

Sebastian KULA\*

## INTERCONNECT ELEMENTS PROPAGATION QUANTITIES IN PIC

This paper describes methods to extract nominal values of propagation quantities for thin-film, straight interconnect elements. The propagation quantities like effective dielectric permittivity  $\epsilon_{\text{eff}}$ , attenuation  $\alpha$ , characteristic impedance  $Z_0$  are calculated with the use of analytical and approximated formulas. These expressions are obtained by transforming and by fitting formulas, typically used for calculation of per unit length transmission line parameters. Methods are verified for typical thin-film interconnect elements dimensions and for typical materials used in the Photonics Integrated Circuits (PIC). The novelty of the study is the parametrization with respect to the geometrical dimensions. The parametrization is based on the approximation expressions in the form of rational polynomials obtained by fitting.

### 1. INTRODUCTION

Nowadays much research is focusing on modelling, simulation, design manufacturing and measuring techniques of Photonics Integrated Circuits (PIC). There is a huge necessity of academic world but also engineers from industry to obtain solutions, providing cost-effective, accurate, universal methods of the PIC design, manufacturing and packaging. One of the proposed way, to achieve these ambitious goals, is to use an existing knowledge in the field of electronic Integrated Circuits (IC) design, manufacturing and packaging, by implementation of this knowledge for PIC devices.

Electronic IC and PIC are very similar devices, essential difference concerns a type of working signal. In PIC devices, the working signal is an electromagnetic signal from optical, microwave frequency range, so usually these signals are with much higher frequencies than in typical electronic IC. What is more, electronic IC devices contain components such like: transistors, capacitors, resistors. In the case of the PIC there are components such like: lasers, attenuators, detectors, modulators, multiplexers and optical amplifiers. The PIC components are fabricated from different materials but currently solutions based on Indium

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\* Kazimierz Wielki University in Bydgoszcz.

\* Chalmers University in Göteborg.

Phosphide InP became a standard. So far, only using InP material in PIC devices can ensure cost-effective mass production.

Interconnect elements play an important role in electronic IC. In the case of PIC the importance of these elements is even stronger, because the working frequency, which is expected in PIC is from few GHz up to tens GHz and even up to several hundreds of GHz. Signals, with such relatively high frequency, which are transmitting through interconnect elements are serious challenge in the design process due to parasitic effect.

The subject of this paper is how nominal and parametric models, for propagation quantities of interconnects can be extracted in a fast and robust manner. The basic interconnects structures, like microstripes were analysed. First, the extraction of the propagation quantities with the use of p.u.l. parameters is presented, and then transforming and fitting formulas are discussed.

Application of PIC devices is very broad, from telecommunication, sensors, scatterometry to medical applications and signal processing. This result, that PICs are very composite devices and during design process of PIC, special and advanced software is required. Proposed here methods can be used in EDAS for PIC, to reduce large computation time and computation effort, typically needed in the interconnect elements simulation or design. The study is made in limited frequency range, from 1GHz up to 50GHz. Approaches proposed are validated by comparing with computer simulation experiments. In general there is a good agreement between results of the approaches proposed and commercial EDAS, especially in the frequency range above 10GHz.

## 2. PROPAGATION QUANTITIES

To design interconnect elements for PIC in efficient and accurate way it is necessary to know values of their propagation quantities. It is essential issue in design process because PIC devices, are working in the high frequency range and propagation quantities  $\epsilon_{\text{eff}}$ ,  $\alpha$  and  $Z_0$  are strongly frequency dependent.

In this chapter the approach to obtain propagation quantities is depicted. The method is based on Transmission Line Theory and per unit length resistance  $R$ , inductance  $L$ , capacitance  $C$ , conductance  $G$ . Per unit length  $R, L, C, G$  values can be calculated from analytical formulas.

### 2.1. TL theory for propagation quantities

For the per unit length line parameters shown in Fig.1 the line impedance and propagation constant are presented in a standard way:

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (1)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2)$$

with commonly used notations:  $\gamma = \alpha + j\beta$ ,  $\epsilon_{\text{eff}} = (\beta/k_0)^2$ ,  $k_0 = 2\pi f/c_0$ ,  $c_0 = 3 \cdot 10^8$  m/s,  $\omega = 2\pi f$  where  $\gamma$  is propagation constant,  $\beta$  is phase constant,  $k_0$  is wave number of a plane wave in free space,  $c_0$  is a speed of light.

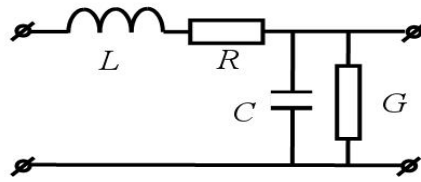


Fig. 1. Equivalent circuit of interconnect element

If we consider that  $C$ ,  $L$ ,  $G$  and  $R$  are known, the impedance  $Z_0$  is calculated with (1), effective dielectric permittivity  $\epsilon_{\text{eff}}$  and losses  $\alpha$  are calculated:

$$\epsilon_{\text{off}} = \left( \frac{\text{Im}(\gamma)}{k_0} \right)^2 \quad (3)$$

$$\alpha = 8.68 \text{Re}(\gamma) \text{length} \quad (4)$$

Propagation constant  $\gamma$  is calculated from (2).

### 3. OVERVIEW OF P.U.L. RLCG PARAMETERS

In this chapter are depicted, already known in literature analytical formulas for calculating p.u.l. RLCG parameters. Presented formulas are dedicated to interconnect, MS-type elements, Fig. 2. Novelty of the paper are coefficients values, introduced in total line capacitance formula.

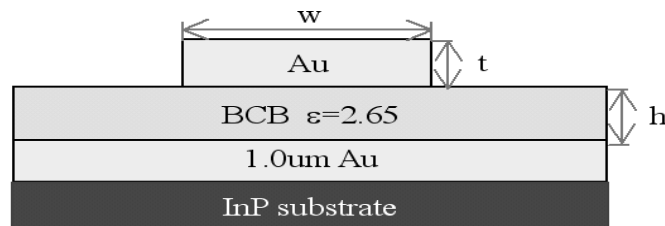


Fig. 2. Cross section of interconnect MS-type element and its geometrical parameters

The MS-type element, from Fig. 2 has on the top metallization layer performed from Au (with width  $w$  and thickness  $t$ ), below there is dielectric layer (with height  $h$ ) made from Benzocyclobutene (BCB). Under BCB is ground metalization, made from Au (thickness  $1 \mu\text{m}$ ) and below InP substrate.

### 3.1. P.u.l. capacitance

Formula for line C is based on publication by Stellari et al [1] where total capacitance of the interconnect MS-type element, is given as a sum of the capacitances between four metal sides of strip and the ground plane. The closed form formulas of these capacitances are based on conformal mapping technique [2] and give their functional dependences on the cross sectional sizes. The fitting parameters, partly given in the original paper, are corrected and extended to match with the results of Momentum and Sonnet simulations.

The partial capacitance of a vertical side wall of the strip is [1]:

$$C_v = \varepsilon_{\text{req}} \frac{\varepsilon_0}{2} \frac{K(k_{\text{vp}})}{K(k'_{\text{vp}})} = 0.715 \frac{\varepsilon(1 + 2.31 \frac{t}{h})}{1 + \frac{t}{h} \varepsilon} \frac{\varepsilon_0}{2} \frac{K(k_{\text{vp}})}{K(k'_{\text{vp}})} \quad (5)$$

where  $\varepsilon_{\text{req}}$  is the equivalent effective dielectric constant and  $\varepsilon$  is the dielectric constant of the substrate,  $K(k_{\text{vp}})$  and  $K(k'_{\text{vp}})$  are complete elliptic integrals of the first kind (they can be calculate using approximations or built-in Matlab function) with modulus  $k_{\text{vp}}$  related to the geometrical dimensions, Fig. 2:

$$k_{\text{vp}} = \sqrt{1 - \left(\frac{h}{h+t}\right)^2} \quad (6)$$

The capacitance of the top side of the strip is given by [1]:

$$C_{\text{top}} = \varepsilon \varepsilon_0 \left( 2 \frac{g(w)}{\pi} - \frac{K(k_{\text{top}})}{K(k'_{\text{top}})} \right) \quad (7)$$

with  $k_{\text{top}} = \tanh\left(\frac{\pi w}{4 h}\right)$

The function  $g(w)$  is defined as:

$$g(w) = 1 + \sqrt{1 + cc(w)^2} - 4(1 + \sqrt{1 + cc(w)^2})^2 \exp(-2(1 + \sqrt{1 + cc(w)^2}))$$

where  $cc(w)$  is a solution to this equation:

$$cc(w) - a \sinh(cc(w)) = \frac{\pi w}{2 h} \quad (8)$$

The capacitance due to the bottom side of the strip including the fringing field:

$$C_{\text{bot}} = \frac{\varepsilon \varepsilon_0 w}{h} + 4 \ln(2) \frac{\varepsilon \varepsilon_0}{\pi} \quad (9)$$

The total line capacitance formula is paper novelty and using (5), (7) and (9) is defined as:

$$C = \frac{\varepsilon\varepsilon_0 W}{h} + 1.1952 \ln(2) \frac{\varepsilon\varepsilon_0}{\pi} + 0.6872C_{\text{top}} + 0.6872C_v \quad (10)$$

### 3.2. Shunt conductance and series resistance

Shunt line G conductance is based on already calculated total line capacitance (10):

$$G = C \cdot 2\pi \cdot f \cdot \tan \delta \quad (11)$$

The series line resistance (Ohmic losses) is given by:

$$R = \frac{1}{(w + 8 \frac{t}{2\pi\varepsilon} (1 + \ln(\frac{8\pi w}{t}))) \sigma \delta (f) (1 - \exp(\frac{-t}{\delta(f)}))} \quad (12)$$

where  $\sigma$  is conductivity of the strip, and  $\delta$  is skin depth,  $\delta = 1/\sqrt{(\pi\mu_0\sigma f)}$ ,  $\mu_0$  is the magnetic constant of vacuum.

### 3.3. P.u.l. inductance

The line inductance is a sum of geometrical (external),  $L_g$ , and internal inductances. The external inductance  $L_g$  takes into account the magnetic field outside of the conductors and it is frequency independent. The internal inductance,  $L_{se}$ , is associated with penetration of the magnetic field into the strip.

$$L = L_g + L_{se} = \frac{1}{c_0^2 C_a} + \frac{R}{2\pi f} \quad (13)$$

where  $c_0 = 1/\sqrt{(\varepsilon_0\mu_0)}$  is the velocity of light in vacuum,  $\mu_0$  and  $\varepsilon_0$  are correspondingly the magnetic and dielectric constants of vacuum,  $C_a$  is the capacitance per unit length of the line without dielectric substrate ( $\varepsilon=1$ ), defined with the use of transformed version of (10):

$$C_a = \frac{\varepsilon_0 W}{h} + 1.9872 \ln(2) \frac{\varepsilon_0}{\pi} + \frac{1.1426C_{\text{top}} + 1.1426C_v}{\varepsilon} \quad (14)$$

Coefficients in (14) are paper novelty comparing to publication [1].

## 4. TRANSFORMING AND FORMULAS FITTING

This chapter depicts approximations, which are improvements of analytical formulas from the previous chapter and are the paper novelty. Obtained formulas include parameterisation of geometrical dimensions. Results were compared with results of ADS Momentum, Sonnet software and also with results from Schnieder publication [3].

### 4.1. P.u.l. CRLG modifications

Conformal mapping (CM) equations, from chapter three were modified, the modification relied on introduction of new coefficients ( $pC$ ,  $pCa$ ) to the formulas

of the total p.u.l. capacitance  $C$  and the capacitance  $C_a$  of the line without dielectric substrate (used during computation of the total p.u.l. inductance  $L$ ). A modified CM formula for the total capacitance:

$$C = \left( \frac{\varepsilon\varepsilon_0 w}{h} + 0.3 \cdot 4 \ln(2) \frac{\varepsilon\varepsilon_0}{\pi} + 0.69C_{\text{top}} + 0.69C_v \right) pC \quad (15)$$

where  $pC$  is new modifying coefficient.

A modified CM formula for the capacitance p.u.l. of the line without dielectric substrate ( $\varepsilon = 1$ ):

$$C_a = \left( \frac{\varepsilon_0 w}{h} + 0.3 \cdot 4 \ln(2) \frac{\varepsilon_0}{\pi} + \frac{0.69C_{\text{top}} + 0.69C_v}{\varepsilon} \right) pC_a \quad (16)$$

where  $pC_a$  is new modifying coefficient.

Values for the new coefficients ( $pC$ ,  $pC_a$ ) were obtained with the use of an empirical method. It has been observed that values of the  $pC$  coefficient depend mainly from the thickness ( $t$ ) and from the height ( $h$ ) of the MS-type interconnect element and that values of  $pC_a$  coefficient depend mainly from the width ( $w$ ) and from the height ( $h$ ) of the MS-type interconnect element. Mathematical description of observed empirical relations  $pC(t, h)$  and  $pC_a(w, h)$  was find with the use of a sftool (Surface Fitting Tool) from the Matlab. New polynomials, which are novelty of the paper for two variables were created. A polynomial formula for the  $pC(t, h)$  coefficient:

$$pC(t, h) = 0.6934 - 0.02203 \cdot 10^6 t + 0.09174 \cdot 10^6 h + 0.00307 \cdot 10^{12} th - 0.006104 \cdot 10^{12} h^2 \quad (17)$$

A polynomial formula for the  $pC_a(w, h)$  coefficient:

$$pC_a = 1.514 - 0.02039 \cdot 10^6 w + 0.09305 \cdot 10^6 h + 0.0003976 \cdot 10^{12} w^2 + 0.000480 \cdot 10^{12} wh - 0.006898 \cdot 10^{12} h^2 \quad (18)$$

The series line resistance is computed exactly the same way as in (12), conductance with (11) formula but with modified  $C$ , formula (15). Inductance with (13) formula but  $C_a$  is modified according to (16).

## 4.2. Validation

The model was compared with ADS Momentum and Sonnet simulations and the estimated relative errors in the specified limits of the parameters are given below:

For  $0.5 \leq w/h \leq 3.3$  and  $0.1 \leq t/h \leq 0.778$  maximum errors at high frequency range, at around 50 GHz are:

$$\text{Max} \left( \frac{\Delta Z}{Z} \right) = 2.9\% \quad \text{Max} \left( \frac{\Delta \varepsilon_{\text{off}}}{\varepsilon_{\text{off}}} \right) = 4.9\% \quad \text{Max} \left( \frac{\Delta \alpha}{\alpha} \right) = 25.4\%$$

For  $0.5 \leq w/h \leq 3.3$  and  $0.1 \leq t/h \leq 0.778$  maximum errors at intermediate

frequency range, at around 10 GHz are:

$$\text{Max}\left(\frac{\Delta Z}{Z}\right) = 6.7\% \quad \text{Max}\left(\frac{\Delta \varepsilon_{\text{off}}}{\varepsilon_{\text{off}}}\right) = 10.3\% \quad \text{Max}\left(\frac{\Delta \alpha}{\alpha}\right) = 13.1\%$$

## 5. CONCLUSIONS

In this paper were presented results of the modified CM formulas dedicated for simple modelling and design of interconnect elements. With the use of modified CM formulas basic quantities of the MS-type interconnect element i.e.  $Z_0$ ,  $\varepsilon_{\text{eff}}$ ,  $\alpha$  were obtained. The modified CM results were compared with Schnieder method and ADS Momentum, Sonnet software. Obtained data for the characteristic impedance  $Z_0$  and the attenuation  $\alpha$  with the use of proposed modified CM formulas, in all analyzed cases fit better to ADS Momentum data than Schnieder formulas. In cases of the effective permittivity constant  $\varepsilon_{\text{eff}}$  for eight examples the modified CM fits better than Schnieder formulas and also for eight examples fits worse than Schnieder. Anyway, differences between Schnieder formulas and the modified CM in the cases of  $\varepsilon_{\text{eff}}$  are small. Taking, all these facts and results into consideration it can be assumed that the modified CM is a simple method giving valid, robust and accurate results. This method is an easy way to obtain propagation quantities of the interconnect elements and can be easily use in EDAS for the PIC devices.

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