COINTEGRATION METHOD FOR TEMPERATURE EFFECT REMOVAL IN DAMAGE DETECTION BASED ON LAMB WAVES

Phong Ba DAO

AGH University of Science and Technology, Faculty of Mechanical Engineering and Robotics, Department of Robotics and Mechatronics, Aleja Mickiewicza 30, 30-059 Krakow, Poland
E-mail: phongdao@agh.edu.pl

Summary

The paper presents an application of the cointegration technique for temperature effect removal in Lamb wave data. The method is based on the analysis of cointegration residuals and stationary statistical characteristics. The experimental results on Lamb wave responses from undamaged and damaged aluminium plates show that the cointegration process can remove undesired temperature effects and accurately detect damage.

Keywords: structural health monitoring, Lamb waves, cointegration, temperature effect removal

WYKRYWANIE USZKODZEŃ PRZY POMOCY FAL LAMBA - KOMPENSACJA WPŁYWU TEMPERATURY OPARTA O METODĘ KOINTEGRACJI

Streszczenie

Fale Lamba stosowane są do wykrywania uszkodzeń w konstrukcjach płytowych. Wpływ temperatury na amplitudę oraz fazę fal Lamba jest jedną z głównych przeszkód w powszechnym zastosowaniu tej metody w praktyce inżynierskiej. W pracy zastosowano metodę kointegracji do kompensacji wpływu temperatury na propagację fal Lamba. Metoda oparta jest na badaniu stacjonarności sygnałów. Zastosowane podejście pokazane jest na przykładzie wykrywania szczelin zmęczeniowych w płytach aluminiowych. Wyniki pokazują, że metoda kointegracji skutecznie usuwa z fal Lamba wszystkie efekty związane z wpływem temperatury, przyczyniając się do wykrycia badanych uszkodzeń.

Słowa kluczowe: wykrywanie uszkodzeń w konstrukcjach, fale Lamba, kointegracja, kompensacja wpływu temperatury

1. INTRODUCTION

Lamb waves are the most widely used guided ulstrasonic waves for structural health monitoring (SHM) applications. Various damage detection methods – based on Lamb waves – have been developed for the last few decades, as reported and reviewed in [1–5]. However, despite many research developments, practical engineering applications are still limited. This is not only due to complex physical wave propagation mechanisms (e.g. multiple modes or dispersive nature) but also due to damage sensitivity that is often affected by operational and environmental conditions [6]. It appears that temperature variability (instantaneous, daily or seasonal) is one of the major problems [7,8]. Therefore it is important for practical applications of Lamb waves to develop methods that are sensitive only to damage but insensitive to operational-environmental conditions, to avoid false-positive and false-negative damage detection scenarios.

Various approaches were developed to deal with undesired effects of operational and environmental variability in SHM data, as presented in [6]. The *cointegration* method – developed originally from the field of econometrics [9] – has been proposed recently in process engineering for abnormality detection [10] and in structural damage detection for the removal of temperature effect from bridge vibration data and Lamb wave responses [11, 12]. The major idea used in these investigations is based on the concept of stationarity of time series. Monitored variables are cointegrated to create a stationary residual whose stationarity represents normal condition. Then any departure from stationarity can indicate that monitored structures are no longer operating under normal condition.

This paper builds upon previous investigations on the cointegration technique for trend removal in Lamb wave data. The major objective is to present a new approach based on cointegration for the removal of temperature variability and damage detection based on Lamb waves. The augmented Dickey-Fuller (ADF) test [13] is used not only to test the degree of non-stationarity of variables, but also to create a damage detection indicator.

2. PREVIOUS STUDY ON TEMPERATURE EFFECT REMOVAL IN LAMB-WAVE-BASED SHM

Lamb-wave-based SHM in principle is based on guided ultrasonic waves introduced into a structure at one point and sensed at a different location. Damage in a structure can be then identified by monitoring changes in the output signal. Signal attenuation and/or mode conversion is sufficient to detect various types of damage such as cracks in metallics or delamination in composites [2]. Lamb-wave-based damage damage techniques have shown great potential for SHM applications since they feature [3]: (1) the ability to inspect large structures as well as insulated structures (e.g. pipeline under water); (2) the ability to examine the entire cross-sectional area of a structure including both internal damage and surface defects; (3) the capability of classifying various types of damage using different wave modes; (4) excellent sensitivity to damage with high precision of identification; and (5) low energy consumption and great cost-effectiveness.

However, as discussed above temperature variability has a dominant effect on Lamb waves therefore various approaches have been proposed to overcome this effect in which some representative works are summarised hereinafter.

The early proposed approaches are based on soft computing [14, 15]. The first publication reported a unique combination of time series analysis, neural networks, and statistical inference techniques. Several methods – such as feature extraction based on signal decomposition and principle component analysis – were investigated in the second publication. Various machine learning tools that could be used in these approaches were reviewed and extensively discussed in [16].

Two different methods were presented in [17]. The first one proposed to record reference signals over a range of temperatures and then to use these signals in the ensemble that best matches a subsequent signal for subtraction. The second method relies on the improvement of sensitivity via an exact compensation scheme for the temperature change. Both methods would require large reference databases in practical applications. This could be often expensive and not always possible. Efficient modeling could ease this task, as demonstrated in [18].

A reference-free approach – based on properties of fundamental Lamb wave mode propagation – has been proposed in [19]. Baseline subtraction methods – based on multiple baselines – have been proposed in [8]. Instead of using a single baseline for subtraction purposes, a series of baselines are used, covering the range of operating conditions of the structure. This approach is known as Optimal Baseline Selection (OBS). More recently, a new Baseline Signal Stretch (BSS) method have been proposed in [20]. A combined strategy that uses both OBS and BSS was also considered in this work.

A model that accounts for relevant temperature-dependent Lamb wave propagation parameters was investigated in [7]. This work underlines the effect of temperature on transducer properties. The role of piezoceramic adhesive layers in structures exposed to elevated temperatures was investigated.

3. COMMON TREND REMOVAL BY USING THE COINTEGRATION METHOD

For the sake of completeness this section briefly introduces the concept of cointegration. Firstly, stationarity and non-stationarity are explained.

For a given time series y_t presented in the form of the first-order Auto-Regressive process $AR(1)$, which is defined as

$$y_t = \phi y_{t-1} + \varepsilon_t \qquad (1)$$

where ε_t is an independent Gaussian white noise process with zero mean, three different time series can be distinguished. These are [21]:

- stationary time series ($|\phi| < 1$) – the process looks irregular, but always oscillates around the mean;
- non-stationary time series ($\phi > 1$) – the process is initially smooth but eventually becomes rough;
- random walk ($\phi = 1$) – the process moves up and down; it behaves as a non-stationary time series, but slowly.

In practice time-invariant behaviour can be indicated by statistical moments of the process. A stationary process would have time invariant moments while a non-stationary process would exhibit some time dependence in moments. It is well known that the most simple stationary time series is the independent Gaussian white noise process [21]. The time series y_t in Eq. (1) is an integrated series of order 1, denoted $I(1)$, if it has the form of random walk (i.e. $\phi = 1$) without a trend [22]. When $\phi = 1$, Eq. (1) yields

$$\Delta y_t = y_t - y_{t-1} = \varepsilon_t \qquad (2)$$

This clearly shows that, the first difference of y_t is just a white noise process ε_t, i.e. a stationary time series. The consequence is that a non-stationary $I(1)$ time series becomes a stationary $I(0)$ time series after the first difference. In a similar way, a non-stationary $I(2)$ time series would require differencing twice to induce a stationary $I(0)$ time series. The number of differences required to achieve stationarity is called the order of integration. Thus time series of order d are denoted as $I(d)$.

Following this short introduction, the concept of cointegration can be introduced. Let

$\mathrm{Y}_t = (y_{1t}, y_{2t}, \ldots, y_{nt})^T$ denote an $(n \times 1)$ vector of $I(1)$ time series. Then Y_t is cointegrated if there exists an $(n \times 1)$ vector $\beta = (1, -\beta_2, \ldots, -\beta_n)^T$ such that

$$\underbrace{\boldsymbol{\beta}^T \mathbf{Y}_t}_{\text{Cointegration residual}} = \underbrace{\begin{bmatrix} 1 & -\boldsymbol{\beta}_2 & \ldots & -\boldsymbol{\beta}_n \end{bmatrix}}_{\text{normalized cointegrating vector}} \underbrace{\begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{bmatrix}}_{\text{cointegrated variables}} \sim I(0)$$

In other words, the non-stationary $I(1)$ time series in Y_t are cointegrated if there exists (at least) a linear combination of them that is stationary or has the $I(0)$ status. The linear combination, denoted $\beta^T \mathrm{Y}_t \sim I(0)$, is referred to as a *cointegration residual* or a long-run equilibrium relationship between time series [22]. The β vector is called a *normalized cointegrating vector*. It can be noticed that the action of creating the cointegration residual ($u_t = \beta^T \mathrm{Y}_t$) is the action of projecting the $(n \times 1)$ Y_t vector on the cointegrating vector β, or in other words applying the cointegrating vector β to the $(n \times 1)$ Y_t vector.

Testing for cointegration is in essence testing for the existence of long-run equilibriums (or stationary linear combinations) among the elements of Y_t [22]. However, before any attempt is made to test for that existence, two constraints (or pre-requirements) related the time series in Y_t need to be fulfilled. Firstly, the analysed time series must *have at least a common trend*. Secondly, the analysed time series must *have the same degree of non-stationarity*, i.e. they must be integrated of the same order.

If the $(n \times 1)$ vector of $I(1)$ time series in Y_t is cointegrated with $0 < r < n$ cointegrating vectors then there are $n - r$ common stochastic trends [22]. For n variables, the cointegration process may create as many as $n - 1$ linearly independent cointegrating vectors. To illustrate the duality between cointegration and common trends, let

$$\mathrm{Y}_t = (y_{1t}, y_{2t})^T \sim I(1) \quad \text{and}$$
$$\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})^T \sim I(0) \tag{3}$$

and suppose that Y_t is cointegrated with the normalized cointegrating vector $\beta = (1, -\beta_2)^T$. This cointegration relationship may be represented as

$$y_{1t} = \beta_2 \sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{3t} \quad \text{and}$$
$$y_{2t} = \sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{2t} \tag{4}$$

where the common trend is $\sum_{s=1}^{t} \varepsilon_{1s}$. As illustrated below, the cointegration process removes the common stochastic trend and the cointegration residual $\beta^T \mathrm{Y}_t$ obtained is a stationary $I(0)$ time series, i.e.

$$\begin{aligned} \beta^T \mathrm{Y}_t &= y_{1t} - \beta_2 y_{2t} \\ &= \beta_2 \sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{3t} - \beta_2 \left(\sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{2t} \right) \\ &= \varepsilon_{3t} - \beta_2 \varepsilon_{2t} \sim I(0) \end{aligned} \tag{5}$$

One of the most common approaches used to determine the existence of cointegration and the number of linearly independent cointegrating vectors (or relationships) among multivariate non-stationary time series in Y_t were developed in [23]. This so-called *Johansen's cointegration procedure* basically is a combination of cointegration and error correction models in a Vector Error Correction Model (VECM) form. This is a sophisticated sequential procedure and thus is not presented in this paper. For a complete description of the Johansen procedure, readers are referred to [23]. In this study, the Johansen's cointegration procedure was realized by using the Econometrics Toolbox [24].

Next, an example that uses the Johansen's cointegration procedure to remove common trends in the simulated data is presented. The Weierstrass-Mandelbrot (W-M) cosine fractal function is used in the analysis because of its non-linear and non-stationary nature. The W-M cosine fractal function is given by [25]

$$W_i = \sum_{j=-N}^{N} \frac{\left(1 - \cos B^j \dfrac{i}{N_p} \right)}{B^{(2-D)j}} \tag{6}$$

where $i = 1 \ldots N_p$, with N_p is the total number of data samples; and the parameter D and B must be in the range $1 < D < 2$ and $B > 1$, respectively.

In this example, three W-M cosine fractal functions (e.g. variables x, y, z) are used (see Fig. 1). Clearly, they share one common stochastic trend, i.e. the positive drift (or upward trending). When these three variables – sharing a common trend – are cointegrated, the cointegration process results in two cointegrating vectors (i.e. $r = 2$), in which
The first cointegrating vector is:

$$\beta_1 = \begin{bmatrix} 1 & -1.62 & 1.99 & 0.16 \end{bmatrix}^T \tag{7}$$

The second cointegrating vector is:

$$\beta_2 = \begin{bmatrix} 1 & 0.12 & -1.21 & 2.03 \end{bmatrix}^T \quad (8)$$

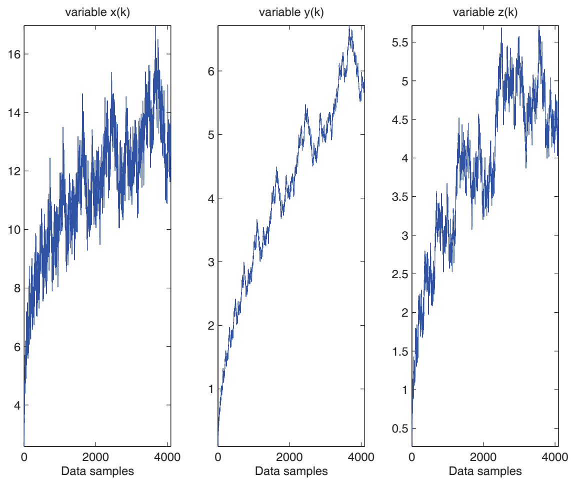

Fig. 1. Three W-M cosine fractal functions (x, y, z).

Next, projecting three variables (x, y, z) on the two obtained cointegrating vectors that results in two cointegration residuals in Fig. 2 and Fig. 3. It is easy to observe that the cointegration residual in Fig. 2 – which uses the first cointegrating vector is more stationary than the one in Fig. 3 – which uses the second cointegrating vector. The results obtained in Fig. 2 and Fig. 3 also showed that the positive drift trending was removed from the W-M variables.

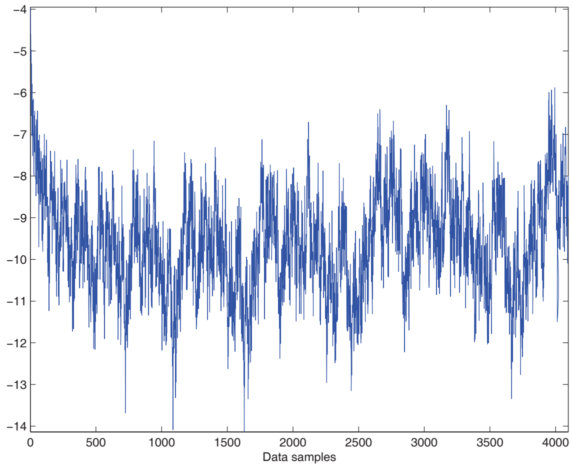

Fig. 2. The first cointegration residual obtained when three W-M variables are projected on the first cointegrating vector.

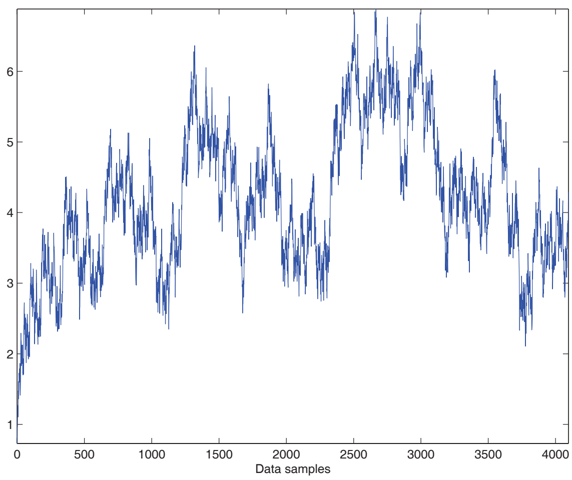

Fig. 3. The second cointegration residual obtained when three W-M variables are projected on the second cointegrating vector.

4. LAMB WAVE EXPERIMENTAL DATA WITH TEMPERATURE TRENDS

Experimental data used in this paper originate from a series of tests described in [15]. Lamb waves were propagated in an aluminium plate (200 x 150 x 2 mm) aluminum plate of 2 mm thickness was used in these experiments. Two low-profile, surface-bonded piezoceramic *Sonox P155* transducers (diameter 10 mm and thickness 1 mm) were used for Lamb wave generation and sensing.

A five-cycle 75 kHz cosine burst signal was used for Lamb wave generation. The excitation signal was enveloped using a half-cosine envelope. The maximum peak-to-peak amplitude of the excitation signal was equal to 10 V. This excitation led to the so-called single Lamb wave mode propagation, i.e. the A0 fundamental Lamb wave mode was generated whereas the amplitude of the S0 mode was negligible. The excitation signal was generated using the *TTi TGA 1230* arbitrary waveform generator. Lamb wave responses were acquired using a digital 4-channel *LeCroy LT264 Waverunner* oscilloscope. Both instruments, i.e. the signal generator and the oscilloscope, were controlled in MATLAB through the General Purpose Control Bus (GPIB) protocol standard, running on a standard PC. Fig. 4. shows a schematic diagram of the experimental arrangement.

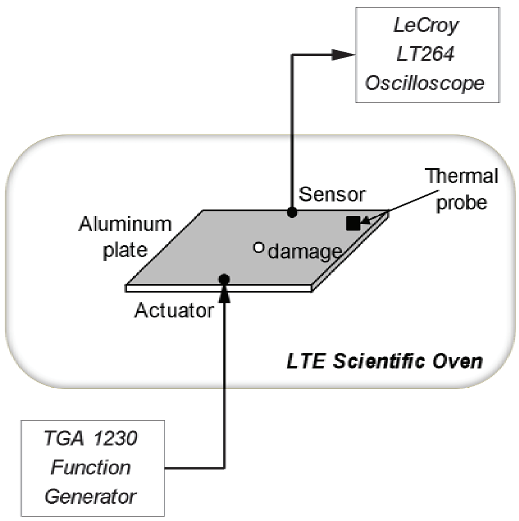

Fig. 4. Experimental arrangement.

The plate was placed in a 100 liter *LTE Scientific* oven to obtain data for various temperature levels. The temperature on the surface of the plate was measured using a thermal probe. Firstly, the experimental tests were performed using the intact plate. The plate was firstly heated up (from 35^{0}C to 70^{0}C) and then cooled down (from 70^{0}C to 35^{0}C) with a step change of 5^{0}C. The heating and cooling cycles were performed twice to address the problem of repeatability and check for possible hysteresis between cycles. Then, a hole was drilled the middle of the plate and the entire experimental work was repeated.

The analysis presented in this paper utilised lamb wave data for the *undamaged plate* and the *damaged plate with 5 mm hole* – measured at four different temperatures (i.e. *35, 45, 60 and 70^{0}C*). Altogether twenty Lamb wave responses were used for each single combined damage-temperature condition and each response measurement consists of 5000 data samples acquired using the sampling rate of 10 MHz. Strong influence of temperature on lamb wave responses was observed, as reported previously in [15, 18].

5. EXPERIMENTAL RESULTS

This section presents the temperature effect removal and damage detection results based on the proposed cointegration-based method. Lamb wave experimental data described in the previous section were used in this analysis.

The cointegration analysis and the ADF test were performed using the Lamb wave experimental data. The entire procedure consists of three schemes:

(1) ADF test on the “pre-cointegrated data”;

(2) Cointegration of Lamb wave responses;

(3) ADF test on the “post-cointegrated data”.

It is well known that whenever the cointegration method is used, the ADF test is firstly performed to measure the degree of stationarity or non-stationarity of the analysed variables. In principle, *the more negative the ADF t-statistic value is obtained, the more stationary the signal is*, as shown in [11,12]. The assumption is that damage introduced to the plate can change stationarity of Lamb wave data. In addition, different severities of damage can produce Lamb wave responses with different stationary characteristics. If this is the case, then not only the existence of damage can be detected, but also its severity can be assessed. Hence, the ADF test can be used not only to test for stationarity but also to detect damage and judge its severity. Following [12], the average ADF t-statistics are used in this study to assess the degree of stationarity of the analysed data and to detect possible damage.

<u>In the first scheme</u>, twenty Lamb wave responses from each single combined damage-temperature condition were directly used in the ADF test. This analysis resulted in ADF t-statistic features, in which each feature consists of twenty ADF t-statistic values that correspond to twenty Lamb wave responses used in the ADF test. The results in Fig. 5 are presented for the undamaged plate and the damaged plate with 5 mm hole at four different temperatures investigated. All statistics vary abruptly and do not exhibit any immediate patterns. Lamb wave responses are corrupted by temperature and the relevant average ADF t-statistics are also influenced by this effect. The average ADF t-statistics for the undamaged plate and the damaged plate are relatively separated when the temperature is equal to 35^{0}C. However, once the temperature increases these statistics are overlapped.

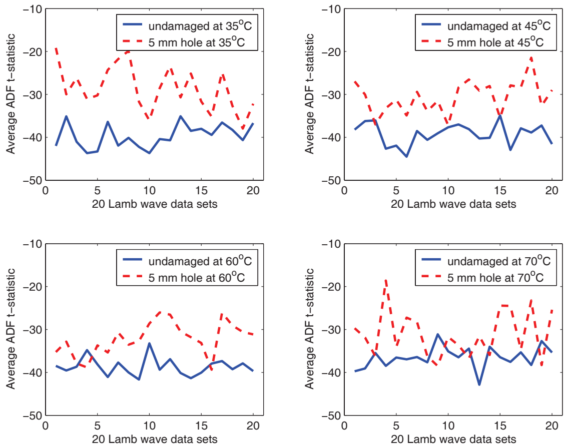

Fig. 5. ADF test results for the pre-cointegrated Lamb wave data.

<u>In the second scheme</u>, twenty Lamb wave responses from each single combined damage-temperature condition were cointegrated by using the Johansen’s cointegration procedure. For the data used the cointegration process creates as many as nineteen linearly independent cointegrating vectors, which subsequently can be used to produce *nineteen cointegrating residuals* (i.e. post-cointegrated Lamb wave responses) after the data projection process.

<u>In the third scheme</u>, nineteen cointegrating residuals obtained from the second scheme were used in the ADF test (or in other words, the ADF test was performed on the post-cointegrated Lamb wave responses. This analysis resulted in *ADF t-statistic features*, in which each feature consists of nineteen ADF t-statistic values that correspond to nineteen cointegrating residuals obtained from the cointegration process.

The results in Fig. 6 for different temperatures investigated show that the average ADF t-statistics for the undamaged plate and the damaged plate are very well separated. Interestingly, these statistics display large negative values (i.e. smaller than -25) for the first cointegrating residual; whereas, the relevant statistics for the remaining residuals are relatively stable and remain between -10 and -5. In this case, the 1st cointegrating vector created the most stationary cointegrating residual. The statistics for the damaged plate with the 5 mm hole increase monotonically from the 2nd to the 19th residual. It is important to emphasize that these statistics are quite similar for all temperatures investigated. This is due to the fact that the temperature effect was purged from the Lamb wave data by the cointegration process. Therefore, the ADF test was applied effectively to the cointegrating residuals that were free from temperature variations. This is why these statistics are relatively stable if compared with the relevant statistics in Fig. 5.

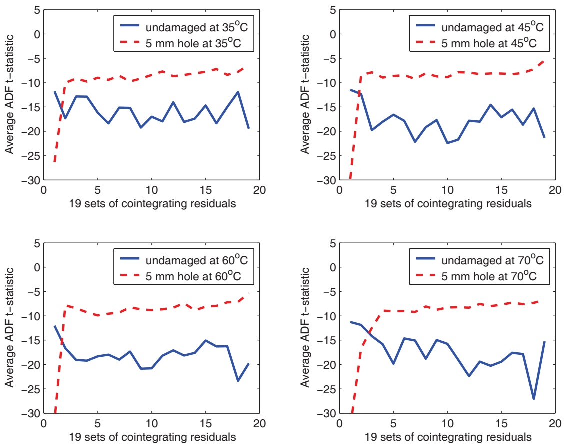

Fig. 6. ADF test results for the post-cointegrated Lamb wave data.

6. CONCLUSIONS

By applying the cointegration analysis, the varying temperature effect is successfully removed from Lamb wave responses and therefore the cointegrating residuals obtained are free from temperature variations.

When the ADF test is applied to the temperature-effect-purged cointegrating residuals, the damaged plate with 5 mm hole can be easily detected.

Although, the results obtained are interesting, further research work is required to confirm these findings. This work should focus on: more complex structures, real damages (e.g. fatigue cracks in metals or delaminations in composites) and different damage detection strategies (wave propagation paths, transducer schemes, etc.).

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Phong Ba DAO received an M. Sc. degree in electrical engineering from Hanoi University of Science and Technology, Hanoi, Vietnam, and a Ph.D. degree in control engineering from the University of Twente, Enschede, The Netherlands in 2003, and 2011, respectively. Since July 2011 he has been an Assistant Professor at the Department of Robotics and Mechatronics, AGH University of Science and Technology in Krakow, Poland. His current research interests are structural health monitoring and damage detection, multi-agent control systems, intelligent control, and mechatronics.