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APPLICATION OF ARTIFICIAL NEURAL NETWORKS TO PREDICT THE DEFLECTIONS OF REINFORCED CONCRETE BEAMS

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Abstract: Nonlinear structural mechanics should be taken into account in the practical design of reinforced concrete structures. Cracking is one of the major sources of nonlinearity. Description of deflection of reinforced concrete elements is a computational problem, mainly because of the difficulties in modelling the nonlinear stress-strain relationship of concrete and steel. In design practise, in accordance with technical rules (e.g., Eurocode 2), a simplified approach for reinforced concrete is used, but the results of simplified calculations differ from the results of experimental studies.

Artificial neural network is a versatile modelling tool capable of making predictions of values that are difficult to obtain in numerical analysis. This paper describes the creation and operation of a neural network for making predictions of deflections of reinforced concrete beams at different load levels. In order to obtain a database of results, that is necessary for training and testing the neural network, a research on measurement of deflections in reinforced concrete beams was conducted by the authors in the Certified Research Laboratory of the Building Engineering Institute at Wrocław University of Science and Technology. The use of artificial neural networks is an innovation and an alternative to traditional methods of solving the problem of calculating the deflections of reinforced concrete elements. The results show the effectiveness of using artificial neural network for predicting the deflection of reinforced concrete beams, compared with the results of calculations conducted in accordance with Eurocode 2. The neural network model presented in this paper can acquire new data and be used for further analysis, with availability of more research results.

Key words: *reinforced concrete beams, research, deflection, artificial neural network*

1. INTRODUCTION

From a structural analysis and design point of view, reinforced concrete is a very complex composite material. It is a combination of two materials (concrete and steel) with entirely different mechanical properties. Due to the nonlinear stress-strain relationship of concrete and steel reinforced concrete cannot be modelled properly by linear elastic behaviour. Moreover, due to the cracking of concrete, even the sectional and therefore the structural properties depend on the nature and magnitude of the applied loads. Cracking of concrete is a significant phenomenon, as the maximum bending moment is usually several times greater than bending moment that causes cracking. Cracks in structural elements cause a change of moment of inertia and therefore the stiffness degradation of the element. The cracked reinforced concrete element shows cracks at certain distances from each other, and their number is finite.

As the load increases the initial distribution of stiffness of the element changes and the number of cracks varies nondeterministically. The distribution of strains and stresses in concrete and steel, along the axis of the element, is irregular.

Accurate determination of deflection of reinforced concrete elements is a computational problem, mainly because of the difficulties in modelling of the nonlinear stress-strain relationship of concrete and steel. There are some scientific publications, where heterogeneous stiffness of the reinforced concrete element is described by using continuum function (linear or nonlinear) [1], as well as using constant value of stiffness in each section of the element [2]. The inelastic characteristics of the structure can be taken into account in the form of spot-localized defects, described by the application of distribution calculus [3].

In design practice, when the deflection of a reinforced concrete beam is calculated, according to the standards [4], [5], two states are analysed: cracked state and uncracked state. The flexural stiffness of a reinforced concrete beam changes from the uncracked state to the cracked state. The equation for the effective moment of inertia is used to calculate a moment of inertia somewhere between the uncracked moment of inertia and the cracked moment of inertia depending on the applied load. The resulting effective moment of inertia can be used in the elastic deflection equations to approximate the actual deflections.

However, this is a simplified approach, which does not allow the deflection of an RC element to be precisely calculated. According to experimental research on bending reinforced concrete beams [6], that was conducted by Kubicki, the difference in mean values of deflections calculated according to [5] and experimentally obtained was 21%, with a coefficient of variation $v = 22.6\%$.

The artificial neural network (ANN) can be an alternative tool to accurately estimate the deflection of reinforced concrete beams. ANN as a modelling tool is suitable for producing prediction systems (such as the prediction of deflections) based on a set of data available from real world observations and experiments. ANN uses the independent parameters as input and predicts the dependent parameters as output. This requires such training of the network that the resultant errors are minimised.

This paper presents the application of artificial neural network in predicting the deflection of reinforced concrete beams, as an effective tool for the analysis of issues in the field of reinforced concrete structures.

2. ARTIFICIAL NEURAL NETWORKS AS A TOOL FOR PREDICTION

Scientists have always endeavored to develop mechanism inspired by a human brain which is capable of machine learning as well as pattern recognition. As an effect of this work artificial neural networks were created. The history of this discipline begins with the development of the first artificial neuron by McCulloch and Pitts in 1943 [7]. Today artificial neural networks are widely used in classification, robotics, data processing and sequence recognition. Main applications are interpolation, approximation, prediction and grouping. In literature, there are plenty of examples how to use neural networks for solving engineering problems such as interpretation of the results of nondestructive testing [8], planning of the construction processes [9] or geotechnical problems [10].

Neural networks can be an alternative for prediction of the behaviour of structural elements. Some of the applications of neural networks in literature, in the field of structural engineering, include prediction of various structural quantities [11], [12]. Papers [13], [14] present the application of ANNs to predict bending moment in continuous composite beams. There are some papers that present the application of artificial neural networks to predict the deflection of structural elements. Neural networks have been used for prediction of deflection in steel-concrete composite bridges incorporating flexibility of shear connectors, shear lag effect and cracking in concrete slabs [15]. Paper [16] presents the application of ANN to predict deep beam deflection using experimental data from eight high-strength-self-compacting-concrete (HSSCC) deep beams. These studies reveal the strength of neural networks in predicting the solutions of different structural engineering problems.

Neural network can consist of many neurons grouped in different count of layers. Layer count is determined by the complexity of the problem to solve [17]. Every artificial neuron receives one or more inputs and sums them to produce an output. Usually the sums of each node are weighted, and the sum is passed through a function known as an activation function. Neural networks are trained using different algorithms such as variable metric methods or back propagation. During the training of different inputs, the weight values are changed dynamically until their values are balanced, so each input will lead to the desired output. Different measures are used to evaluate the efficacy of the neural network. The most popular one is MSE – mean squared error (1) or RMSE – root mean squared error (2). The error is calculated simultaneously for training and testing data in the course of the training process.

$$\mathrm{MSE}(P) = \frac{1}{P}\sum_{i=1}^{P}(z_i - y_i)^2, \tag{1}$$

$$\mathrm{RMSE}(P) = \sqrt{\frac{1}{P}\sum_{i=1}^{P}(z_i - y_i)^2}, \tag{2}$$

where

y_i – predicted values of output, $i = (1, ..., P)$,
z_i – actual (measured) values of output,
P – count of elements in database.

Mainly used neural network architecture is MLP – Multi-Layered Perceptron. The foundation of this networks training is the back propagation algorithm. MLP networks can estimate many complex mappings. Network structure is shown in Fig. 1.

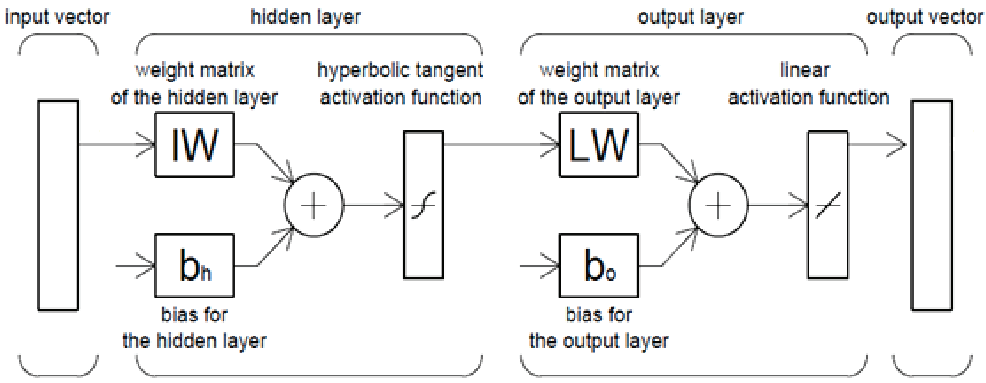

Fig. 1. The structure of MLP neural network used for prediction

The input of the network is vector **x**. It is multiplied by the weight matrix of the hidden layer **IW**. In the next step bias $\mathbf{b_h}$ is added to the result vector. After that hyperbolic tangent activation function is used. It can be described using the following equation

$$\mathbf{y_h} = \tanh(\mathbf{IW} \cdot \mathbf{x} + \mathbf{b_h}) . \tag{3}$$

Vector $\mathbf{y_h}$ (3) is the output of the hidden layer. It is multiplied by the weight matrix of the output layer **LW**. Analogously bias $\mathbf{b_0}$ is added. The only difference is the activation function – in that case a linear function is used. Final output of the network is given by the equation

$$y = \mathbf{LW} \cdot \mathbf{y_h} + \mathbf{b_0} . \tag{4}$$

3. RESEARCH

3.1. PURPOSE, SCOPE AND RESEARCH PROGRAM

Experimental research on bending reinforced concrete beams was conducted in order to provide a database used for training and testing artificial neural network. The analysis of three beams on a lab scale had been planned beforehand.

Experimental investigations carried out on RC beams were preceded by an experimental determination of material properties, which included:

- determining the average value of Young's modulus of reinforcing steel,
- determining the average value of Young's modulus of concrete.

Properties of reinforcing steel were determined in the state of axial tension and PN-EN10002-1:2004 standard [18] was followed. The average value of Young's modulus of concrete was determined as a result of cyclic loading of cylindrical samples. Material properties were determined one day prior to the research on RC beams.

3.2. RESEARCH ON REINFORCED CONCRETE BEAMS

Three reinforced concrete beams with rectangular cross-section (100 × 200 mm) and the same length were prepared for the tests. RC beams differed in the degree of reinforcement. There were two tensile reinforcement rods with a diameter of 10 mm (beam

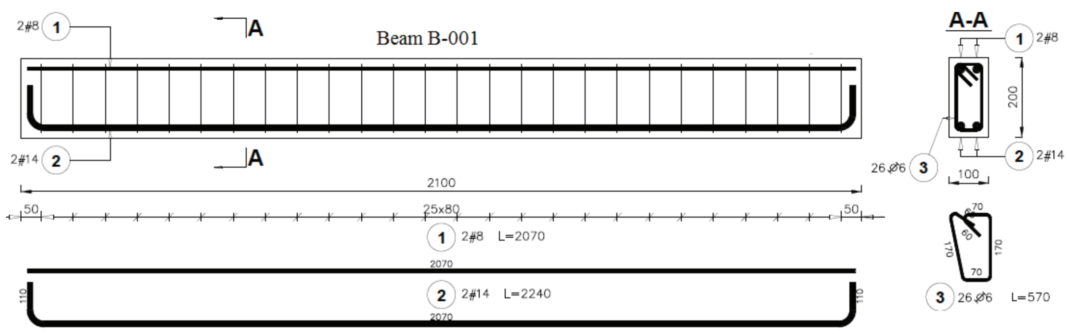

Fig. 2. Beam B-001

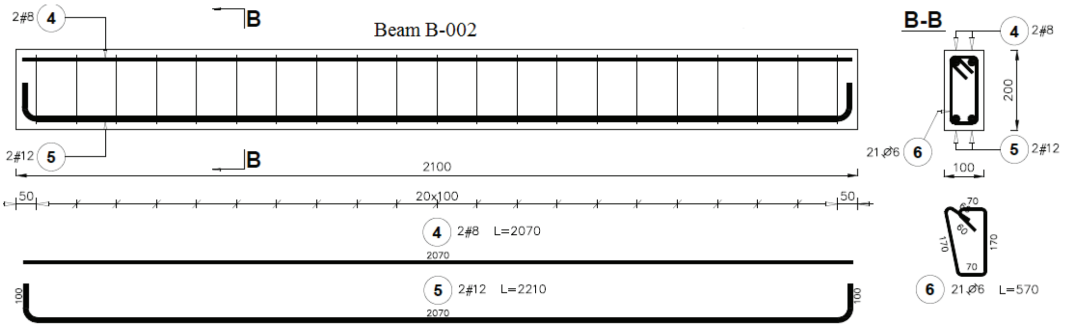

Fig. 3. Beam B-002

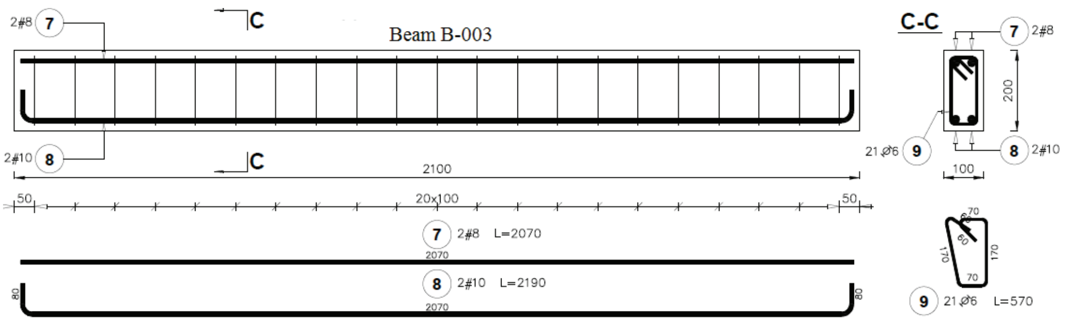

Fig. 4. Beam B-003

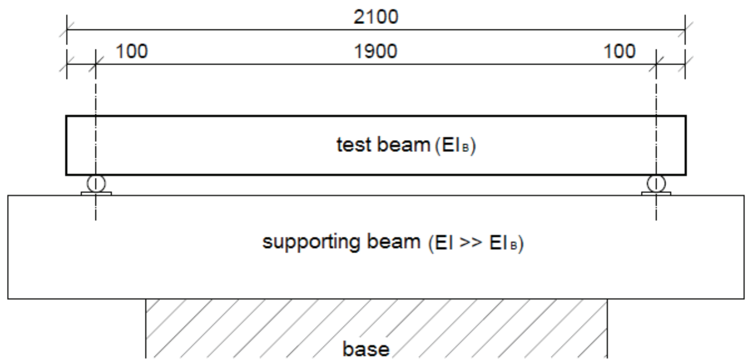

Fig. 5. Test stand. EI – stiffness of supporting beam, EI_B – stiffness of test beam

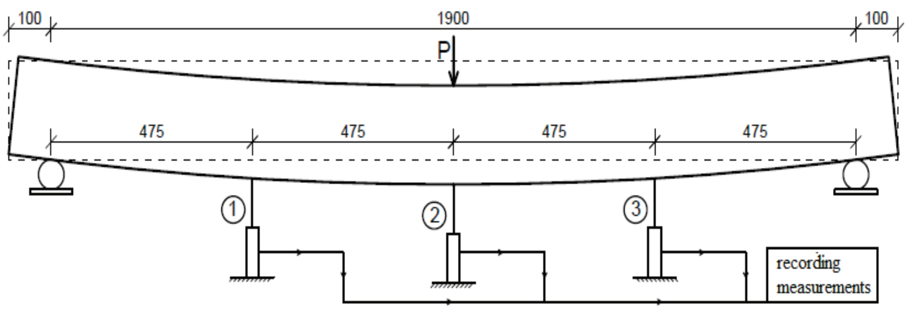

Fig. 6. Scheme of the measurement system

B-001), 12 mm (beam B-002) and 14 mm (beam B-003). The beams that were prepared for the tests are presented in Figs. 2–4.

The study assumed a static diagram of a simply supported beam. Steel rollers provided freedom of rotation on the ends of the beam. Research methodology assumed a three-point bending scheme. A scheme of the test stand is presented in Fig. 5.

Deflections were measured by using inductive sensors, accurate to 0.001 mm. A scheme of the measurement system is shown in Fig. 6.

Deflections were measured for each load level. At the next load level the force was increased by 0.4 kN. Finally, there were measured 293 values of deflection (74 values of deflection of beam B-001, 96 values of deflection of beam B-002 and 123 values of deflection of beam B-003). Table 1 presents the tabulation of measured values of deflection.

Table 1. Measured values of deflection

Number of test	Force [kN]	Bending moment M [kNm]	Deflection a [mm]
Beam B-001			
1	0.5	0.238	0.057
2	1.0	0.475	0.106
3	1.5	0.713	0.152
...	...	...	...
72	36.0	17.100	12.417
73	36.5	17.338	12.681
74	37.0	17.575	13.035
Beam B-002			
75	0.5	0.238	0.038
76	1.0	0.475	0.073
77	1.5	0.713	0.104
...	...	...	...
168	47.0	22,325	12,140
169	47.5	22.563	12.582
170	48.0	22.800	13.184
Beam B-003			
171	0.5	0.238	0.022
172	1.0	0.475	0.042
173	1.5	0.713	0.059
...	...	...	...
291	60.5	28.738	12.492
292	61.0	28.975	12.857
293	61.5	29.213	13.529

The graph (Fig. 7) presents a comparison of deflections a in the middle of the span versus the value of the bending moment M for all three RC beams.

The comparison of deflections a in the middle of the span versus the value of the bending moment M demonstrates the difference in the stiffness of three RC beams tested.

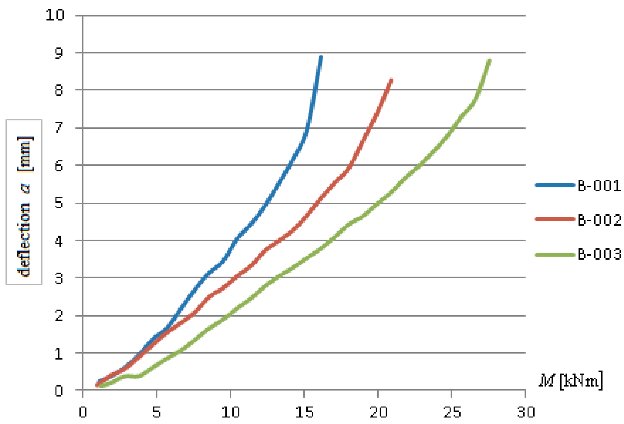

Fig. 7. Graph of deflections of RC beams versus value of the bending moment M

4. APPLICATION OF ARTIFICIAL NEURAL NETWORKS

A database made up of the results of experimental investigations carried out on RC beams (the values of the deflections in the middle of the span) was used to train and test the network. Finally, 293 patterns, corresponding to 293 tests performed on beams (sets of input and output data) were obtained. The database was randomly divided into two parts: a training set (197 patterns – 67%) and a testing set (96 patterns – 33%).

In order to predict the deflection of beams, as a result of training and testing, a unidirectional Multi-Layered Perceptron network (MLP) with error back propagation algorithm was constructed. This type of network was chosen since it is the most suitable for solving the problem considered. Input vector **x** consisted of four elements: surface area of tension reinforcement A_s, the value of Young's modulus of reinforcing steel E_s, the value of Young's modulus of concrete E_c and the value of bending moment M in the cross section.

$$\mathbf{x} = \{A_s, E_c, E_s, M\}. \tag{5}$$

The deflection of reinforced concrete beam also depends on the geometry of the element (cross-sectional dimensions, span) and the static scheme. In view of the fact that all of the test beams have the same geometrical dimensions and the same static scheme, these components are omitted in the input vector. For the analysis of more diverse beams vector with a larger number of inputs should be adopted in the structure of the artificial neural network.

A neural network was designed by experimentally determining its structure. First, the data which the neural network was trained on were input and the ability to reproduce the training patterns was tested. Then the testing data were input and the identification was checked for correctness. The value of the deflection of RC beam was obtained at the segment's output.

The following quantities: the number of hidden layers, the number of neurons in the hidden layer and the number of training epochs were chosen experimentally. The structure of the neural network that was determined is shown in Fig. 8.

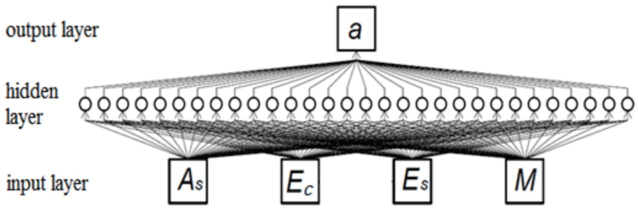

Fig. 8. Structure of the neural network

Architecture of neural network that is used in this paper: I-H-O: 4-30-1 [Input–Hidden–Output] means that this unidirectional neural network consists of 4 input neurons, one hidden layer with 30 neurons and one output neuron (the result – predicted value of deflection).

Figure 9 shows a line graph of mean square error MSE for training the neural network depending on the number of training epochs.

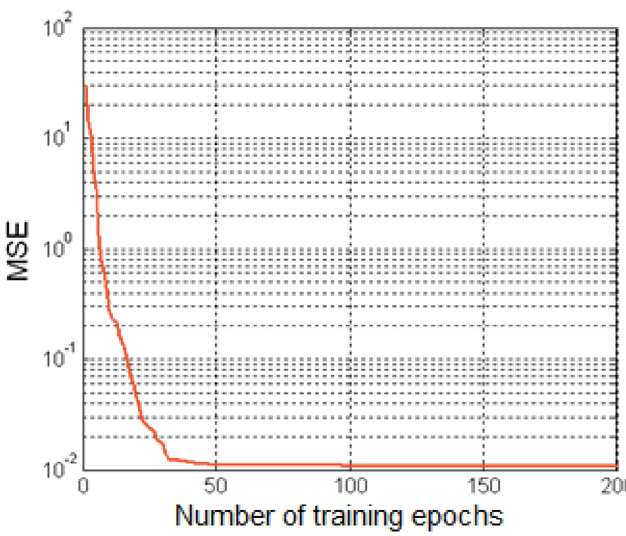

Fig. 9. Mean square error versus number of training epochs

The number of epochs used to train the neural network was 200, which provided satisfactory minimization of the MSE (MSE $\leq 10^{-2}$).

The accuracy of prediction of deflection by ANN was assessed by calculating the mean square error (1), the square root of the mean square error (2) and the Pearson correlation coefficient PCC

$$PCC(y_i, z_i) = \frac{E(y_i \cdot z_i) - (E(y_i) \cdot E(z_i))}{\sigma_{yi} \cdot \sigma_{zi}} \tag{6}$$

where

$E(y_i)$, $E(z_i)$ – the expected value of the estimated and real element of the output vector,

σ_{yi}, σ_{zi} – standard deviation of the estimated and real element of the output vector.

Table 2 presents the obtained values of MSE (1), RMSE (2) and the Pearson correlation coefficient PCC (6) for the training and testing set.

Table 2. Parameters of the effectiveness of the neural network

MSE [–]		RMSE [–]		PCC [–]	
training set	testing set	training set	testing set	training set	testing set
0.0118	0.0627	0.1086	0.2504	0.9996	0.9979

Graphs (Fig. 10) show the correlation between the values of deflections of reinforced concrete beams obtained in the laboratory and the values of deflections obtained by using an artificial neural network. Graphs present the values predicted by ANN versus real deflections for training set (Fig. 10a) and testing set (Fig. 10b). The convergence of the position of points with line $y = x$ indicates identification of values with very high accuracy.

Figure 11 shows the results obtained from the neural network in the form of histograms of the relative error $\varDelta$. The vertical axis represents the frequency FR of a given relative error $\varDelta$

$$\varDelta = \left| \frac{z_i - y_i}{z_i} \right| \cdot 100\,[\%], \tag{7}$$

$$\mathrm{FR}_{\varDelta \in (a,b)} = \frac{\text{the number of results with } \varDelta \in (a,b)}{\text{the number of all results}} \cdot 100\,[\%]. \tag{8}$$

The accuracy of prediction of deflections is very high. For the training set, the value of the relative

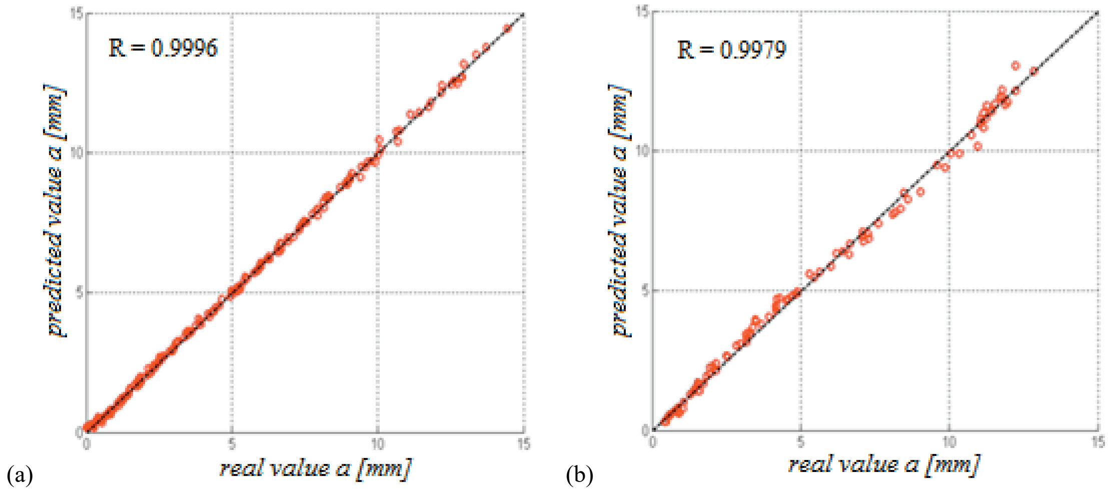

Fig. 10. Correlation between the values obtained in the laboratory and by using ANN
(a) training set, (b) testing set

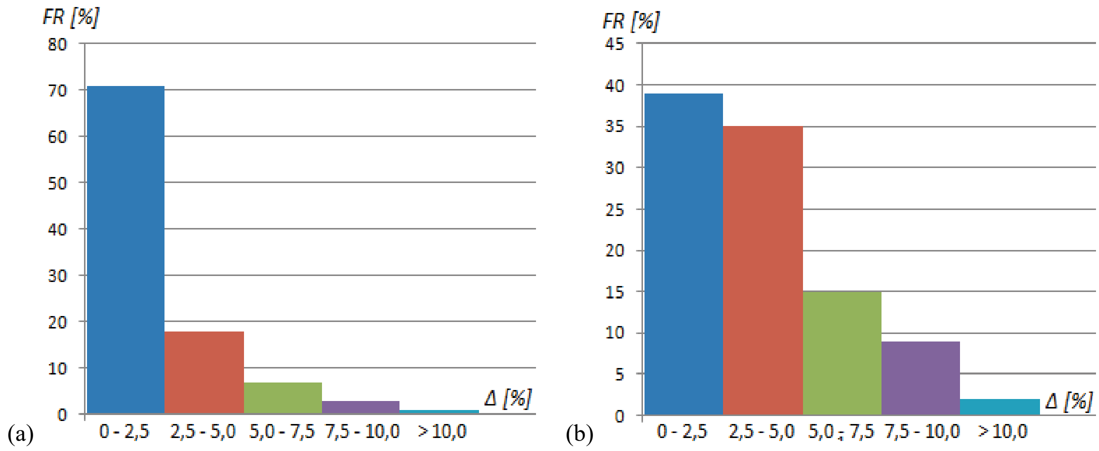

Fig. 11. Histograms of the relative error Δ: (a) training set, (b) testing set

error Δ was less than 2.5% in over 70% of the cases. For the testing set, the value of the relative error Δ was less than 5.0% in over 70% of the cases.

5. COMPARISON OF THE EFFECTIVENESS OF NEURAL NETWORK WITH THE RESULTS OF CALCULATION ACCORDING TO EUROCODE 2

Calculations of deflections according to PN-EN 10002-1:2004 standard [5] was done for different load levels in order to compare the effectiveness of deflection prediction of reinforced concrete beams by using artificial neural network. Material properties (the value of Young's modulus of reinforcing steel and concrete) used for calculation were obtained from laboratory tests.

Graph (Fig. 12a) shows the correlation between the values of deflections of reinforced concrete beams obtained in the laboratory and the values of deflections calculated according to the standard [5]. The position of points in comparison to line $y = x$ indicates that values of deflection were calculated with accuracy which was not as high as in the case of prediction by using neural network.

Figure 12b shows the results calculated according to the standard [5] in the form of histograms of the relative error Δ.

The accuracy of calculations of deflections is not high. For the values calculated according to the stan-

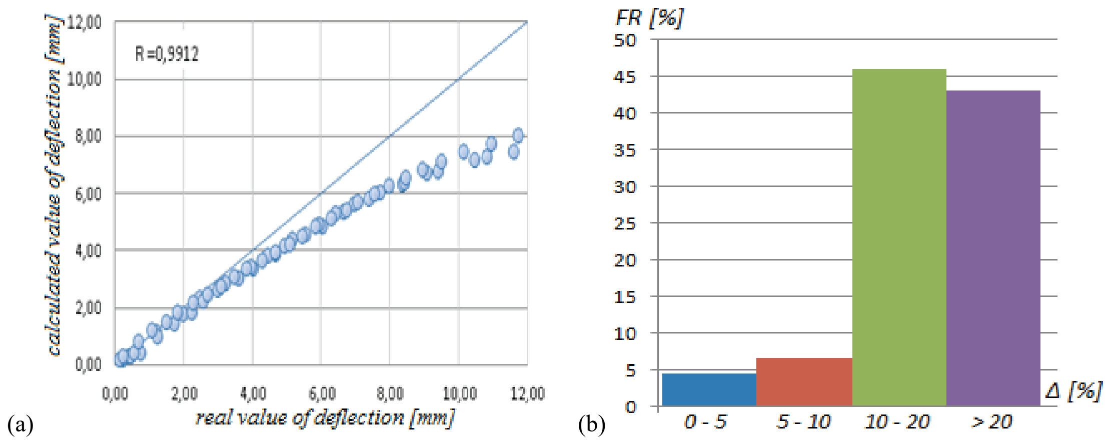

Fig. 12. Comparison of deflections a obtained from studies with values calculated according to the standard [5]: (a) correlation diagram of results, (b) histogram of the relative error

dard [5], the value of the relative error Δ was greater than 20% in over 40% of the cases.

6. ANALYSIS OF THE SIGNIFICANCE OF THE INPUT PARAMETERS

Each component of the input vector of the neural network has different information content and therefore has different effect on the value of the output vector. For this reason, it is worth considering the question of which of the inputs is the most important in the prediction. This is possible by using sensitivity analysis for minimization of input data dimension described in [19]. This type of analysis is carried out after artificial neural network training.

The sensitivity $S_i^{(p)}$ of the output $\mathbf{y}$ (it is assumed that the network has only one output) for the i-th component of the input vector $\mathbf{x}$, belonging to the pattern p, is defined by the formula

$$S_i^{(p)} = \frac{\partial y}{\partial x_i}. \tag{9}$$

This section presents the analysis of the significance of the input parameters for the designed artificial neural network.

The sensitivity $S_i^{(p)}$ of the output vector $\mathbf{y}$ for the x_i input parameter (element of the input vector $\mathbf{x}$), belonging to the pattern p, is described by the formula

$$S_i^{(p)} = \Sigma_{k=1}^{K}(LW_{(1,k)} \cdot IW_{(k,i)} \cdot \operatorname{sec}h^2(IW_{(k,1)} \cdot x_1 + IW_{(k,2)} \cdot x_2 + IW_{(k,3)} \cdot x_3 + IW_{(k,4)} \cdot x_4 + b_{h_k}). \tag{10}$$

When the results (10) for all P training patterns are obtained, the average sensitivity of the neural network for the i-th element of the input vector can be determined, e.g., using the Euclidean norm

$$S_{i,av} = \sqrt{\sum_{p=1}^{P} \frac{[S_i^{(p)}]^2}{P}}. \tag{11}$$

After calculating the average values (11), they were scaled by upholding the minimum and maximum values for each input parameter, and the minimum and maximum values of the output vector

$$S_{i,av,sc} = S_{i,av} \frac{\max_{p=1,\dots,P}\{x_i^{(p)}\} - \min_{p=1,\dots,P}\{x_i^{(p)}\}}{\max_{p=1,\dots,P}\{y^{(p)}\} - \min_{p=1,\dots,P}\{y^{(p)}\}}. \tag{12}$$

The values of $S_{1,av,sc}$, $S_{2,av,sc}$, $S_{3,av,sc}$, $S_{4,av,sc}$ express the relative importance of each of the input parameters. According to formula (5) elements of the input vector correspond to the following parameters: $\{x_1, x_2, x_3, x_4\} = \{A_s, E_c, E_s, M\}$. Table 3 presents the obtained values of $S_{i,av,sc}$.

Table 3. The average sensitivity of the neural network for the i-th input parameter

i [–]	$S_{i,av,sc}$ [–]
1	0.4232
2	0.4757
3	0.2262
4	1.8714

It may be noted that the significance of the fourth entry, corresponding to the value of the bending moment M is the highest, while the significance of the third input parameter, corresponding to Young's modulus of reinforcing steel E_s is the lowest.

The analysis of the significance of the input parameters demonstrates that the most important for the prediction of the value of deflection is the bending moment M and Young's modulus of concrete E_c. This is consistent with expectations – load change implies a change in the deflection while Young's modulus of concrete directly affects the beam stiffness. The lower significance of the Young's modulus of concrete in this case is caused by a small variety of input data – set of training data contained only three types of concrete, differing in the value of Young's modulus, but the differences were not exceeding 5 GPa.

The significance of the first entry is lower than the significance of the second and the fourth entry. This confirms that the area of reinforcement As has not as high impact on the beam stiffness as the area of concrete, for example, for a typical rectangular cross-section of a beam, the moment of inertia resulting from the reinforcement is approx. 10–20% of the total moment of inertia.

The lowest significance of the third entry confirms that the effect of Young's modulus of reinforcing steel E_s is small. This is due to the low diversity of this variable, which is in the range of 203 Gpa $\leq E_s \leq$ 207 GPa. In engineering calculations E_s is considered as a constant (E_s = 205 GPa).

7. CONCLUSIONS AND FUTURE RESEARCH

This paper presents the application of artificial neural network for making predictions of deflections of reinforced concrete beams at different load levels. The accuracy of the prediction of deflection is very high, over 70% of all analysed cases were determined without error or with the relative error ± 2.5% for the training set and ± 5% for the testing set. The results of prediction are in good agreement with real values obtained in the laboratory.

Correlation between calculated values of deflection (according to standard [5]) and real values obtained in the laboratory is not as high as in the case of prediction by using neural network. The relative error of deflection in over 80% of the cases exceeds ± 10%.

Compared to conventional digital computing techniques neural networks are advantageous because they can learn from example and generalize solutions to new renderings of a problem, can process information rapidly and can adapt to fine changes in the nature of a problem. Neural networks have, however, some disadvantages, especially, a lack of precision, no guarantee of success in finding an acceptable solution in each case, limited theory to assist in their design and a limited ability to rationalize the solutions provided. Moreover, the success of a neural network implementation is dependent on the quality of the data used for training. Despite their limitations, neural networks are a powerful tool that might be used for solving poorly defined problem (for example, the stiffness of cracked RC beam).

However, experimental studies, providing results for the database required for training and testing the neural network presented in this paper, were carried out on the limited material available for the test. Artificial neural network was applied only to three beams with the same section and loading condition (3-point bending configuration). The testing set of data and the training set of data were obtained only from the same three kinds of experimental tests, thus the artificial neural network, that was used in this paper, can predict only the deflections of RC beams with the same rectangular section and loaded with a concentrated force in the middle of the span (but with different types of reinforcement, concrete properties and the value of bending moment).

A first step of future research should be the analysis of more diverse RC beams with different geometrical dimensions and static scheme. Then input vector with a larger number of input neurons should be adapted in the structure of the artificial neural network. In order to get predictive results input vector $\mathbf{x}$ (13) should consist of the following elements: surface area of tension reinforcement A_s, the value of Young's modulus of reinforcing steel E_s, the value of Young's modulus of concrete E_c, the value of bending moment M in the cross-section, cross-sectional dimensions (for rectangular cross section: height h and width b), loading conditions and the static scheme *static*, length of span L and load duration t (short-term, long-term). Static scheme could be included by the susceptibility of supports of beam.

$$\mathbf{x} = \{A_s, E_c, E_s, M, b, h, static, L, t\}. \qquad (13)$$

The structure of the neural network that should be used to get predictive results is shown in Fig. 13.

The results presented in this paper confirm the possibility of application of unidirectional multilayer error back propagation neural networks in predicting the value of deflection of RC beams with the same rectangular section and loaded with a concentrated force in the middle of the span. It can be concluded that having a data set consisting of the material properties (Young's modulus of reinforcing steel and concrete), surface area of tension reinforcement, the value of bending moment in the cross-section, geometrical dimensions and static scheme enables training artificial neural network and then using it for the reliable prediction of the value of deflection.

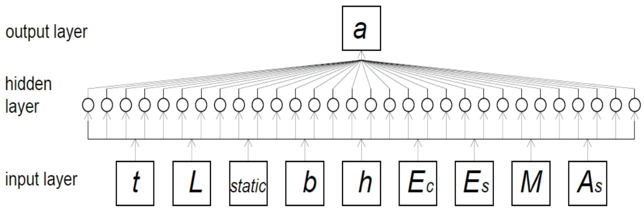

Fig. 13. Structure of the neural network that can be used in future research

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