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ANALYSIS OF HIGH FREQUENCY VIBRATION OF TRAM MONOBLOC WHEEL

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Abstract: *European Environmental Agency estimates that about 120 million people in the EU (over 30% of the total population) are exposed to traffic noise above 55 Ldn dB. It is estimated that 10% of the EU population is exposed to noise associated with the rail traffic. The two main sources of traffic noise comes from vehicles engines and the noise generated in the contact between the wheel and the road. In the latter the considerable part the noise is due to phenomena occurring in a wheel tram – rail system. Therefore, the problem of reducing the noise generated by railway vehicles is the subject of many studies, both experimental and theoretical. Commonly used wheel trams so called "resilient wheels" are equipped with layer made of a resilient material, e.g. rubber, between the tread and the wheel disc. But the monobloc tram wheel is the standard design against which should be carried out the studies on reduction of noise in wheel-rail system. This paper presents the results of calculations related to eigenforms, eigenfrequencies and Frequency Response Function of a three-dimensional model of a monobloc tram wheel. The calculations were carried out using the finite element method. Vibration analysis was performed for the range to 5 kHz. Analysis carried out has shown that the wheel tread plays a more important role in the generation of high-frequency vibrations.*

Key words: *noise, tram wheel, FEM analysis.*

1. Introduction

Trams are reliable means of public transport and moreover largely independent of the traffic congestion,. Unfortunately, they are one of the causes of troublesome noise. This noise reduction is an important task related to improving the quality of life in large urban areas. The noise source is the running engine, the impact of aerodynamic turbulences, squealing wheels on curves or when braking, but most of all are the sound effects, called a rolling noise, which appear in the wheel-rail contact system.

Good understanding of the noise propagation mechanism is associated with the analysis of tram wheel vibrations in the range of high frequencies.

Currently used wheel trams are equipped with resilient layer, which isolates the wheel tread from the wheel disc, but its aim mainly is the reduction of the impact of unsprung mass on the track. Due to the fact that the uses of such wheels not radically reduce the noise there was a need to start the work on new wheels designs.

The monobloc tram wheel is the standard construction, against which should be carried out the further studies. Thorough analysis of its behaviour should also be helpful in deciding whether research on the new wheel design should focus on the damping of vibration in the wheel-rail system or to concentrate on a reduction of noise radiation from this system to the environment.

It is necessary to mention that the use of wheels with resilient layer in high speed trains was forbidden after the catastrophe at Enschede.

The paper presents the results of calculations related to eigenforms and eigenfrequencies of a three-dimensional model of a tram wheel. The calculations were carried out using the finite element method. Vibration analysis was performed for monobloc wheel in the range of approx. 5 kHz. This range covers the vibrations associated with rolling noise and partly with wheel squeal. The main subject of this article study is preceded by a short description of the phenomenon of noise and its classification.

2. Noise in wheel – rail system

2.1. General description

The literature on the generation of noise in a wheel-rail system is very extensive and cited in this study certainly does not exhaust the subject. The mechanism of noise generation is extremely complex and for this reason the works on the analysis, which have started in the 70s of the last century, is still ongoing and they are far to be complete.

General classification of noise has been proposed in the course of those studies. It distinguishes the rolling noise, impact noise, the wheel squeal and howl. It should be emphasized that the source of the noise are rails irregularities, different from the classified e.g. (Kardas-Cinal et. al., 2009; Sowinski, 2013) irregularities affecting the dynamics of rail vehicle in the low frequency range of vibrations.
The rolling noise is the result of irregularities existing on the surfaces of the rolling wheels and rails, which could include corrugated rail wear, loss of roundness on wheels as well as accidental damage of their rolling surfaces.
Rail corrugation also causes excessive rolling noise of very high level. This noise is called "roaring rail" or "wheel howl" or "wheel/rail howl". The wheel howl vibrations are contained within a frequency band ranging from about 250 to 750 Hz (Transit Cooperative Research Program Report 23, 1997).
The impact noise is caused by large amplitude irregularities that can occur on the rails, wheels or rails flats and special sections of track e.g. crossings and turnouts.
Squeal of wheels is a specific noise that occurs when the vehicles are running through curves with a small radius. In general this phenomenon appears, due to the slipping of wheel tread on the rail head, what is always accompanied with friction. That results in wheel vibration in a perpendicular direction to wheel disc.
In details, the cause of squeal can be attributed to three phenomena typical to the movement along an arc:
- longitudinal slip caused by the movement of wheel tread along the rail,
- lateral slip accompanied to a non-zero yaw angle,
- contact of the wheel flange with the rail head.

2.2. The rolling noise

Rolling noise is the most widespread noise and therefore has the greatest interest in existing studies. In 1998 ERRI (European Railway Research Institute) published a report (ERRI Committee C 163, 1998) summarizing the work on rolling noise generation models. These models are used to create a package of programs - TWINS ("Track-Wheel Interaction Noise Software"). The models were mostly based on the work (Remington, 1976, 1987). General scheme of the model used in software TWINS is shown in Figure 1 and 2.

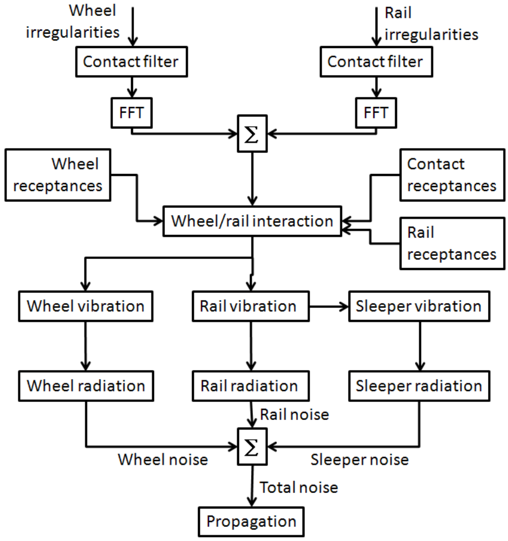


Fig. 1. TWINS model for rolling noise generation
Source: Thompson, Hemsworth & Vincent (1996).

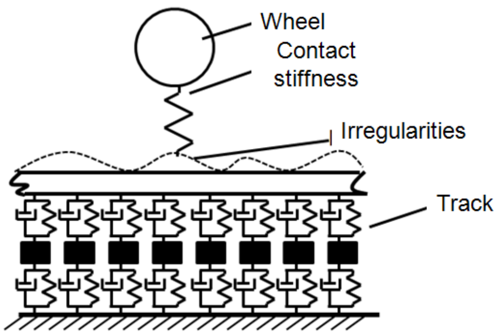


Fig. 2. Scheme of the wheel rail system excited by rolling surfaces roughness

The spectrum of rolling noise, according to data published in the report (Transit Cooperative Research Program Report 23, 1997), is broadband and is in the range of from 250 to 2000 Hz but it should be mentioned here that e.g. (Thompson, 2011) gives this range as from 100 to 5000 Hz.
As the factors causing rolling noise one can mention:
- rail and wheel roughness,
- parameter variation, or moduli heterogeneity,
- creep,
- aerodynamic noise.

Wheel and rail surface roughness are the most significant cause of wheel/rail noise. Short waves of irregularities (lengths less than 5 mm) are filtered through the contact area of the rail and wheels. In practice, it is assumed that the reasons of oscillation are wavelengths from about 5 to 200 mm (Thompson, 2003; Thompson, 2014). Taking into account the specific conditions related to trams – smaller ellipse of contact caused by fact that radius of tram wheel is less than railway wheel radius and less normal pressure, tram generated rolling noise spectrum range should be increased to approx. 3000 Hz. This fact can be justified by following brief discussion considering the length of the waves of roughness (wavelengths between about 6 and 500 mm) and the speed achievable by trams (60 km/h). A roughness of wavelength λ traversed at a speed v excites vibration at a frequency $f = v/\lambda$ i.e. approximately up to 3 kHz.
Parameter variation refers to the variation of rail and wheel steel moduli (Young's and Kirchhoff modulus), rail support stiffness, and contact stiffness between the wheel and the rail due to variation in rail head transverse radius of curvature of wheels and rails at the point of contact.
Longitudinal slips are usually ignored as a cause of rolling noise. It is assumed that longitudinal slip occurs mainly during braking and acceleration of the vehicle and while driving along curves, and then it causes wheel squeal. Lateral slips are small, when the vehicle motion along a tangent track is analyzed, but in practice, they cannot be neglected due to the presence of the track centre line irregularities.
Aerodynamic noise is caused by turbulent boundary layer noise about the wheel circumference as it moves forward.

“Standard” mathematical model of the system shown in Figure 2 can be represented in the form (Kisilowski & Sowiński, 1991):

Equation (1) is a mathematical model of rail displacement (Euler Bernoulli beam with bending stiffness EJ [Nm^2], the mass of unit of length m [kg/m], and an axial force N [N], stiffness of the rail pad k_r [N/ m] and damping c_r [Ns/m]).
Equation (2) is a model of wheel vibration (concentrated mass m_w [kg], k_H [N/m] Hertz’s contact stiffness between the wheel and the rail).
Equations (3) describe sleepers vibration (concentrated masses m_{sn} resting on elastic-damping foundation represented by k_b [N/m] and c_b [Ns/m]).
Each of models (1 – 3) can be developed up to obtain a 3-dimensional model solved e.g. by FEM.
Currently, work on the rolling noise are intensively developed in the framework of German-French (project DEUFRACO - STARDAMP - Standardization of damping technologies for the reduction of railway noise).

2.3. Impact Noise

The cause of the noise impact is passing through a place on the track where exists high amplitude roughness e.g. rail joints, rail defects or other discontinuities in the rail running surface.
The source of the impact noise can also be a local flat of the wheel and drive through the special sections of the track like a cross or a switch frogs.
The main issue, arising in the analysis of noise impact, is the construction of a suitable model of the interaction between wheel and rail. The first works, analyzing occurrence of impact noise, are works (Ver, Ventres & Myles, 1976; Remington, 1988). The phenomenon of temporary separation of wheel rolling surface and rail is included in the model of contact in (Remington, 1988). Classification of impact noise, taking into account the type of irregularity, the direction of ride of the vehicle and its speed was made in work (Ver, Ventres & Myles, 1976). Such works as e.g. (Wu, Thompson, 2002; Steenberger, 2006; Yang, 2012) are further development of contact models mentioned before.

$$EJ\frac{\partial^4 z}{\partial x^4} + N\frac{\partial^2 z}{\partial x^2} + m\frac{\partial^2 z}{\partial t^2} + \sum_{n=-\infty}^{n=\infty}\left(k_r(z - z_{sn}) + c_r\left(\frac{\partial z}{\partial t} - \dot{z}_{sn}\right)\right)\delta(x - nl) + k_H(z - z_w + \eta)\delta(x - vt) \tag{1}$$

$$m_w\ddot{z}_w + k_H(z_w - z(vt,t) + \eta(vt)) = 0 \tag{2}$$

$$m_{sn}\ddot{z}_{sn} - k_r(z - z_{sn}) - c_r\left(\frac{\partial z(nl,t)}{\partial t} - \dot{z}_{sn}\right) + k_b z_{sn} + c_b\dot{z}_{sn} = 0, \quad n = -\infty,...,\infty \tag{3}$$

In Wu & Thompson (2002) a relative displacement excitation is introduced between the wheel and rail that differs from the geometric form of the wheel flat due to the finite curvature of the wheel. The model allows the possibility of loss of contact between the wheel and the rail.
Model, shown in Steenberger (2006), takes into account the multi-point transient contact of wheel and rail, which may appear on the flattened wheel.
Interaction model proposal, efficient from the point of view of calculation time, between wheel and rail was reported in Yang (2012). This model was used to investigate the noise characteristics of impact.

2.4. Wheel squeal

Generally it is assumed that wheel squeal covers the range of vibrations up to 10 kHz, e.g. (Thompson, 2011). Publication (Rudd, 1976) should be cited as one of the first works related to the study of the phenomenon of wheel squeal. The model, presented in this work, relates to generate noise for rail vehicles moving along arcs with small curvature radius.
The model assumes that wheel squeal is due to the lateral movement of the wheels across the rail head – so called stick-slip motion. This movement is described as the effect of a negative damping coefficient and causes vibrations increasing up to their amplitudes saturation.
Another cause of wheels squeal is a flange contact of the wheel with the rail which appears on the outside wheel of the vehicle during ride along a curve with a large curvature (Thompson & Jones, 2006).
Extensive studies on the phenomenon of wheels squeal are presented in (Vincent et al., 2006; Koch, et al., 2006; Chiello et al., 2006). The results of experimental tests performed on metro vehicles, trams and measurements made with the scale of 1: 4 model on the test rig dedicated to generate wheel squeal associated with the movement of the vehicle on a curve are described in those articles. The model took into account the tangential and normal forces in wheel contact and the rail, and the dynamics of the wheel. Despite the long-lasting work, the phenomenon of squealing is still far to give a full explanation. This is due to the fact that the wheel squeal is the complex result of a number of mechanical, acoustic and tribological effects (Brunel et al., 2006).

3. Tram monobloc wheel model

Commonly used tram's wheels have resilient wheels with a layer made of resilient material, for example rubber between the tread and the wheel disc. In the case considered in this article, the wheel hub, disc and the wheel tread are made of steel. The theoretical studies on such construction are based on circular plates and curved rods dynamics. As examples of the early works on this subject may be mentioned articles (Timoshenko & Woinowsky-Kreiger, 1959; Suzuki, 1971; Rao & Sundararajan, 1969; Hawkings, 1977; Mallik & Mead, 1977; Bert & Chen, 1978; Lin & Soedel, 1988), and this issue is still popular e.g. (Wu & Parker, 2006; Noga, Markowski & Bogacz, 2012).
This study, on high-frequency noise generation, is based on the use of the finite element method, and it takes into account the three-dimensional model. The key works in this field were the works (Thompson, 1993a,b,c,d,e).
Considered in this section wheel model is shown in Figure 3.

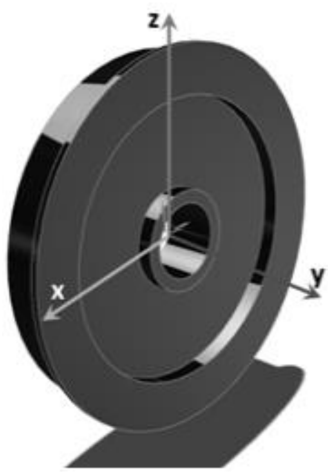


Fig. 3. Monobloc tram wheel and the associated reference frame (y – axis is perpendicular to the wheel plane)

Basic dimensions of the wheel are shown in Figure 4, and the material constants, inertial and the definition of the mesh in Table 1.

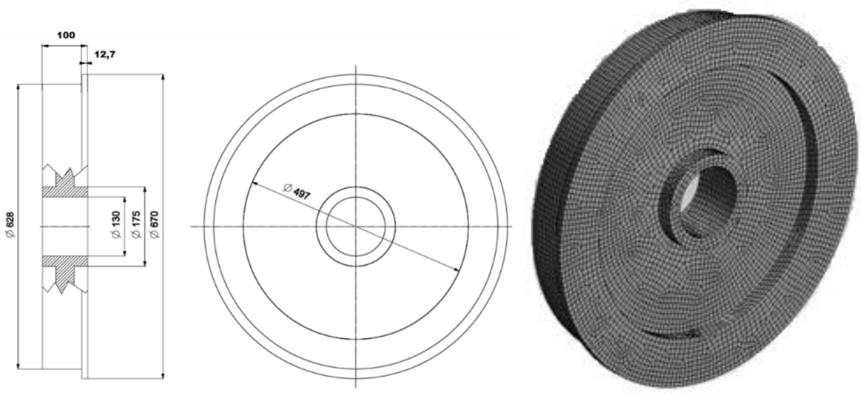


Fig. 4. Basic dimensions of wheel model and adopted mesh

Table 1. The model material constants and inertial parameters

Material constants and inertial parameters	
Volume	0,019973 m^3
Mass	160,38 kg
Moment of inertia I_y	9,762 $kg \cdot m^2$
Moment of inertia I_x	4,9807 $kg \cdot m^2$
Moment of inertia I_z	4,9807 $kg \cdot m^2$
Density (Steel)	8030 kg/m^3
Young's modulus	210 GPa
Poisson's ratio	0,3
Helmholtz modulus	175 GPa
Kirchhoff modulus	80,77 GPa
Mesh	
nodes	153680
elements	39308
max. element dimension	10^{-2}m

Calculations were performed examining the eigenfrequencies and eigenforms in the range up to 5 kHz. Besides that, the special attention was paid to the Frequency Response Function (FRF) of the wheel vibration at a point situated on the wheel flange (Fig. 5a). Two cases were studied: first one when the wheelset is in the central position (Fig. 5b) (harmonic excitation is applied to the contact point corresponding to this position) and the second one when the wheelset is in flange contact with rail (Fig. 5c) (in this case, the harmonic force is applied to the wheel flange). The amplitude value of the harmonic forces is equal to 10 kN in both cases.

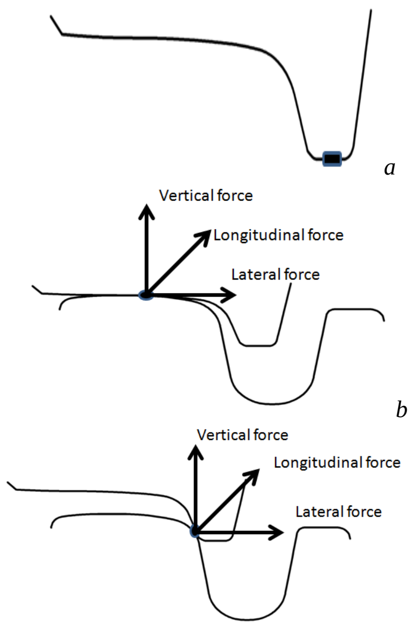


Fig. 5. *a* – point of Frequency Response Function study, *b* – contact point corresponding wheelset central position, *c* – flange contact point

During railway vehicle motion the force applied to wheel is three dimensional vector. Due to the fact, FRF was calculated for longitudinal, vertical and lateral excitation separately.

Simplifying boundary conditions, assuming constraining movement on the inner surface of the wheel hub were adopted for calculations. Calculations were performed using Matlab package.

4. Calculations results

4.1. Model eigenfrequencies and eigenforms

In this section are presented eigenfrequencies and eigenforms of studied model and results connected with FRF values.

Calculations were performed in the frequency range from 1 to 5 kHz. Results regarding to eigenforms and eigenvalues are listed in table 2.

We can conclude, analyzing the vibrations eigenforms that they can be divided into four groups. The first group contains the modes in which the nodes form diameters, the second group contains the modes in which the nodes form circles, the third group is a group of modes, wherein the nodal points appear in mentioned before both cases and the fourth group includes the in plane eigenforms of vibrations (table 3).

We can observe, taking into account previously carried out considerations and eigenvalues frequencies corresponding to eigenforms, that modes numbered from 1 to 7 are associated with rolling noise. Mostly, those modes represent the movement of the wheel tread and even in the case of mode 6 it is a movement in the ZX plane (Fig. 3).

Table 2 .Summary of monobloc wheels modes

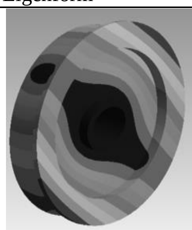
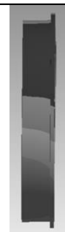
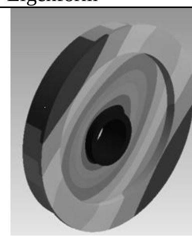
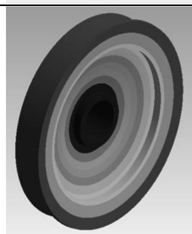
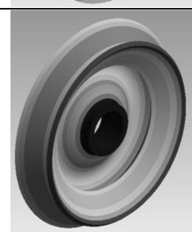
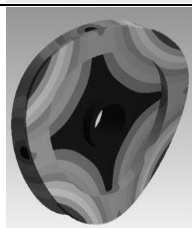
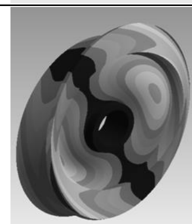
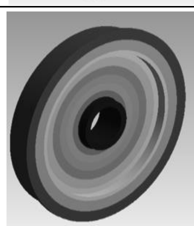
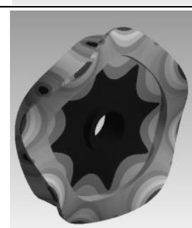
Mode	Eigenfreq. [Hz]	Eigenform		Mode	Eigenfreq. [Hz]	Eigenform	
1	384			6	2032		
2	512			7	2798		
3	721			8	3045		
4	767			9	3083		

Table 2 .Summary of monobloc wheels modes (cont.)

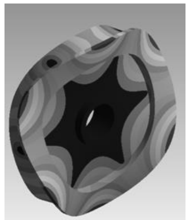
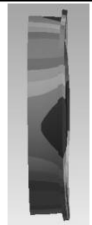
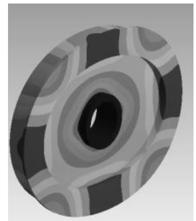
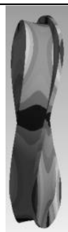
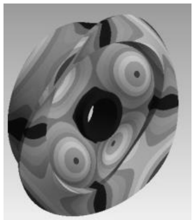
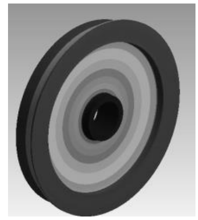
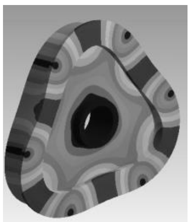
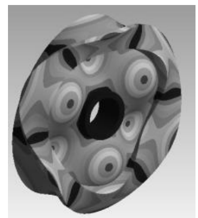
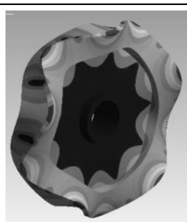
5	1761			10	3222		
11	3773			14	4870		
12	4418			15	4924		
13	4533						

Table 3. **Vibration characteristics of wheel**

Nodes create diameters			Nodes create circles		Nodes create diameters and circles			In plane modes	
Mode No	Eigenfreq. [Hz]	diameters number	Mode No	Eigenfreq. [Hz]	Mode No	Eigenfreq. [Hz]	diameters number	Mode No	Eigenfreq. [Hz]
1	384	1	2	512	8	3045	1	6	2032
3	721	2	4	767	11	3773	2	10	3222
5	1761	3	7	2798	15	4924	3	12	4418
9	3083	4	14	4870					
13	4533	5							

4.2. Analysis of model frequency characteristics

This section presents the study of wheel model frequency characteristics. Calculations are performed for the wheel position close to the central position (Figure 5*b*) and the position corresponding to a flange contact (Figure 5*c*). Frequency Response Function values, at the point of the wheel shown in Figure 5*a*, are the subject of analysis. The FRF values were calculated in steps of 5 Hz. In the case of three-dimensional model, the displacement of that point $\vec{U}(x,y,z)$ and the FRF is a vector:

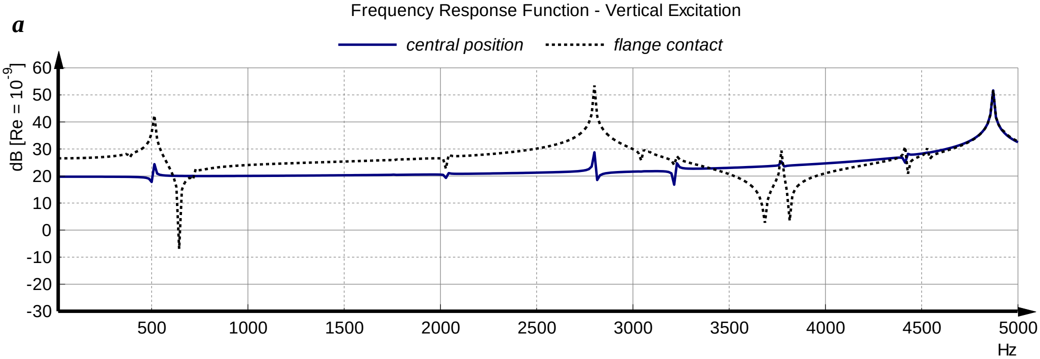
$$\vec{U}(x,y,z) = [u_x(x,y,z), u_y(x,y,z), u_z(x,y,z)] .$$

This vector absolute value is the subject of analysis which consists with the comparison of FRF values and its course depending on the point of application of excitation force.

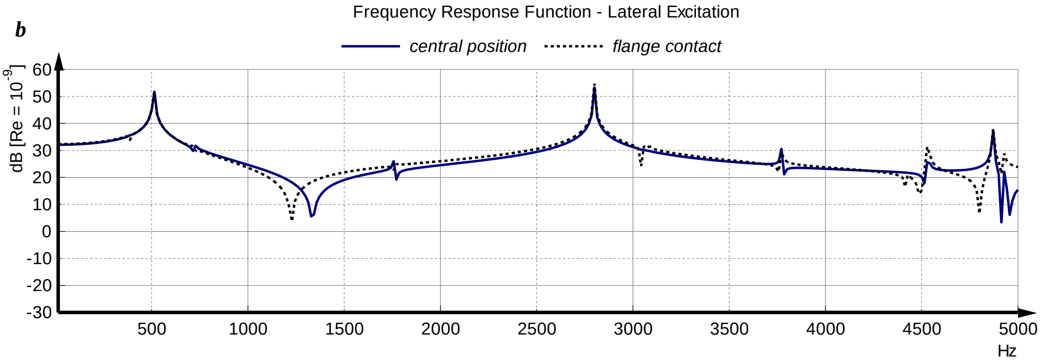
Figures 6 *a*,*b*,*c* are the graphs of the FRFs due to the vertical (Figure 6*a*), lateral (Figure 6*b*) and the longitudinal force (Figure 6*c*). The graphs are presented in decibel scale i.e. dB = 10log10 (W/W_{ref}), where $W_{ref} = 10^{-9}$ m.

Preliminary analysis shows that the point of exciting force application of vibration can change the FRF.

a



b



c

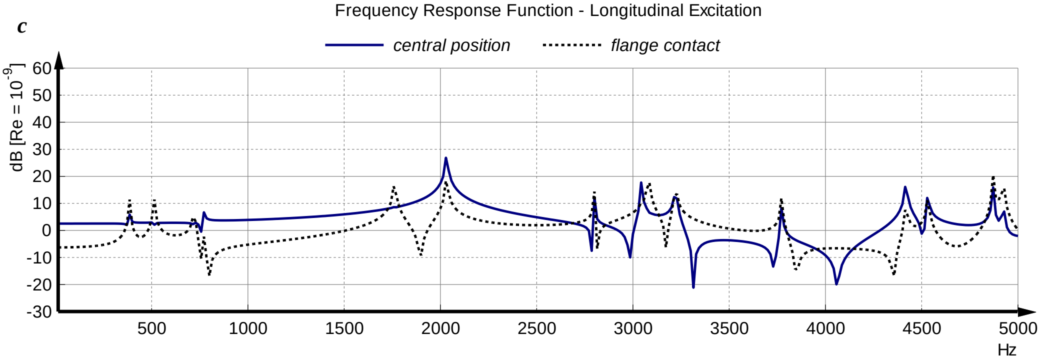


Fig. 6. Frequency Response Functions due to the vertical (Figure 6a), lateral (Figure 6b) and the longitudinal force (Figure 6c)

4.3. Analysis of calculations results

When examining calculation results obtained from the wheel model primarily we would consider its technical aspect. We will focus on the values of a frequency corresponding to the eigenforms of vibrations, the peaks of FRFs and "band width" accompanying the peaks of FRF in the graph. Value of the peak of the FRF indicates in some way simultaneously the accompanying bandwidth that is associated with this peak. This is due to the assumed transmittance values calculation step. Theoretically all peaks of FRF are infinite because the lack of damping in model (steel).

Taking into account made previously considerations and eigenfrequencies corresponding them eigenforms, the rolling noise will be associated with modes the numbered from 1 to 7.

Those modes, above all, represent the motion of the wheel rim, and even in the case of mode 6, it is a movement in the XZ plane (Fig. 3).

We discuss now the FRF peaks values and their eigenforms.

Mode number 1 generally may be described as a bending of the wheel relative to its axis of symmetry lying in its plane and its clear occurrence in FRF graph dependents on the position of the exciting force application. Mode exposes as the effect of longitudinal force (Fig. 6*c*). The value of this peak in comparison to other peaks appearing in the graph is small and therefore this peak is accompanied by comparatively narrow bandwidth.

Mode 2 shows motion of the tread in a direction perpendicular to the plane of the wheel. This mode is obviously the consequence of the lateral force but it is also demonstrated in FRF graphs for longitudinal and vertical forces. Albeit in those two cases the values of the peaks are significantly smaller.

Mode 3 belongs to group of modes where nodes of eigenforms create diameters. The value of the peak is small. Peak appears on the FRF graphs depends on longitudinal and lateral forces.

Mode 4 shows the movement of the tread in its plane. It represents tension and compression of the tread. Mode appears on the graph of the FRF when the system is excited by longitudinal force. Mode depends on the point of force application.

Mode 5 appears only once a FRF graph when the system is excited by longitudinal force. Its appearance depends on the position of the applied force. Peak value is small and accompanying bandwidth is narrow.

Mode 6 illustrates the movement of the wheel in the plane (XZ) - wheel adopts an elliptical shape. The presence of the peak corresponding with this mode in FRF graph is associated with the vertical and longitudinal forces. Peak value is small.

Mode 7 appears in the FRF graph when the wheels vibrations are caused by any forces but FRF peak reaches the greatest value for lateral force. Then the bandwidth is accompanied by a broad band. Mode shows the movement of the tread in a direction perpendicular to the plane of the wheel.

Finally, the peak values of FRF regarding to forces occurring in contact between the wheel and the rail in the range up to 3000 Hz should be discussed. We may consider the ratio of the lateral force to vertical force during ride on a tangent track i.e. without extreme situations when the high frequency phenomena occur. Under these conditions it can be assumed that the vertical force values are ten times greater than the lateral force values. This ratio, for longitudinal force and vertical force is even greater.

The modes 2 and 7 relating to vibration of the tread in a direction perpendicular to the plane of the wheel are dominant, even under those conditions. It is worth to mention that during the ride on regular arc the lateral force values are close to vertical force values.

Previously discussed ratio of the lateral to vertical force is accepted in the analysis of modes in the range from 3000 to 5000 Hz. The largest value and the widest bandwidth is associated with the modes numbered 10, 11 and 14. These modes are primarily demonstrated in graphs as an effect of the vertical force action but also occur in the FRF graphs associated with the longitudinal force. Other modes are associated with the effects of longitudinal forces.

5. Conclusions

The article focuses on the study of steady state motion of monobloc wheels tram in frequency domain. The scope of the analysis has been established arbitrarily and includes the vibration range up to 5 kHz. The eigenforms and eigenvalues of wheel vibration were examined. Frequency Response Function was calculated at the point of the wheel lying on its flange. Calculations were performed using the finite element method.

From the point of view of the construction of the object, in which one may distinguish a disc and a tread this last plays a more important role in the generation of high-frequency vibrations (up to 5 kHz). The modes numbered 2, 4, 7 and 14 are clearly related to a tread movement. Those modes belong to modes group in which nodes create circles.

The calculations showed that the "intensity" of the vibration in the specific frequency bands depends on the place where the exciting force is applied. Changes in the values of Frequency Response Function, depending on the point of force application, relate primarily to the frequency range up to 2000 Hz.

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