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USING ELECTRICAL IMPEDANCE TOMOGRAPHY IN LINEAR ARRAYS OF MEASUREMENT

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Abstract. *The article presents an application to the topology optimization in electrical impedance tomography using the level set method. The level set function is based on shape and topology optimization for areas with partly continuous conductivities. The finite element method has been used to solve the forward problem. The proposed algorithm is initialized using topological sensitivity analysis. Shape derivative and material derivative have been incorporated with the level set method to investigate shape optimization problems. The coupled algorithm is a relatively new procedure to overcome this problem. Using the line measurement model is very useful to solve the inverse problem in the copper-mine ceiling and the flood embankment.*

Keywords: inverse problem, level set method, electrical impedance tomography

ZASTOSOWANIE ELEKTRYCZNEJ TOMOGRAFII IMPEDANCYJNEJ W UKŁADACH POMIARU LINIOWEGO

Streszczenie. *W artykule przedstawiono aplikację do optymalizacji topologicznej w elektrycznej tomografii impedancyjnej przy użyciu metody zbiorów poziomicowych. Funkcja poziomicowa oparta jest optymalizacji topologii i kształtu dla obszarów z częściowo ciągłymi konduktywnościami. Metoda elementów skończonych została wykorzystana do rozwiązania tego problemu. Proponowany algorytm jest inicjalizowany przy użyciu topologicznej analizy wrażliwościowej. Pochodna kształtu i pochodna topologiczna zostały zaimplementowane z metodą zbiorów poziomicowych do rozwiązania problemu optymalizacji. Sprzężony algorytm jest stosunkowo nową procedurą do rozwiązania tego zadania. Zastosowanie modelu pomiaru tablicowego jest bardzo użyteczne w celu rozwiązania problemu odwrotnego m.in. w chodniku kopalni miedzi i wałach przeciwpowodziowych*.

Słowa kluczowe: zagadnienie odwrotne, metoda zbiorów poziomicowych, tomografia impedancyjna

Introduction

Numerical methods of the shape and the topology optimization were based on the level set representation and the shape differentiation [1, 2, 13, 14]. Possible changes made during the topology optimization process. Level set methods have been applied very successfully in many areas of the scientific modelling [3-5, 10]. This approach is based on the sensitivity of the shape and includes a form of the edge of interface. There are two features that make these methods are suitable for optimization of the topology. The structure is represented by an implicit function. The zero level set defines the boundary of the object. This function is often discretized on a regular grid with the finite element mesh. The next valid feature is the simple update of the implicit function using the Hamilton-Jacobi equation, where the velocity function is determined by the shape sensitivity of the structure. The discussed technique can be applied to the solution of inverse problems in the electrical tomography [6-9, 11, 12]. There were implemented the new algorithms to identify unknown conductivities. The purpose of the presented method is obtaining the better image reconstruction than gradient methods. Next advantage there is accelerating the iterative process by using different shapes of the zero-level set function.

1. Level Set Method

In the level set method, $\phi(x,t)$ is always a function. Numerical simulations can be carried out with the use of discrete grid points in x and substitute a finite approximation of differential for spatial derivatives. The derivative of the spatial approximation of the differential time it is possible by a suitable finite differential operator. Iterative process starts by defining a zero level set function ϕ (time dependent). The function $\phi(x,t)$ is assigned to the area, where the test object is belonged (x - point of space, t - time). Initialization the level set function ϕ starts at t = 0, and the solution can be approximated by the function value in small increments of time (Fig. 1).

The function $\phi(x,t)=0$ (where x is a point in the area R^n) is defined as follows:

$$\phi(x,t)=\pm \mathrm{d} \tag{1}$$

where: d – the distance between point x, and the edge of the closed surface Ω(t=0), sign +/- represents the location of the point x, respectively inside or outside the area.

The state in which the zero-level set always to this area are:

$$\phi(x(t),t)=0 \tag{2}$$

Calculating the derivative of this function by the time:

$$\phi' + v|\nabla\phi| = 0 \tag{3}$$

where: v – is a function of the velocity vector Γ.

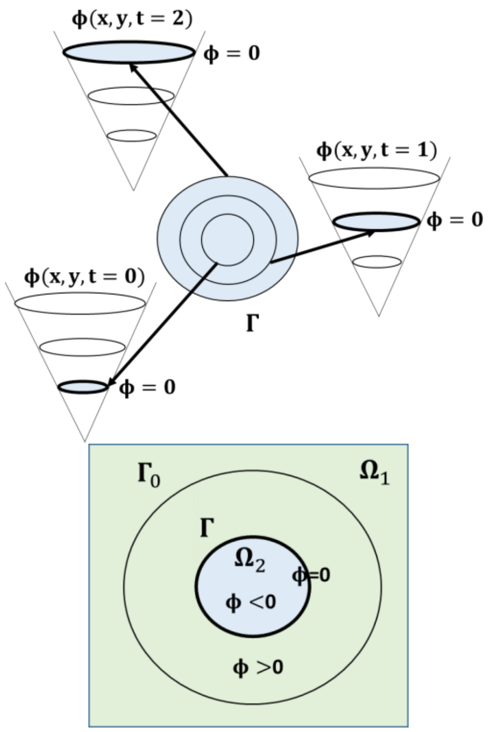


Fig. 1. The idea of the level set function

Properties of the edge of tested area are depend on the level set function ϕ, where at each point of the edge normal vector to this point is given by:

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} \tag{4}$$

The curvature in two dimensions on a plane is described by the equation:

$$\kappa = \nabla\cdot\frac{\nabla\phi}{|\nabla\phi|} = \frac{\phi_{zx}\phi_y^2 - 2\phi_y\phi_z\phi_{xy} + \phi_{xy}\phi_x^2}{\left(\phi_x^2+\phi_y^2\right)^{3/2}} \tag{5}$$

If the surface is growing in three dimensions, the curvature can be described as a mean value of curvature κ_M, or as a Gaussian curvature κ_G. Under the terms of the level set function, these two values are given appropriate expression:

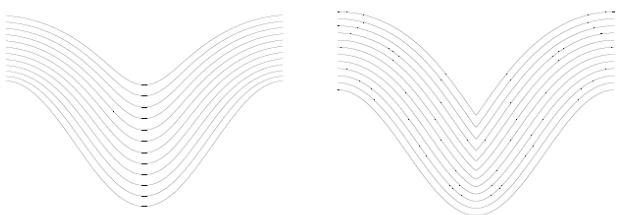
$$\kappa_M = \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} = \frac{\left\{ \begin{matrix} (\phi_{yy}+\phi_{zz})\phi_x^2+(\phi_{xx}+\phi_{zz})\phi_y^2+(\phi_{xx}+\phi_{yy})\phi_z^2+ \\ -2\phi_x\phi_x\phi_{xy}-2\phi_x\phi_z\phi_{xz}-2\phi_y\phi_z\phi_{yz} \end{matrix} \right\}}{(\phi_x^2+\phi_y^2+\phi_z^2)^{3/2}} \quad (6)$$

$$\kappa_G = \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} = \frac{\left\{ \begin{matrix} \phi_z^2(\phi_{yy}\phi_{zz}-\phi_{yz}^2)+\phi_y^2(\phi_{xx}\phi_{zz}-\phi_{xz}^2)+ \\ \phi_z^2(\phi_{xx}\phi_{yy}-\phi_{xy}^2)+ \\ -2\begin{bmatrix} \phi_x\phi_y(\phi_{xz}\phi_{yz}-\phi_{xy}\phi_{zz})+ \\ \phi_y\phi_z(\phi_{xy}\phi_{xz}-\phi_{yz}\phi_{xx})+ \\ \phi_x\phi_z(\phi_{xy}\phi_{yz}-\phi_{xz}\phi_{yy}) \end{bmatrix} \end{matrix} \right\}}{(\phi_x^2+\phi_y^2+\phi_z^2)^{3/2}} \quad (7)$$

The curvature of each level set is determined by calculating the divergence of the vector (4):

$$\kappa = \nabla \cdot \vec{n} = \nabla \frac{\nabla\phi}{|\nabla\phi|} \quad (8)$$

When function creates sharp edges, or a sudden collapse, the approximating function cannot be estimated by partial derivative. In such cases, the methods for smoothing sharp edges are used. Level set methods are derived from the mathematical theory of " viscosity solution ". The smoothing function is done by adding to the equation " viscous term" ε. Figure 2 shows the effect of the regularization coefficient on the process function.



ε = 0.005 ε = 0

Fig. 2. The impact factor ε limiting viscosity on the process function

Extraction of the border viscosity is to examine the behavior of potential solutions during their search. The equation of motion in such case is described in the following:

$$\Psi_t = v(1+\Psi_x^2)^{\frac{1}{2}} \quad (9)$$

where Ψ_t is the value of the propagation function of time t, therefore $\Psi(x,0) = f(x)$.

For such a waveform function, an initial shape for the calculation is defined as "V" (Fig. 3).

$$\Psi(x,0) = f(x) = \begin{cases} \frac{1}{2} - x & x \le \frac{1}{2} \\ x - \frac{1}{2} & x > \frac{1}{2} \end{cases} \quad (10)$$

Approximation of the above tasks is based on the central calculating of the difference equation:

$$\Psi_t = \frac{\Psi_i^{n+1}-\Psi_i^n}{\Delta t} = \left[1 + \left[\frac{\Psi_{i+1}^n - \Psi_{i-1}^n}{2\Delta x}\right]^2\right]^{\frac{1}{2}} \quad (11)$$

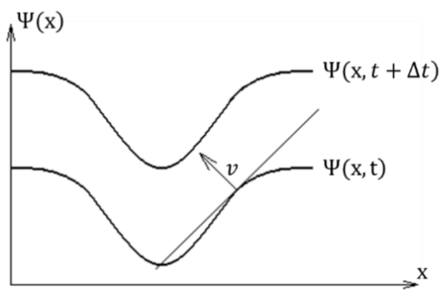


Fig. 3. Variables for the plot propagation

For all $x \neq 1/2$ there is a solution to one value Ψ_i. However, accurate determination of the value $x = 1/2$ (in which the edge shape is not defined) is not possible. Taking the difference reveals unexpected occurrence of oscillations around the point x = 1/2, depending on the time step (Fig. 4).

Proper solution $\Delta t = 0.005$ $\Delta t = 0.0005$

Fig. 4. Approximation of the central difference method according to the Δt

Focusing on the gradient component: $(1+\Psi_x^2)$ - it can be approximated as follows:

$$\Psi_x^2 \approx [\max(D_i^{+x}\Psi, 0)^2 + \min(D_i^{-x}\Psi, 0)^2] \quad (12)$$

where

$$D_i^{-x}\Psi = \frac{\Psi_i - \Psi_{i-1}}{h}$$

$$D_i^{+x}\Psi = \frac{\Psi_{i+1} - \Psi_i}{h}$$

Ψ_i – value of the function Ψ at the point with step h.

The assumptions can be applied to the level set function where the formula for the initial level set method in three-dimensional space becomes:

$$\phi_{ij}^{n+1} = \phi_{ij}^n - \Delta t[(\max(v_{ij},0))\nabla^+ + (\min(v_{ij},0))\nabla^-] \quad (13)$$

where:

$$\nabla^+ = \Big[(\max(D_{ij}^{-x}\phi_{ij}^n,0))^2 + (\min(D_{ij}^{+x}\phi_{ij}^n,0))^2 + (\max(D_{ij}^{-y}\phi_{ij}^n,0))^2 + (\min(D_{ij}^{+y}\phi_{ij}^n,0))^2\Big]$$

$$\nabla^- = \Big[(\max(D_{ij}^{+x}\phi_{ij}^n,0))^2 + (\min(D_{ij}^{-x}\phi_{ij}^n,0))^2 + (\max(D_{ij}^{+y}\phi_{ij}^n,0))^2 + (\min(D_{ij}^{-y}\phi_{ij}^n,0))^2\Big]$$

This is the equation of partial derivatives and it means that the velocity v is defined for all sets of contour lines, not just for a set of zero corresponds to the interface. The consequence of this is the need to upgrade the level set function at each grid point, using the velocity v.

2. Electrical Impedance Tomography

The electrical impedance tomography (EIT) is a non-destructive imaging technique which has various applications. Its purpose is to reconstruct the conductivity of hidden objects inside a medium with the help of boundary field measurements. Efficient algorithms for solving forward and inverse problem have to be developed in order to use this approach for practical tasks. Moreover, it is necessity to improve performance of selected numerical methods. Typical problem in EIT requires the identification of the unknown internal area from near-boundary measurements of the electrical potential. It is assumed that the value of the conductivity is known in subregions whose boundaries are unknown. The forward problem in EIT is described by Laplace's equation:

$$\nabla \cdot (\gamma\nabla u) = 0 \quad (14)$$

where γ denotes conductivity. Symbol u represents electrical potential. Function u is taken under Dirichlet condition in boundary points adjacent to electrodes and Neumann condition on remaining part of the boundary. Problem can be reduced to determination of the minimum value of the functional:

$$I[u] = \frac{1}{2}\int_\Omega \gamma|\nabla u|^2 dxdy \quad (15)$$

For the minimization problem iterative coupling of the level set method and the finite element method has been proposed.

3. Numerical results

In examples, two numerical models with different discretization elements are presented. Additionally, it is shown different geometries of the boundary shape. The conductivity of searched objects is known. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method. In forward problem, which is given by Laplace's equation, there was used the finite element method.

Figures 5 and 6 display: square measurement system with the inserted object, the image reconstruction with the original objects, the zero level set function, the process reconstruction and the image reconstruction. Figures 7 and 8 show similar object with the one side of measurements.

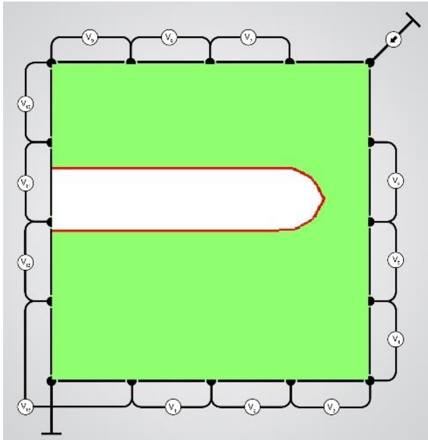


Fig. 5. The square measurement system with the inserted object

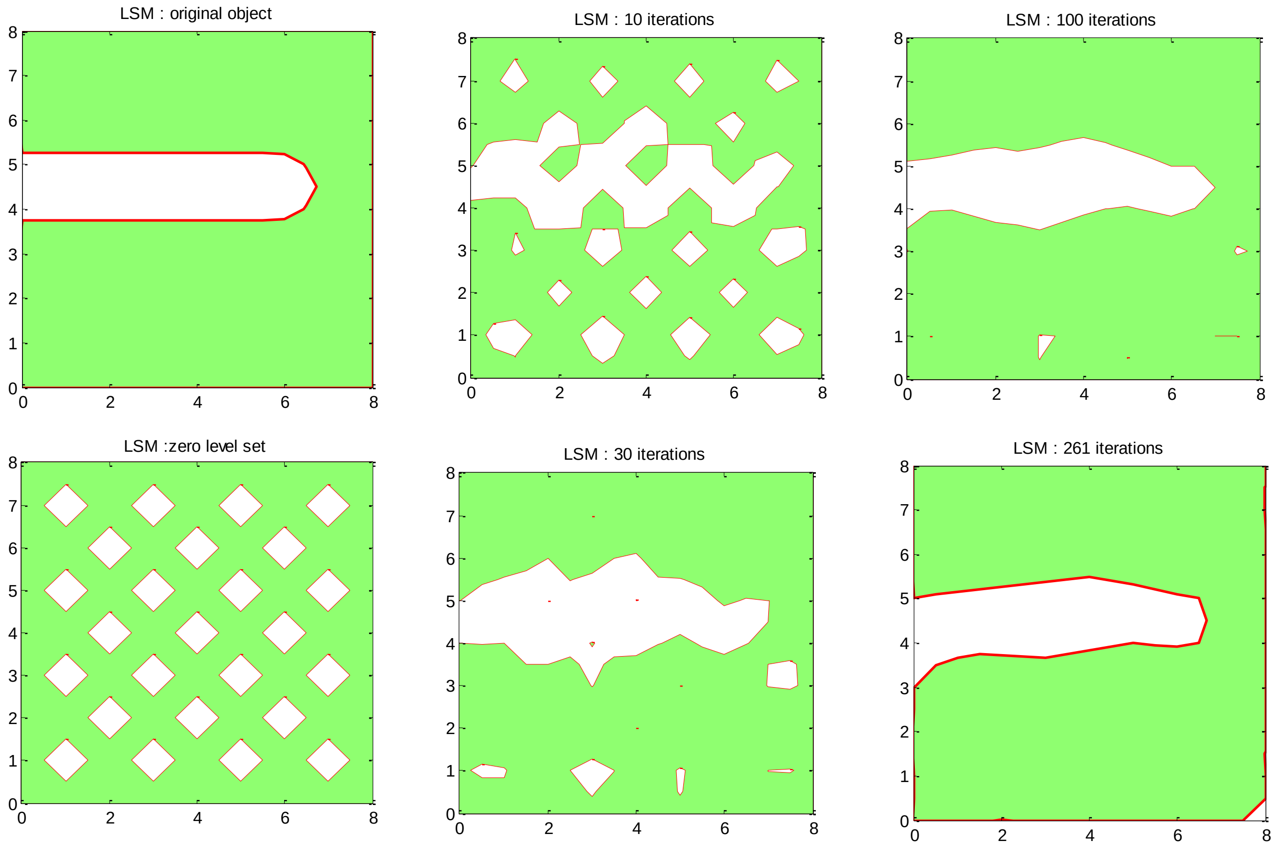


Fig. 6. The image reconstruction - the original object, the zero level set function, the process reconstruction and the image reconstruction

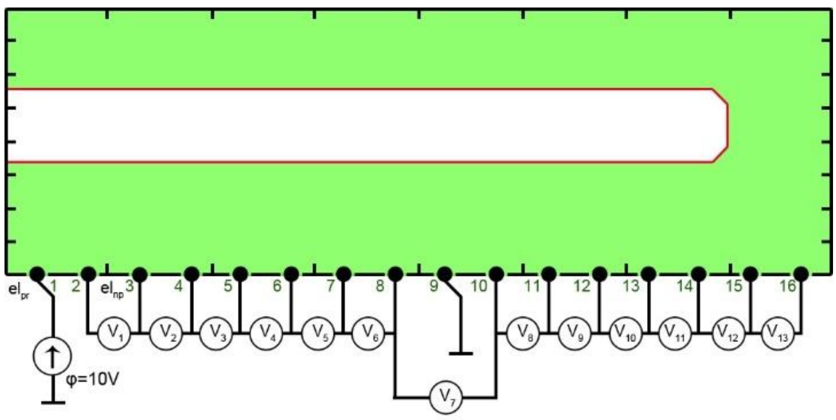


Fig. 7. The rectangular measurement system with the "line" object

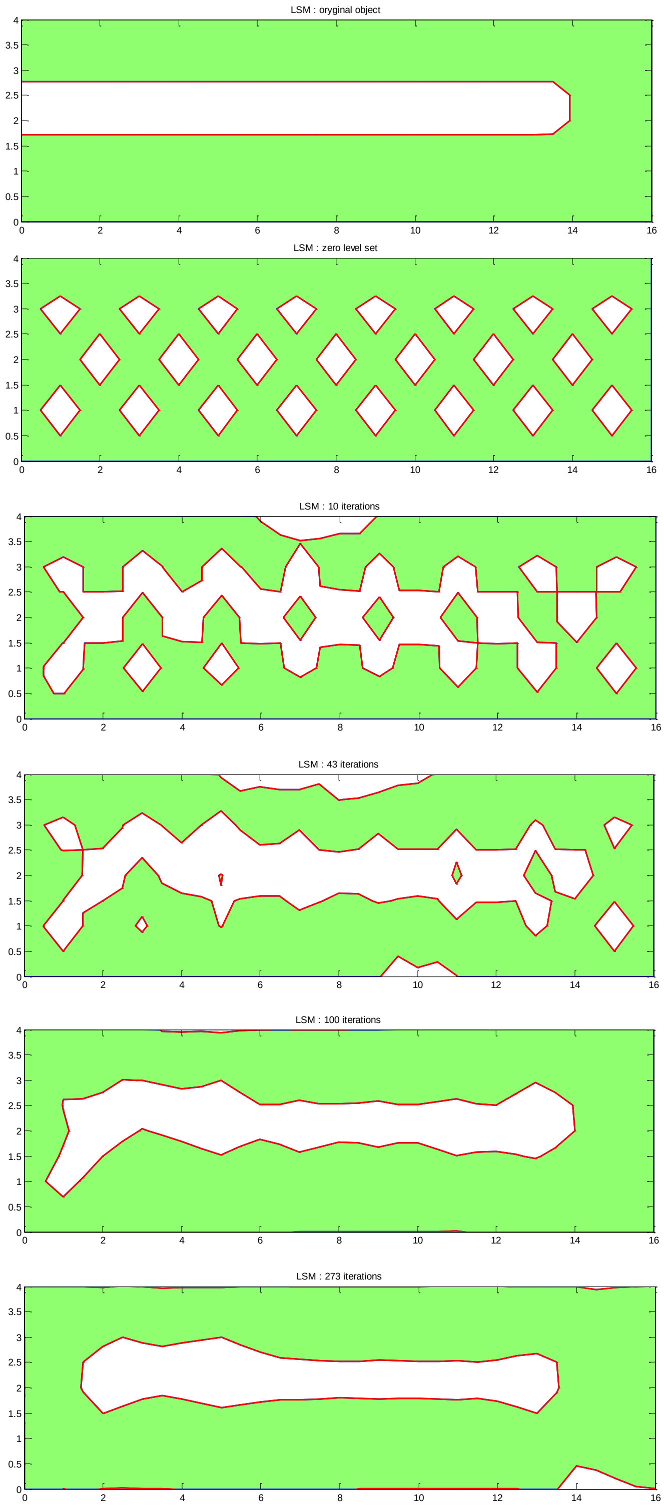


Fig. 8. The image reconstruction with the one side of measurements - the original objects, the zero level set function, the process reconstruction and the image reconstruction

4. Conclusion

The algorithm based on the level set method have been proposed in this work. It is iterative algorithm where repeatedly the shape boundary evolves smoothly and the object is detected. An efficient algorithm for solving the forward and inverse problems would also improve a lot of the numerical performances of the proposed method. In model problem from EIT is required to identify unknown conductivities from near-boundary measurements of the potential. The number of iterations determine the position and shape of zero level set functions. In this algorithm, there can control the process of the image reconstruction. The level set function techniques have been shown to be successful to identify the unknown boundary shapes. The accuracy of the image reconstruction is better than gradient methods. Applying the line measurement model is very effective to solve the inverse problem in the copper-mine ceiling and flood embankment. The right selection of the zero level set function gives the better results, reduces the time of reconstruction process and improves the quality of the image reconstruction.

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