ship accidents

If we decide to accept the density function $h^{31}(t)$ of the conditional sojourn time θ^{31} of the process of initiating events given by (36), then applying the general formula (22) for the mean value it is possible to find its mean value evaluation:

$$M^{31} = \int_0^{\infty} t h^{31}(t)dt =$$

$$\int_0^{14397.5} 0.0000664367 t dt + \int_{14397.5}^{71987.5} 0.0000000755 t dt =$$

$$= \frac{1}{2} 0.0000664367 [(14397.5)^2 - 0^2] +$$

$$\frac{1}{2} 0.0000000755 [(71987.5)^2 - (14397.5)^2]$$

$$\cong 70736.$$

Similarly in the case of the conditional sojourn time θ^{26} of the process of initiating events given by (38), we have

$$M^{26} = \int_0^{\infty} t h^{26}(t)dt = \int_{0.5}^{1.5} 1 t dt = \frac{1}{2}[(1.5)^2 - (0.5)^2] = 1.$$

Because of the lack of sufficient numbers of realizations of the process of initiating events conditional sojourn times at the operation states, it is not possible to identify statistically their distributions. In those cases of not identified distributions, using formula (12), it is possible to find the approximate empirical values of the mean values $M^{kl} = E[\theta^{kl}]$ of the conditional sojourn times at the particular states that are given in *Appendix 1*.
Farther, applying (25), it is possible to evaluate the approximate mean values M^k of the unconditional sojourn times variables θ^k :

$M^1 = 0.5000 \cdot 151372.8 + 0.2500 \cdot 151372.8$
$+ 0.2500 \cdot 151372.8 = 151372.8,$

$M^2 = 0.2059 \cdot 50.1 + 0.0147 \cdot 60 + 0.3824 \cdot 1$
$+ 0.2500 \cdot 1 + 0.0294 \cdot 10 + 0.1029 \cdot 1 +$
$+ 0.0147 \cdot 1 = 12.24,$

$M^3 = 0.3594 \cdot 3513.2 + 0.0469 \cdot 1 + 0.0313 \cdot 1$
$+ 0.2969 \cdot 1 + 0.1875 \cdot 1 + 0.0156 \cdot 1 +$
$+ 0.0312 \cdot 120.5 + 0.0312 \cdot 1 = 1267.01,$

$M^4 = 0.0909 \cdot 10 + 0.5000 \cdot 1 + 0.2727 \cdot 5.8$
$+ 0.0455 \cdot 1 + 0.0455 \cdot 1 + 0.0454 \cdot 1 = 3.13,$

$M^5 = 0.1622 \cdot 142.7 + 0.4324 \cdot 9 + 0.0270 \cdot 1$
$+ 0.1081 \cdot 1 + 0.1081 \cdot 117.5 + 0.1622 \cdot 1 =$
$= 40.04,$

$M^6 = 0.6364 \cdot 632.1 + 0.0909 \cdot 1440 + 0.0909 \cdot 10$
$+ 0.0909 \cdot 225 + 0.0909 \cdot 10 = 555.43,$

$M^7 = 0.0555 \cdot 10 + 0.2222 \cdot 35 + 0.1111 \cdot 720.5$
$+ 0.0556 \cdot 20 + 0.2222 \cdot 43360.5$
$+ 0.1666 \cdot 511.7 + 0.0556 \cdot 60 + 0.0556 \cdot 10$

$+ 0.0556 \cdot 10 = 9813.89,$

$M^8 = 0.1000 \cdot 240 + 0.2000 \cdot 1830 + 0.2000 \cdot 10$

$+ 0.1000 \cdot 15 + 0.3000 \cdot 43.3 + 0.1000 \cdot 10$

$= 407.49,$

$M^9 = 0.1111 \cdot 10 + 0.6667 \cdot 796.8 + 0.1111 \cdot 1$

$+ 0.1111 \cdot 10 = 533.56,$

$M^{10} = 1 \cdot 10 = 10,$

$M^{11} = 1 \cdot 10 = 10,$

$M^{12} = 0.2857 \cdot 180 + 0.2857 \cdot 414.5 + 0.1429 \cdot 60$

$+ 0.1429 \cdot 10 + 0.1428 \cdot 10 = 181.28,$

$M^{13} = 1 \cdot 4140 = 4140,$

$M^{14} = 0.6 \cdot 751.3 + 0.2 \cdot 10 + 0.2 \cdot 10 = 454.78,$

$M^{15} = 0.5 \cdot 2527.5 + 0.5 \cdot 5.5 = 1266.50,$

$M^{16} = 1 \cdot 120 = 120,$

$M^{17} = 1 \cdot 21480 = 21480,$

$M^{18} = 0.500 \cdot 180 + 0.500 \cdot 270 = 225,$

$M^{19} = 0.5000 \cdot 30 + 0.5000 \cdot 10 = 20$

$M^{20} = 1 \cdot 1 = 1,$

$M^{21} = 1 \cdot 22 = 22,$

$M^{22} = 1 \cdot 1 = 1,$

$M^{23} = 1 \cdot 240 = 240,$

$M^{24} = 1 \cdot 120 = 120,$

$M^{25} = 1 \cdot 120 = 120,$

$M^{26} = 1 \cdot 120 = 120,$

$M^{27} = 1 \cdot 9126 = 9126,$

$M^{28} = 1 \cdot 10 = 10,$

$M^{29} = 1 \cdot 120 = 120,$

$M^{30} = 1 \cdot 120 = 120,$

$M^{31} = 1 \cdot 120 = 120,$

$M^{32} = 1 \cdot 1 = 1,$

$M^{33} = 1 \cdot 120 = 120,$

$M^{34} = 0.6666 \cdot 61.5 + 0.3334 \cdot 60 = 60,$

$M^{35} = 0.6667 \cdot 120 + 0.3333 \cdot 40320 = 13518.66,$

$M^{36} = 1 \cdot 30 = 30,$

$M^{37} = 1 \cdot 120 = 120,$

$M^{38} = 1 \cdot 120 = 120,$

$M^{39} = 1 \cdot 120 = 120,$

$M^{40} = 1 \cdot 120 = 120,$

$M^{41} = 1 \cdot 120 = 120,$

$M^{42} = 1 \cdot 10 = 10,$

$M^{43} = 1 \cdot 120 = 120.$ (40)

To find the limit values of the transient probabilities p^k, $k = 1,2,...,43$, of the process of initiating events $E(t)$ at its particular states, first we have to solve the system of equations (27) that in our case takes the following form

$$\begin{cases} [\pi^k]_{1x43} = [\pi^k]_{1x43}[p^{kl}]_{43x43} \\ \sum_{k=1}^{43} \pi^k = 1, \end{cases}$$

where

$$[\pi^k]_{1x43} = [\pi^1, \pi^2, ..., \pi^{43}]$$

and the elements of the matrix $[p^{kl}]_{43x43}$ are given by (30).

$0.3594\ \pi^3 + 0.1622\,\pi^5 + 0.1000\,\pi^8 = \pi^1$

$0.1111\,\pi^9 + 0.5000\,\pi^{19} = \pi^2$

$0.5\,\pi^1 + 0.2059\,\pi^2 + 0.0909\,\pi^4 + 0.6667\,\pi^9 + \pi^{10}$

$+ \pi^{11} = \pi^3$

$0.25\,\pi^1 = \pi^4$

$0.0147\,\pi^2 + 0.2857\,\pi^{12} = \pi^5$

$0.3824\pi^{2} + 0.0469\pi^{3} + 0.5000\pi^{4} + 0.6\pi^{14} = \pi^{6}$

$0.2857\pi^{12} + \pi^{27} = \pi^{7}$

$0.2500\pi^{2} + 0.0313\pi^{3} + 0.2727\pi^{4} + 0.1429\pi^{12}$

$+ 0.2\pi^{14} + 0.5\pi^{15} + \pi^{17} + \pi^{21} + \pi^{25} + \pi^{29} + \pi^{30}$

$+ \pi^{31} + \pi^{33} + 0.8333\pi^{35} + \pi^{39} + \pi^{40} + \pi^{43} = \pi^{8}$

$0.0294\pi^{2} + 0.1429\pi^{12} = \pi^{9}$

$0.25\pi^{1} + \pi^{26} + \pi^{37} = \pi^{12}$

$0.2000\pi^{8} + 0.1111\pi^{9} = \pi^{13}$

$0.2969\pi^{3} + 0.6364\pi^{6} + \pi^{28} = \pi^{14}$

$0.1875\pi^{3} + 0.0555\pi^{7} + 0.2000\pi^{8} + 0.5\pi^{18} = \pi^{15}$

$\pi^{32} = \pi^{16}$

$0.0156\pi^{3} = \pi^{17}$

$\pi^{20} + 0.1667\pi^{35} = \pi^{18}$

$0.1029\pi^{2} + 0.0312\pi^{3} + 0.0909\pi^{6} + 0.1000\pi^{8}$

$+ \pi^{16} + \pi^{22} + \pi^{38} + \pi^{41} = \pi^{19}$

$0.2222\pi^{7} = \pi^{20}$

$0.4324\pi^{5} + 0.3000\pi^{8} = \pi^{21}$

$0.1111\pi^{7} = \pi^{22}$

$0.0270\pi^{5} = \pi^{23}$

$\pi^{23} = \pi^{24}$

$\pi^{24} = \pi^{25}$

$0.0455\pi^{4} + 0.0556\pi^{7} = \pi^{26}$

$0.1081\pi^{5} + 0.2222\pi^{7} = \pi^{27}$

$0.0909\pi^{6} + \pi^{36} = \pi^{28}$

$0.1428\pi^{12} = \pi^{29}$

$0.0312\pi^{3} + 0.5000\pi^{15} + 0.5000\pi^{19} = \pi^{30}$

$0.0147\pi^{2} + 0.0455\pi^{4} + 0.1111\pi^{9} = \pi^{31}$

$0.1667\pi^{7} = \pi^{32}$

$0.1081\pi^{5} = \pi^{33}$

$0.1622\pi^{5} = \pi^{34}$

$0.0454\pi^{4} + 0.0556\pi^{7} + 0.5\pi^{18} + 0.6666\pi^{34} = \pi^{35}$

$0.0909\pi^{6} = \pi^{36}$

$0.1000\pi^{8} = \pi^{37}$

$0.3334\pi^{34} = \pi^{38}$

$0.2\pi^{14} = \pi^{39}$

$0.0909\pi^{6} = \pi^{40}$

$0.0556\pi^{7} = \pi^{41}$

$0.0556\pi^{7} = \pi^{42}$

$\pi^{42} = \pi^{43}$

$$\sum_{k=1}^{43}\pi^{k} = 1. \quad (41)$$

Solving this system of equations we get

$\pi^{1} = 0.0548,\ \pi^{2} = 0.0204,\ \pi^{3} = 0.0835,$
$\pi^{4} = 0.0137,\ \pi^{5} = 0.0112,\ \pi^{6} = 0.0657,$

$\pi^{7} = 0.0156,\ \pi^{8} = 0.2297,\ \pi^{9} = 0.0061,$

$\pi^{10} = 0,\ \pi^{11} = 0,\ \pi^{12} = 0.0382,\ \pi^{13} = 0.0466,$

$\pi^{14} = 0.0785,\ \pi^{15} = 0.0646,\ \pi^{16} = 0.0026,$

$\pi^{17} = 0.0013,\ \pi^{18} = 0.0043,\ \pi^{19} = 0.0394,$

$\pi^{20} = 0.0035,\ \pi^{21} = 0.0738,\ \pi^{22} = 0.0017,$

$\pi^{23} = 0.0003,\ \pi^{24} = 0.0003,\ \pi^{25} = 0.0003,$

$\pi^{26} = 0.0015,\ \pi^{27} = 0.0047,\ \pi^{28} = 0.0119,$

$\pi^{29} = 0.0055,\ \pi^{30} = 0.0546,\ \pi^{31} = 0.0016,$

$\pi^{32} = 0.0026,\ \pi^{33} = 0.0012,\ \pi^{34} = 0.0018,$

$\pi^{35} = 0.0048,\ \pi^{36} = 0.0060,\ \pi^{37} = 0.0230,$

$\pi^{38} = 0.0006,\ \pi^{39} = 0.0157,\ \pi^{40} = 0.0060,$

$\pi^{41} = 0.0009,\ \pi^{42} = 0.0009,\ \pi^{43} = 0.0009.$

Hence, according to (26) and considering (40), we get the approximate limit values of the transient probabilities at the particular states of the process of initiating events

$p^1 = 0.9057,\ p^2 = 0.0003,\ p^3 = 0.0116,$

$p^4 = 0.000005,\ p^5 = 0.00005,\ p^6 = 0.0040,$

$p^7 = 0.0167,\ p^8 = 0.0102,\ p^9 = 0.0004,$

$p^{10} = 0.0000,\ p^{11} = 0.0000,\ p^{12} = 0.0008,$

$p^{13} = 0.0211,\ p^{14} = 0.0039,\ p^{15} = 0.0089,$

$p^{16} = 0.00003,\ p^{17} = 0.0031,\ p^{18} = 0.0001,$

$p^{19} = 0.0001,\ p^{20} = 0.0000004,\ p^{21} = 0.0002,$

$p^{22} = 0.0000002,\ p^{23} = 0.000008,\ p^{24} =$ 0.000004,

$p^{25} = 0.000004,\ p^{26} = 0.00002,\ p^{27} = 0.0047,$
$p^{28} = 0.00001,\ p^{29} = 0.0001,\ p^{30} = 0.0001,$

$p^{31} = 0.00002,\ p^{32} = 0.0000003,\ p^{33} = 0.00002,$

$p^{34} = 0.00001,\ p^{35} = 0.0071,\ p^{36} = 0.00002,$

$p^{37} = 0.0003,\ p^{38} = 0.000008,\ p^{39} = 0.0002,$

$p^{40} = 0.0001,\ p^{41} = 0.00001,\ p^{42} = 0.000001,$

$p^{43} = 0.00001.$ (42)

Further, by (28) and considering (42), the approximate mean values of the sojourn total times $\hat{\theta}^k$ of the process of initiating events $E(t)$ in the time interval $\theta = 1$ month = 43200 minutes at the particular states e^k expressed in minutes are:

$\hat{M}^1 = 39126.2,\ \hat{M}^2 = 12.96,\ \hat{M}^3 = 501.12,$

$\hat{M}^4 = 0.216,\ \hat{M}^5 = 2.16,\ \hat{M}^6 = 172.8,$

$\hat{M}^7 = 721.44,\ \hat{M}^8 = 440.64,\ \hat{M}^9 = 17.28,$

$\hat{M}^{10} = 0,\ \hat{M}^{11} = 0,\ \hat{M}^{12} = 34.56,\ \hat{M}^{13} = 911.52,$

$\hat{M}^{14} = 168.48,\ \hat{M}^{15} = 384.48,\ \hat{M}^{16} = 1.296,$

$\hat{M}^{17} = 133.92,\ \hat{M}^{18} = 4.32,\ \hat{M}^{19} = 4.32,$

$\hat{M}^{20} = 0.01728,\ \hat{M}^{21} = 8.64,\ \hat{M}^{22} = 0.00864,$

$\hat{M}^{23} = 0.3456,\ \hat{M}^{24} = 0.1728,\ \hat{M}^{25} = 0.1728,$

$\hat{M}^{26} = 0.864,\ \hat{M}^{27} = 203.04,\ \hat{M}^{28} = 0.432,$

$\hat{M}^{29} = 4.32,\ \hat{M}^{30} = 4.32,\ \hat{M}^{31} = 0.864,$

$\hat{M}^{32} = 0.01296,\ \hat{M}^{33} = 0.864,\ \hat{M}^{34} = 0.432,$

$\hat{M}^{35} = 306.72,\ \hat{M}^{36} = 0.864,\ \hat{M}^{37} = 12.96,$

$\hat{M}^{38} = 0.3456,\ \hat{M}^{39} = 8.64,\ \hat{M}^{40} = 4.32,$

$\hat{M}^{41} = 0.432,\ \hat{M}^{42} = 0.0432,\ \hat{M}^{43} = 0.432.$ (43)

The last results (42) and (43) can play an essential and practically important role in the minimization of critical infrastructure accident consequences and the losses mitigation.

4. Conclusion

The model of the process of the environment degradation initiating events generated by the critical infrastructure accident or exceeding its safety critical level presented in the paper is a first part of a general model of critical infrastructure accident consequences that are dangerous for its operation environment. The procedure of its practical application is illustrated in the modelling, identification and prediction of the initiating events process caused by the critical infrastructure accident, i.e. the exceeding a critical safety level by the ship operating at Baltic Sea waters. The approximate mean time to the exceeding the critical safety level by the considered ship is taken from [12]. The noncomplete yet statistical data concerned with the realizations of the initiating events process often are either not enough numerous to be sufficient for the identification of this process unknown parameters with a good accuracy or are coming from experts as a result of their arbitrary approximate evaluation.

Therefore, at this stage of study, the results obtained should be treated as an illustration of the proposed approach.

Presented in this paper model and tools are supposed to be very useful in the critical infrastructure accident consequences modelling, identification, prediction, optimization and mitigation the losses. It is also expected that the results will be significantly developed in the scope of the project EU-CIRCLE concerned with the strengthening critical infrastructure resilience to climate change that is just going to start [21].

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Appendix 1. The realizations of the conditional lifetimes at the states of the process of initiating events

transitions $e^k \to e^l$	transition time θ^{kl} [min]	number of transitions n^{kl}	mean value $\bar{\theta}^{kl}$ [min]
$e^1 \to e^3$	151372.8, 151372.8	2	151372.8
$e^1 \to e^4$	151372.8	1	151372.8
$e^1 \to e^{12}$	151372.8	1	151372.8
$e^2 \to e^3$	40, 30, 10, 240, 10, 10, 10, 1, 10, 10, 10, 10, 10, 300	14	50.1
$e^2 \to e^5$	60	1	60
$e^2 \to e^6$	all equal to 1	26	1
$e^2 \to e^8$	all equal to 1	17	1
$e^2 \to e^9$	10, 10	2	10
$e^2 \to e^{19}$	all equal to 1	7	1
$e^2 \to e^{31}$	1	1	1
$e^3 \to e^1$	15, 9702, 228, 300, 1905, 150, 120, 1440, 10, 1440, 22, 420, 57600, 1440, 120, 702, 1320, 1440, 120, 1320, 300, 810, 300	23	3513.2
$e^3 \to e^6$	1, 1, 1	3	1
$e^3 \to e^8$	1, 1	2	1
$e^3 \to e^{14}$	all equal to 1	19	1
$e^3 \to e^{15}$	all equal to 1	12	1
$e^3 \to e^{17}$	1	1	1
$e^3 \to e^{19}$	240, 1	2	120.5
$e^3 \to e^{30}$	1, 1	2	1
$e^4 \to e^3$	10, 10	2	10
$e^4 \to e^6$	all equal to 1	11	1
$e^4 \to e^8$	1, 30, 1, 1, 1, 1	6	5.8
$e^4 \to e^{26}$	1	1	1
$e^4 \to e^{31}$	1	1	1
$e^4 \to e^{35}$	1	1	1
$e^5 \to e^1$	60, 186, 10, 240, 300, 60	6	142.7
$e^5 \to e^{21}$	1, 1, 1, 1, 1, 1, 1, 1, 1, 10, 1, 1, 1, 1, 1, 10	16	9
$e^5 \to e^{23}$	1	1	1
$e^5 \to e^{27}$	1,1,1,1	4	1
$e^5 \to e^{33}$	360, 10, 90, 10	4	117.5
$e^5 \to e^{34}$	1, 1, 1, 1, 1, 1	6	1
$e^6 \to e^{14}$	10, 10, 10, 10, 4320, 10,55	7	632.1
$e^6 \to e^{19}$	1440	1	1440
$e^6 \to e^{28}$	10	1	10
$e^6 \to e^{36}$	225	1	225
$e^6 \to e^{40}$	10	1	10
$e^7 \to e^{15}$	10	1	10
$e^7 \to e^{20}$	60,10,10,60	4	35
$e^7 \to e^{22}$	1, 1440	2	720.5
$e^7 \to e^{26}$	20	1	20
$e^7 \to e^{27}$	172800, 240, 300, 102	4	43360.5
$e^7 \to e^{32}$	10, 1515, 10	3	511.7
$e^7 \to e^{35}$	60	1	60
$e^7 \to e^{41}$	10	1	10
$e^7 \to e^{42}$	10	1	10
$e^8 \to e^1$	240	1	240
$e^8 \to e^{13}$	3480, 180	2	1830
$e^8 \to e^{15}$	10, 10	2	10
$e^8 \to e^{19}$	15	1	15
$e^8 \to e^{21}$	90, 10, 30	3	43.3
$e^8 \to e^{37}$	10	1	10
$e^9 \to e^2$	10	1	10

$e^9 \to e^3$	66, 10, 60, 10, 315, 4320	6	796.8
$e^9 \to e^{13}$	1	1	1
$e^9 \to e^{31}$	10	1	10
$e^{10} \to e^3$	10	1	10
$e^{11} \to e^3$	10	1	10
$e^{12} \to e^5$	300, 60	2	180
$e^{12} \to e^7$	1, 828	2	414.5
$e^{12} \to e^8$	60	1	60
$e^{12} \to e^9$	10	1	10
$e^{12} \to e^{29}$	10	1	10
$e^{13} \to e^3$	4140	1	4140
$e^{14} \to e^6$	2184, 10, 60	3	751.3
$e^{14} \to e^8$	10	1	10
$e^{14} \to e^{39}$	10	1	10
$e^{15} \to e^8$	5040, 15	2	2527.5
$e^{15} \to e^{30}$	1, 10	2	5.5
$e^{16} \to e^{12}$	120	1	120
$e^{17} \to e^8$	21480	1	21480
$e^{18} \to e^{15}$	180	1	180
$e^{18} \to e^{35}$	270	1	270
$e^{19} \to e^2$	30	1	30
$e^{19} \to e^{30}$	10	1	10
$e^{20} \to e^{18}$	1	1	1
$e^{21} \to e^8$	20, 10, 40, 10, 30	5	22
$e^{22} \to e^{19}$	1	1	1
$e^{23} \to e^{24}$	240	1	240
$e^{24} \to e^{25}$	120	1	120
$e^{25} \to e^8$	120	1	120
$e^{26} \to e^{12}$	120	1	120
$e^{27} \to e^7$	9126	1	9126
$e^{28} \to e^{14}$	10, 10	2	10
$e^{29} \to e^8$	120	1	120
$e^{30} \to e^8$	120	1	120
$e^{31} \to e^8$	120	1	120
$e^{32} \to e^{16}$	1	1	1
$e^{33} \to e^8$	120	1	120

$e^{34} \to e^{35}$	3, 120	2	61.5
$e^{34} \to e^{38}$	60	1	60
$e^{35} \to e^8$	120, 160, 140, 100, 80	5	120
$e^{35} \to e^{18}$	40320	1	40320
$e^{36} \to e^{28}$	30	1	30
$e^{37} \to e^{12}$	120	1	120
$e^{38} \to e^{19}$	120	1	120
$e^{39} \to e^8$	120	1	120
$e^{40} \to e^8$	120	1	120
$e^{41} \to e^{19}$	120	1	120
$e^{42} \to e^{43}$	10	1	10
$e^{43} \to e^8$	120	1	120