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AN INSTALLATION ANGLE ERROR CALIBRATION METHOD IN AN ULTRA-SHORT BASELINE SYSTEM BASED ON A DUAL TRANSPONDER

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ABSTRACT

The installation error of an acoustic transceiver array is one of the important error sources in an ultra-short baseline (USBL) system. In a USBL system with a positioning accuracy of 0.5%, an installation error angle of 1° will lead to a positioning error of 1.7% times the slant distance. In this paper, a dual transponder-based installation angle error calibration method for USBL is proposed. First, the positioning errors induced by various installation angles are deduced and analysed using the linear measurement of seafloor targets. Then, an iterative algorithm is proposed that estimates the rolling alignment error, pitching alignment error, and heading alignment error, in that order. The simulation and experienced results show that, after three iterations, the estimates of the three alignment errors can converge quickly, all of the estimates converge to within 0.001° and the estimated values are very close to the true values. The horizontal positioning error caused by the installation error angle can be reduced by nearly 75%. The method has good effectiveness and robustness, and can greatly improve the positioning accuracy of the USBL system.

Keywords: USBL, installation angle error, misalignment calibration, numerical algorithm, hydrolocation

INTRODUCTION

The benefits of an ultra-short baseline positioning system (USBL) are a small array size, easy installation, adaptability, and low cost. As a result, it has significant social value and practical potential and can be instrumental in modern ocean mapping, resource exploration, the development of marine national security, and the detection of undersea targets (1). The installation error of an acoustic transceiver array is one of the important error sources in USBL systems (1). The installation error can be divided into coordinate offset error and angle rotation error, and the angle rotation error has a great influence on the positioning accuracy of USBL. Relevant literature shows that, in a USBL system with a positioning accuracy of 0.5%, an installation error angle of 1° will lead to a positioning error of 1.7% times the slant distance, so it is important to accurately calibrate the installation angle before using the USBL system (2). Moreira et al. (2007) (3) first proposed the need to calibrate the error angle before USBL positioning. Yu (2010) (4) applied GPS-RTK (Global Positioning System real-time Kinematic) to the angle error calibration and used the least square method to calculate the installation error angle, but this method had special requirements for the selection of sampling points. Li et al. (2013) (5) verified the feasibility of the least square method to experimentally estimate the error angle of USBL installations in the sea, and the accuracy of the positioning system after error compensation could reach 5‰ of the slant distance. Morgado et al. (2013) (6) pointed out that the installation error angle of the USBL system can be estimated by the multi-circle detour method with different navigation radii. However, this method requires the centre of the mother ship's orbit trajectory to be located above the underwater transponder, which is difficult to be directly applied in practical engineering. Chen (2008) (7) studied the USBL installation error calibration method for ships sailing along a straight line and (8) verified the feasibility of the straight track method through theory and experiment. Jinwu et al. (2018) [(10)] proposed a fast calibration algorithm for USBL installation error angle based on a laid out multi-transponder but this algorithm has strict requirements on the placement of transponders, limiting its application.

It is often difficult for ships to perform complex manoeuvres, and so they typically follow circular or straight paths in order to collect USBL, GPS, or other gyro compass observation data. Both the circular path and the straight path are simple manoeuvres but the disadvantage of the circular path scheme is that the rotational acceleration will reduce the performance of the gyro compass and motion sensor, and it needs to use different radii to circumnavigate many times, which is complicated. As a result, this paper suggests a dual transponder-based installation angle error calibration method for the USBL system. The positioning errors caused by various installation angles are first deduced and analysed. Based on this, an iterative approach is proposed, correcting the positioning errors of each angle sequentially by first estimating the rolling alignment error, then the pitching alignment error, and, finally, the heading alignment error.

PRINCIPLE AND ERROR ANALYSIS OF USBL POSITIONING COORDINATE SYSTEMS

The schematic of a ship navigating along a straight line path to locate an underwater transponder is shown in Fig. 1, where d represents the horizontal distance between the transponder and the ship track, θ represents the ship heading, (d, l) represents the position of O_s relative to the origin O_a, and the depths of transponder 1 and 2 are h_1 and h_2, respectively.

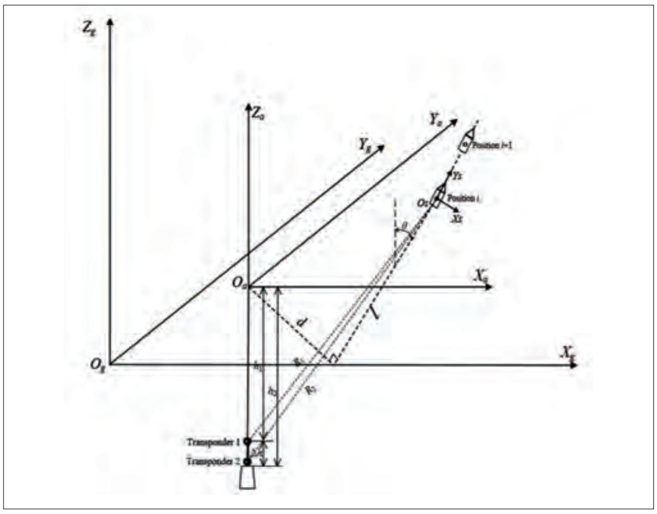

Fig. 1. The geometry of a straight path survey for positioning an underwater transponder

This paper employs several coordinate systems. The global coordinate system $O_gX_gY_gZ_g$ can obtain the absolute coordinate position of the object and $O_sX_sY_sZ_s$ is the base coordinate system, including the gyro compass and motion sensor. The auxiliary coordinate system $O_aX_aY_aZ_a$ is defined to simplify the derivation and its origin is located on the sea surface, above the transponder. The global coordinate system $O_gX_gY_gZ_g$ is a common earth-fixed coordinate system, e.g. the world geodetic system 1984 (WGS-84). The auxiliary coordinate system $O_aX_aY_aZ_a$ is an imaginary local coordinate system with the set point as the origin. The USBL acoustic array is installed under the base array and, due to the installation error, the base coordinate system and the acoustic array coordinate system cannot completely coincide. To study the impact of installation error on the USBL positioning, the USBL acoustic array coordinate system $O_tX_tY_tZ_t$ is defined as shown in Fig. 2, where the alignment errors of heading, pitch, and roll are α, β and γ, respectively.

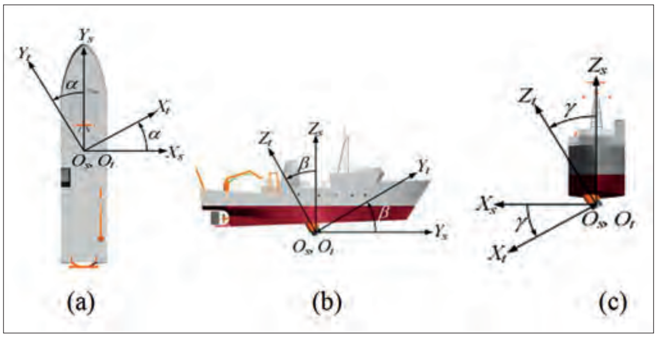

Fig. 2. The USBL acoustic array coordinate system $O_tX_tY_tZ_t$. (a) heading misalignment, (b) pitch misalignment, and (c) roll misalignment

THE BASIC POSITIONING PRINCIPLE

The USBL positioning system sends an inquiry signal to an underwater target transponder by a transducer array. The transponder receives the signal and sends a positioning signal; the transducer array measures the phase difference or delay difference between the base array elements to determine the target orientation. The slant distance between the measured target and the transducer array is then calculated by measuring the round-trip time between the query signal and the positioning signal. By measuring the orientation and slant distance, the underwater target location may be finalised. The commonly used geometric diagram of the USBL positioning principle is shown in Fig. 3.

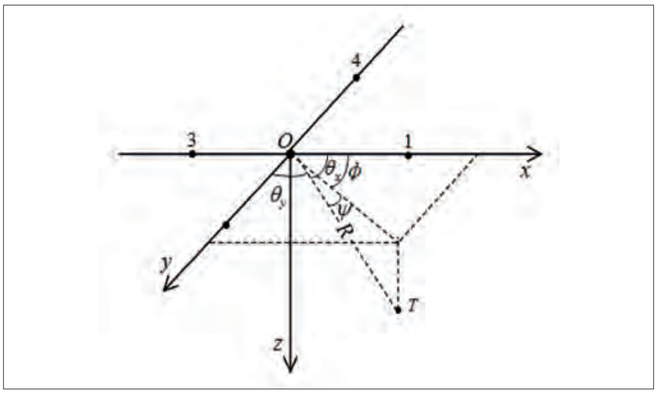

Fig. 3. The geometric diagram of the USBL positioning principle

The North-East-Earth rectangular coordinate system (x, y, z) is used, with the array's centre at coordinate origin O. The No. 1 and No. 3 transducers are located on the x-axis, and the No. 2 and No. 4 transducers are located on the y-axis. The included angles between the target T and the x-and y-axes are θ_x and θ_y, respectively, and the slant range between the coordinate origin O and the target T is R.

Then, the coordinates of target T can be expressed as:

$$P_t = [R \cdot \cos\theta_x, R \cdot \cos\theta_y, R \cdot \sqrt{1-(\cos^2\theta_x + \cos^2\theta_y)}\,] \tag{1}$$

The distance between the target and the coordinate origin is much larger than the array spacing, so the signal can be considered to be the far-field plane wave propagation. Then, the delay difference τ_{13} of the No. 1 and No. 3 transducers, and the delay differences τ_{24} of the No. 2 and No. 4 transducers are, respectively:

$$\tau_{13} = \frac{d_{13}\cos\theta_x}{c}, \quad \tau_{24} = \frac{d_{24}\cos\theta_y}{c}, \tag{2}$$

where c is the sound velocity in water, d_{13} is the distance between the No. 1 and No. 3 transducers, and d_{24} is the distance between the No. 2 and No. 4 transducers. Substituting Eqs. (2) into Eq. (1) gives the coordinates of T as:

$$P_t = \left[\frac{cR\tau_{13}}{d_{13}}, \frac{cR\tau_{24}}{d_{24}}, R \cdot \sqrt{1-c^2[(\tfrac{\tau_{13}}{d_{13}})^2 + (\tfrac{\tau_{24}}{d_{24}})^2]}\,\right] \tag{3}$$

When the baseline length and sound speed in seawater are known, Eq. (3) states that the coordinate position of the target may be determined by computing the time delay and slant range, allowing for USBL positioning of the underwater target.

In addition, the slant range (R), bearing (ϕ), and depression angle (ψ) from the underwater transponder to the USBL centre can also be used to calculate the position. The coordinate of the transponder T is shown in Eq. (4). It can be seen that, by this method, the position of the transponder in the USBL acoustic array coordinate can be determined by simply measuring the slant range, horizontal bearing, and depression angle.

$$P_t = [R\cos\psi\cos\phi, R\cos\psi\sin\phi, -R\sin\psi] \tag{4}$$

AN INSTALLATION ANGLE ERROR CALIBRATION BASED ON A DUAL TRANSPONDER

USBL POSITIONING ERRORS DUE TO ANGULAR MISALIGNMENTS

The position of transponder 1 in the $O_aX_aY_aZ_a$ coordinate system can be written as: $\boldsymbol{P}_a = [0, 0, -h_1, 1]$. When there is no alignment error between the attitude sensor and USBL acoustic array, $O_tX_tY_tZ_t$ and $O_sX_sY_sZ_s$ completely coincide, and the position of the seabed transponder in the $O_tX_tY_tZ_t$ coordinate system is: $\boldsymbol{P}_t = \boldsymbol{T}_{sa}(d, l, \theta) \cdot \boldsymbol{P}_a = [-d, -l, -h_1, 1]$. When there is a heading alignment error α between the $O_tX_tY_tZ_t$ coordinate system and $O_sX_sY_sZ_s$ coordinate system, the position of the seabed transponder is:

$$\boldsymbol{P}_t^{\alpha} = \boldsymbol{T}_{ts}(\alpha)\, \boldsymbol{T}_{sa}(d, l, \theta)\, \boldsymbol{P}_a \begin{bmatrix} -d\cos\alpha - l\sin\alpha \\ d\sin\alpha - l\cos\alpha \\ -h_1 \\ 1 \end{bmatrix} \tag{5}$$

Similarly, when there is pitch alignment error β and roll alignment error γ, between the USBL acoustic base coordinate system and the carrier base coordinate system, respectively, the position of the transponder in the $O_tX_tY_tZ_t$ coordinate system is:

$$\boldsymbol{P}_t^{\beta} = \boldsymbol{T}_{ts}(\beta)\, \boldsymbol{T}_{sa}(d, l, \theta)\, \boldsymbol{P}_a \begin{bmatrix} -d \\ -l\cos\beta - h_1\sin\beta \\ l\sin\beta - h_1\cos\beta \\ 1 \end{bmatrix},$$

$$\boldsymbol{P}_t^{\gamma} = \boldsymbol{T}_{ts}(\gamma)\, \boldsymbol{T}_{sa}(d, l, \theta)\, \boldsymbol{P}_a \begin{bmatrix} -d\cos\gamma + h_1\sin\gamma \\ -l \\ -d\sin\gamma - h_1\cos\gamma \\ 1 \end{bmatrix} \tag{6}$$

In order to analyse the influence of alignment errors on the USBL positioning in the $O_tX_tY_tZ_t$ coordinate system, the positioning error caused by the heading misalignment α is:

$$\boldsymbol{\varepsilon}^{\alpha} = \boldsymbol{P}_t^{\alpha} - \boldsymbol{P}_t = \begin{bmatrix} -d\cos\alpha - l\sin\alpha + d \\ d\sin\alpha - l\cos\alpha + l \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

Similarly, the positioning errors of transponders $\boldsymbol{\varepsilon}^{\beta}$ and $\boldsymbol{\varepsilon}^{\gamma}$, caused by the pitch and roll misalignments under the coordinate system $O_tX_tY_tZ_t$, are, respectively:

$$\boldsymbol{\varepsilon}^{\beta} = \boldsymbol{P}_t^{\beta} - \boldsymbol{P}_t = \begin{bmatrix} 0 \\ -l\cos\beta - h_1\sin\beta + l \\ l\sin\beta - h_1\cos\beta + h_1 \\ 0 \end{bmatrix},$$

$$\boldsymbol{\varepsilon}^{\gamma} = \boldsymbol{P}_t^{\gamma} - \boldsymbol{P}_t = \begin{bmatrix} -d\cos\gamma + h_1\sin\gamma + d \\ 0 \\ -d\sin\gamma - h_1\cos\gamma + h_1 \\ 0 \end{bmatrix} \tag{8}$$

To analyse the influence of angular misalignments on positioning accuracy, the following simulation was conducted. Suppose the depth of transponder 1 is 100 m, the horizontal distance d from the transponder to the ship's straight track is 50, 100, 200, and 300 m, l is –100 m to 100 m, and the heading, pitch, and roll misalignments are all set to 10°. Figs. 4-6 show the positioning error diagram caused by the misalignment of heading, pitch, and roll.

$\boldsymbol{\varepsilon}_x^{\alpha}$ and $\boldsymbol{\varepsilon}_y^{\alpha}$ are the positioning errors in the x-direction and y-direction caused by heading misalignment, respectively. According to Eq. (8), we can obtain:

$$\boldsymbol{\varepsilon}_x^{\alpha} = \cot\frac{\alpha}{2} \cdot \boldsymbol{\varepsilon}_y^{\alpha} + \csc\frac{\alpha}{2} \cdot d \tag{9}$$

It can be seen from Eq. (9) and Fig. 4 that the positioning error in the x-direction, caused by heading misalignment, has a linear relationship with the positioning error in the *y*-direction, and the error increases with the increase of horizontal distance *d*, while there is no positioning error in the *z*-direction.

Similarly, $\boldsymbol{\varepsilon}_y^\beta$ and $\boldsymbol{\varepsilon}_z^\beta$ are the positioning errors in the *y*-direction and *z*-direction caused by pitch misalignment, and $\boldsymbol{\varepsilon}_x^\gamma$ and $\boldsymbol{\varepsilon}_z^\gamma$ are the positioning errors in the *x*-direction and z-direction caused by roll misalignment. According to Eq. (8), we can obtain:

$$\boldsymbol{\varepsilon}_y^\beta = \tan\frac{\beta}{2}\cdot\boldsymbol{\varepsilon}_z^\beta + \sec\frac{\beta}{2}\cdot h,\quad \boldsymbol{\varepsilon}_x^\gamma = -\tan\frac{\gamma}{2}\cdot\boldsymbol{\varepsilon}_z^\gamma + \sec\frac{\beta}{2}\cdot h \tag{10}$$

It can be seen that the positioning error in the *y*-direction, caused by pitch misalignment, has a proportional linear relationship with the error in the *z*-direction, and the error is independent of the horizontal distance *d*, while there is no positioning error in the *x*-direction. In addition, the positioning errors in the *x*-direction and *z*-direction, caused by roll misalignment, are independent of *l*, and the errors increase with the increase of horizontal distance *d*, while there is no positioning error in the *y*-direction.

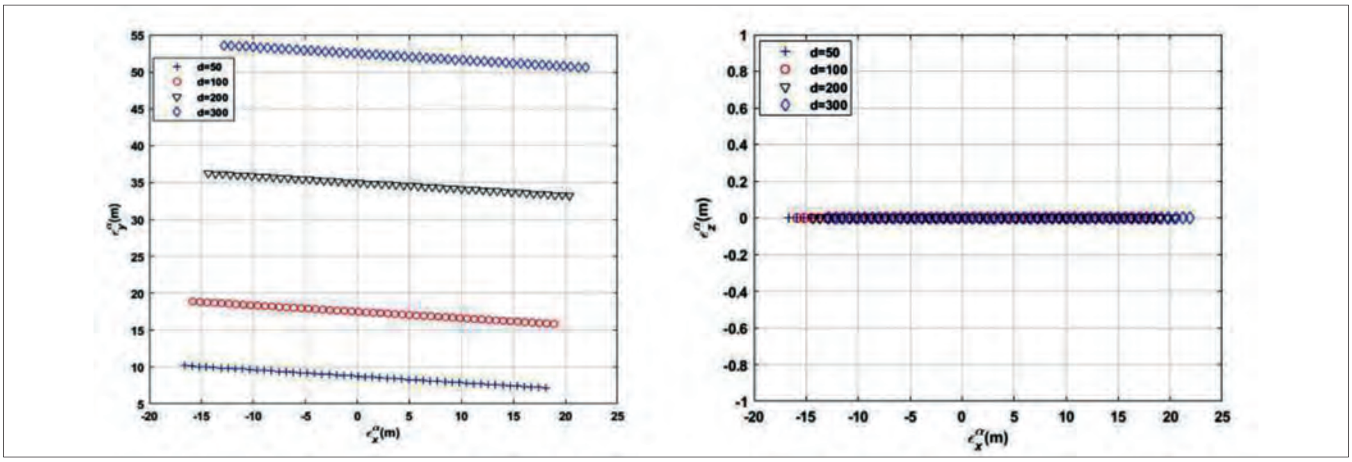

Fig. 4. The positioning error diagram caused by heading misalignment

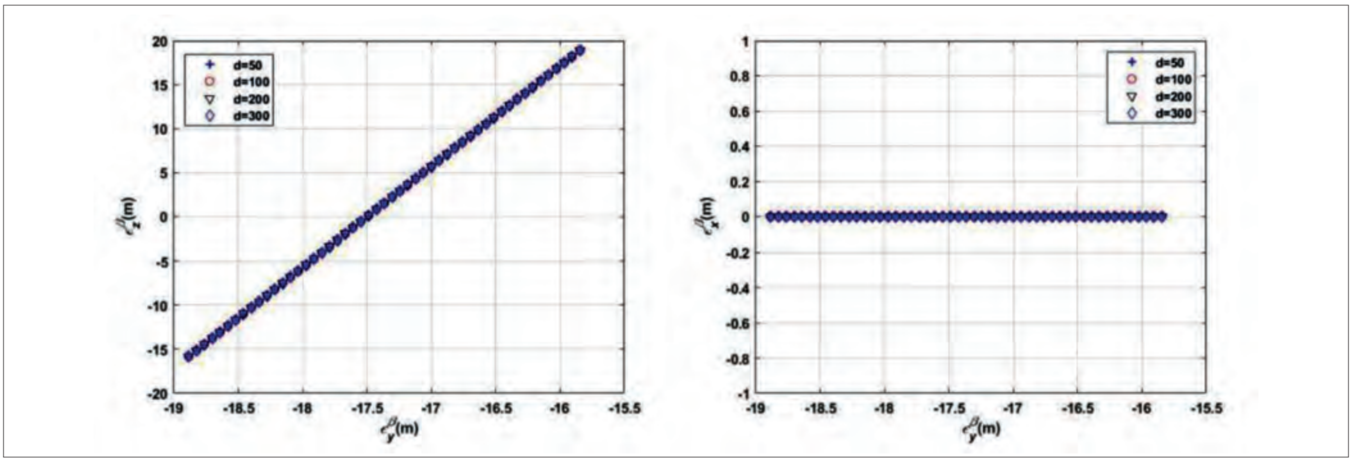

Fig. 5. The positioning error diagram caused by pitch misalignment

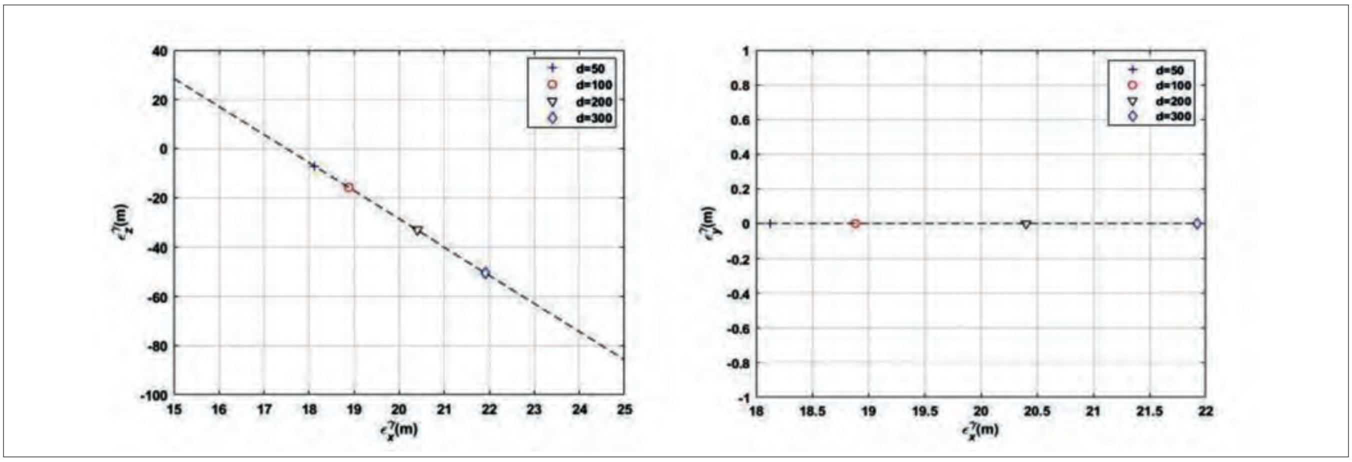

Fig. 6. The positioning error diagram caused by roll misalignment

ITERATIVE ALGORITHM

When the USBL acoustic coordinate system and the carrier base coordinate system have alignment errors of heading, pitch, and roll, the position of the seabed transponder 1 is:

$$\boldsymbol{P}'_{t1} = \boldsymbol{T}_{ts}(\alpha)\ \boldsymbol{T}_{ts}(\beta)\ \boldsymbol{T}_{ts}(\gamma)\ \boldsymbol{T}_{sa}(d, l, \theta)\ \boldsymbol{P}_a\ = \tag{11}$$

$$= \begin{bmatrix} -d\cos\gamma\cos\alpha - l\cos\gamma\sin\alpha + h_1\sin\gamma \\ -d(\sin\beta\sin\gamma\cos\alpha - \cos\beta\sin\alpha) - l(\sin\beta\sin\gamma\sin\alpha + \cos\beta\cos\alpha) - h_1\sin\beta\cos\gamma \\ -d(\sin\gamma\cos\beta\cos\alpha + \sin\beta\sin\alpha) - l(\sin\gamma\cos\beta\sin\alpha - \sin\beta\cos\alpha) - h_1\cos\beta\cos\gamma \\ 1 \end{bmatrix}$$

Similarly, the coordinate of the seabed transponder 2 is:

$$\boldsymbol{P}'_{t2} = \begin{bmatrix} -d\cos\gamma\cos\alpha - l\cos\gamma\sin\alpha + h_2\sin\gamma \\ -d(\sin\beta\sin\gamma\cos\alpha - \cos\beta\sin\alpha) - l(\sin\beta\sin\gamma\sin\alpha + \cos\beta\cos\alpha) - h_2\sin\beta\cos\gamma \\ -d(\sin\gamma\cos\beta\cos\alpha + \sin\beta\sin\alpha) - l(\sin\gamma\cos\beta\sin\alpha - \sin\beta\cos\alpha) - h_2\cos\beta\cos\gamma \\ 1 \end{bmatrix} \tag{12}$$

For the characteristics of positioning errors caused by each installation angle, an algorithm for iterative calculation and estimation of heading, pitch, and roll misalignments is proposed. The specific steps are as follows:

(1) The installation error angle of roll is solved by the error component in the x-direction of the $O_tX_tY_tZ$ coordinate system. According to Eq. (11) and Eq. (12), when the ship sails in a straight line, the x-axis values of transponder 1 and transponder 2 are:

$$\begin{aligned} \boldsymbol{P}'_{t1\_x} &= -d\cos\gamma\cos\alpha - l\cos\gamma\sin\alpha + h_1\sin\gamma\,, \\ \boldsymbol{P}'_{t2\_x} &= -d\cos\gamma\cos\alpha - l\cos\gamma\sin\alpha + h_2\sin\gamma \end{aligned} \tag{13}$$

From Eq. (13), we can get the following equation:

$$\boldsymbol{P}'_{t1\_x} - \boldsymbol{P}'_{t2\_x} = (h_1 - h_2)\sin\gamma \tag{14}$$

Then, the roll misalignment is given as:

$$\gamma = \arcsin\frac{\boldsymbol{P}'_{t1\_x} - \boldsymbol{P}'_{t2\_x}}{h_1 - h_2} = \arcsin\frac{\boldsymbol{P}'_{t2\_x} - \boldsymbol{P}'_{t1\_x}}{\Delta h} \tag{15}$$

where $\boldsymbol{P}'_{t1\_x}$ and $\boldsymbol{P}'_{t2\_x}$ represent the x-axis coordinate values of transponder 1 and transponder 2, respectively, in the USBL array coordinate system. Δh is the depth difference between transponder 1 and transponder 2.

(2) After correcting the roll error, it can be found that the pitch installation error angle can be solved by the error component in the y-direction. According to Eq. (11) and Eq. (12), the y-axis values of transponder 1 and transponder 2 are:

$$\begin{aligned} \boldsymbol{P}'_{t1\_y} &= -d(\sin\beta\sin\gamma\cos\alpha - \cos\beta\sin\alpha) - \\ &-l(\sin\beta\sin\gamma\sin\alpha + \cos\beta\cos\alpha) - h_1\sin\beta\cos\gamma \end{aligned} \tag{16}$$

$$\begin{aligned} \boldsymbol{P}'_{t2\_y} &= -d(\sin\beta\sin\gamma\cos\alpha - \cos\beta\sin\alpha) - \\ &-l(\sin\beta\sin\gamma\sin\alpha + \cos\beta\cos\alpha) - h_2\sin\beta\cos\gamma \end{aligned} \tag{17}$$

When there is no roll error, it can be seen from Eq. (16) and Eq. (17) that:

$$\boldsymbol{P}'_{t1\_y} - \boldsymbol{P}'_{t2\_y} = (h_2 - h_1)\sin\beta \tag{18}$$

Then, the pitch misalignment is calculated as:

$$\beta = \arcsin\frac{\boldsymbol{P}'_{t1\_y} - \boldsymbol{P}'_{t2\_y}}{h_2 - h_1} = \arcsin\frac{\boldsymbol{P}'_{t1\_y} - \boldsymbol{P}'_{t2\_y}}{\Delta h} \tag{19}$$

where $\boldsymbol{P}'_{t1\_y}$ and $\boldsymbol{P}'_{t2\_y}$ represent the y-axis coordinate values of transponder 1 and transponder 2, respectively, in the USBL array coordinate system.

(3) After both the roll and pitch misalignments are corrected, it should be noted that the heading error angle can be solved by combining the error component in the x-direction and y-direction of a transponder. When there is no roll and pitch error, it can be seen that:

$$\boldsymbol{P}'_{t1\_x} = -d\cos\alpha - l\sin\alpha,\ \ \boldsymbol{P}'_{t1\_y} = d\sin\alpha - l\cos\alpha \tag{20}$$

Then, the heading misalignment is given as:

$$\alpha = \arcsin\frac{\boldsymbol{P}'_{t1\_y}\cdot d - \boldsymbol{P}'_{t1\_x}\cdot l}{d^2 + l^2} \tag{21}$$

(4) However, the above analysis is carried out under the assumption that the deviations of the other two angles have been corrected when calculating each installation error angle, which is not the case in engineering practice. Therefore, the above method cannot directly estimate the error angle and so the above steps must be repeated. An incremental iterative algorithm is proposed to estimate the error angle, as shown in Fig. 7. The iterative algorithm starts from $\alpha = \beta = \gamma = 0$ and, in each iteration, the increments $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ of the installation error angle are calculated and accumulated. Then the original positioning data of the positioning observations are modified with the updated estimation of roll, pitch and heading. The iterative process is continued until all estimates of the angular deviations converge.

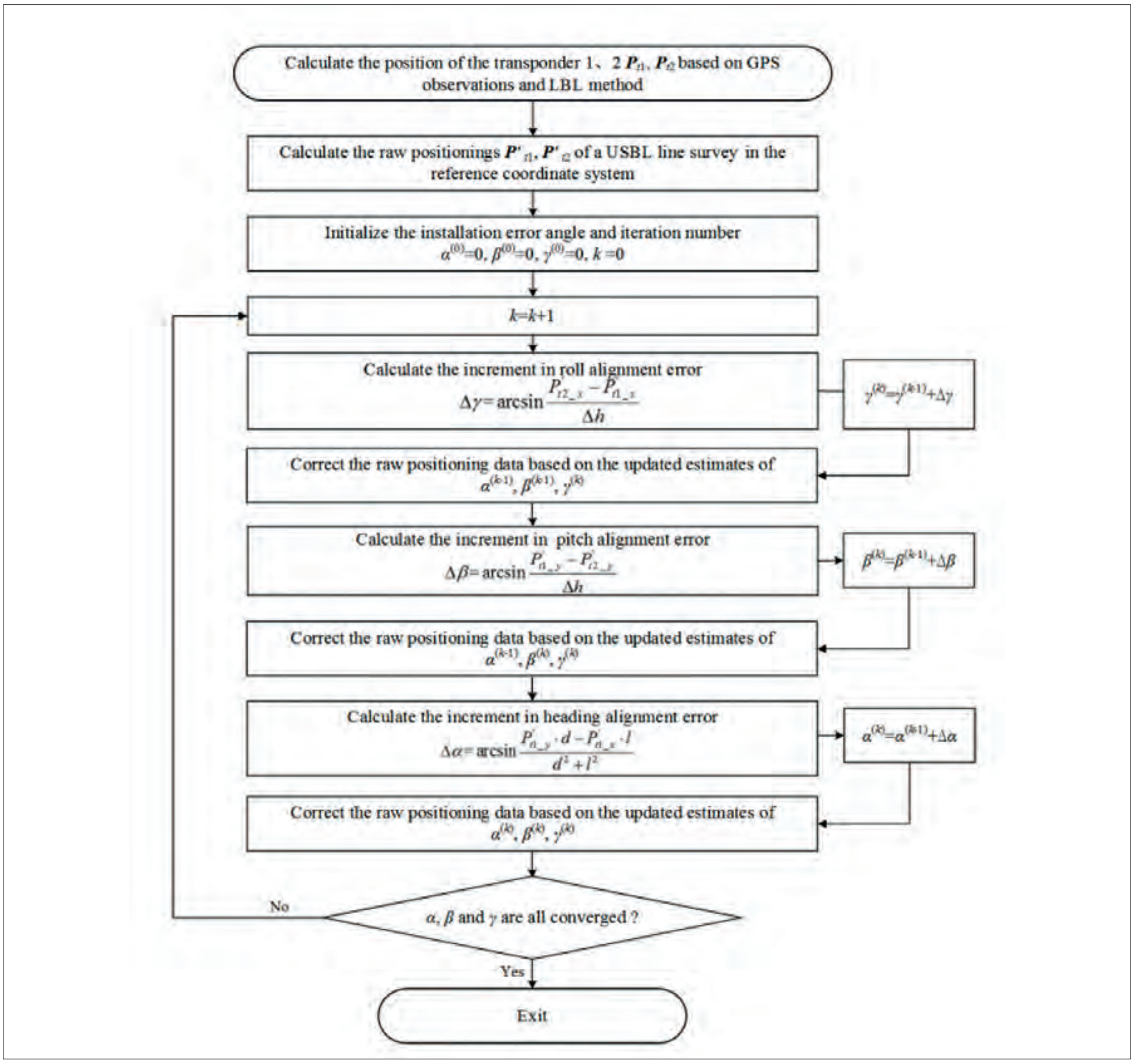

Fig. 7. The incremental iterative algorithm

NUMERICAL SIMULATIONS

According to Eq. (4), the position of the transponder can be calculated by measuring the oblique distance R, bearing ϕ, and depression angle ψ. The effectiveness and robustness of the new algorithm to noise are analysed by adding random Gaussian noise. The simulation conditions are as follows: the depths of transponder 1 and 2 are 100 m and 150 m, respectively, and the horizontal distance d from the transponder to the ship's straight track is 50 m, l is −100 m to 100 m, the values of three installation angle errors α, β, and γ are 7°, 3°, and 5°, respectively. The values of R, ϕ, and ψ all follow a normal distribution with zero mean, the standard deviation of R is 0.25 m, and the standard deviation of ϕ and ψ is 0.2°.

When the alignment error is not corrected, the x and y coordinates of the two transponders are as shown in Fig. 8(a), the position after one iteration is shown in Fig. 8(b), and the result after five iterations is shown in Fig. 8(c). It can be seen that, after correction, the x and y coordinates of the two transponders tend to be approximately the same as the number of iteration increases. The number of iterations ranges from one to five and the iterative estimations of the alignment error for 200 observed positions are calculated and averaged according to Eqs. (22), (26), and (29), and the results are shown in Table 1. It can be seen from the table that the estimates of roll, pitch, and heading converge to their true values with a small error. In addition, the algorithm is robust, even with measurement errors, and all of the estimates converge to within 0.001° after three iterations.

Tab. 1. The iterative estimates of alignment errors

Iteration number	$\Delta\alpha$	$\Delta\beta$	$\Delta\gamma$	α	β	γ
1	6.968°	2.495°	4.512°	6.968°	2.495°	4.512°
2	0.024°	0.426°	0.443°	6.992°	2.921°	4.955°
3	0.008°	0.079°	0.045°	7.000°	3.000°	5.000°
4	3.6e-5°	6.8e-4°	5.3e-4°	7.000°	3.000°	5.000°
5	1.5e-7°	2.2e-5°	3.8e-5°	7.000°	3.000°	5.000°

EXPERIENCE AND ANALYSIS

A USBL localisation experiment was conducted to further verify the effectiveness of the proposed installation angle error calibration method in practical applications. The test equipment

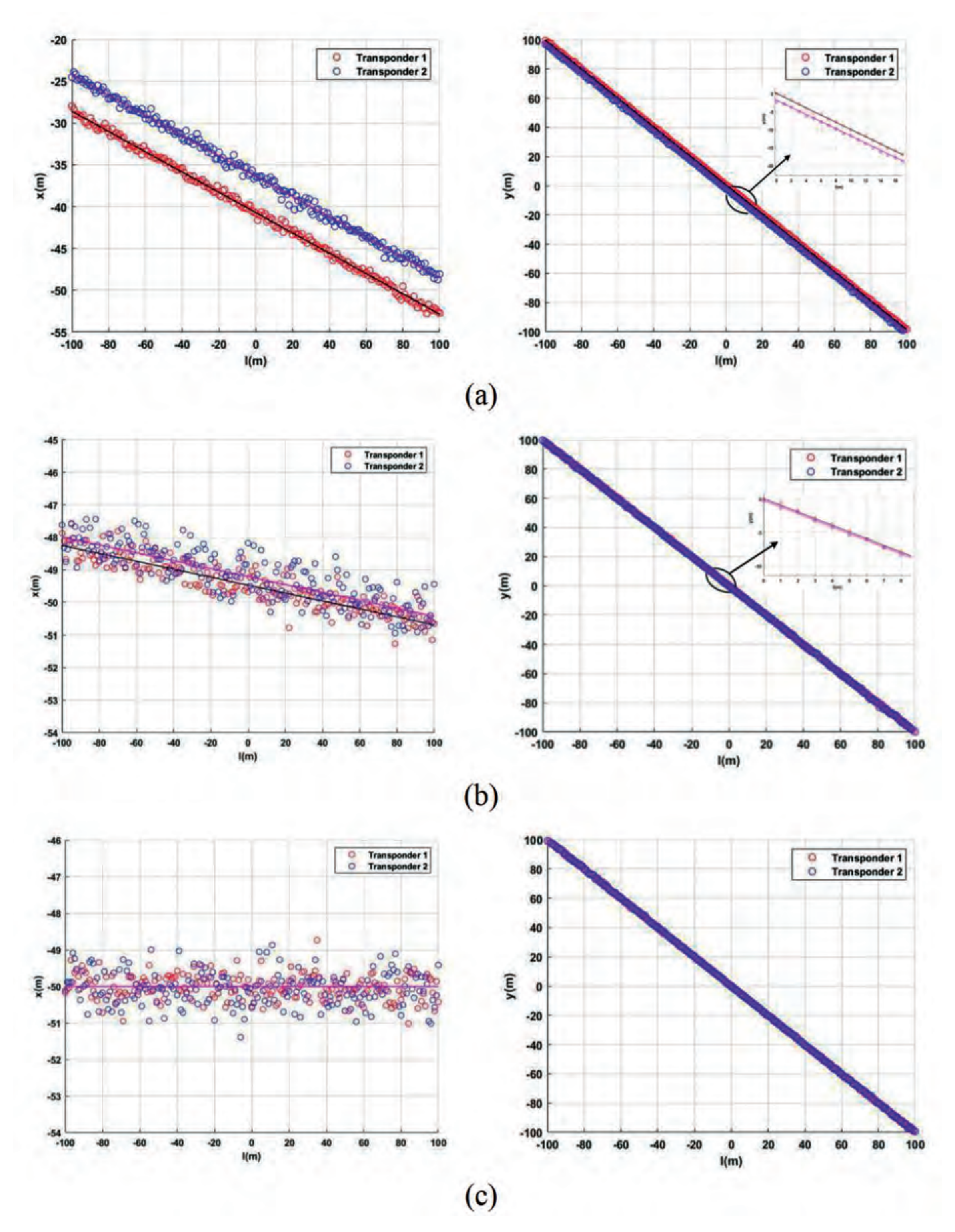

Fig. 8. *The x and y coordinates of the two transponders. (a) No iteration; (b) one iteration; (c) five iterations*

included the USBL system, Real Time Kinematical Global Positioning System (RTK-GPS), and Attitude and Heading Reference System (AHRS). The transducer array has a four-element planar array structure, with four elements located on the horizontal axis of the acoustic array, and the transmitter head is located at the geometric centre of the four arrays. The transducer array is mounted on the starboard side of the mother ship at a depth of 2 m and is rigidly connected to the connecting rod. The RTK-GPS is used to measure the horizontal position of the two transponders and take it as the origin. Since the accuracy of RTK reaches the centimetre level, this coordinate could be taken as the true value of algorithm verification. In the experiment, the ship sailed in a straight line near the transponders, and eight navigation tracks were seen (Fig. 9).

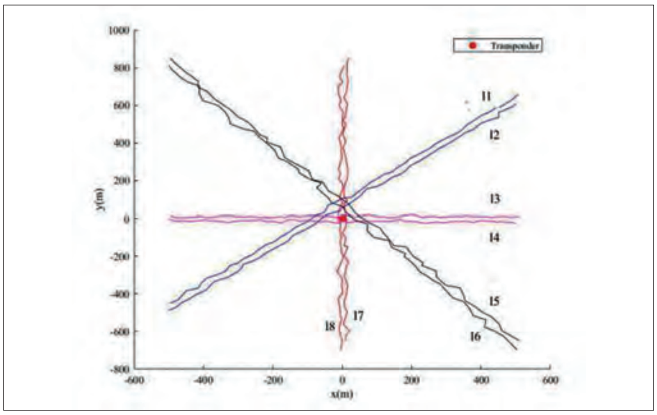

Fig. 9. The eight navigation tracks

The pitch, roll, and heading alignment errors were estimated using the USBL observations measured along path *l*1, according to the installation angle error calibration method proposed in the previous section and shown in Fig. 10. As can be seen from the figure, the estimates of the three alignment errors can converge quickly, and the estimates are very close to the true values when the number of iterations is three. Then, these three alignment errors were estimated using the USBL observations collected along paths *l*1–18 and the results are shown in Table 2. It can be seen that the standard deviations of the pitch, roll and heading alignment error estimates are small, which indicates that the algorithm is robust for all three error estimates. It should be noted that the USBL installation angle error can be caused by surface ambient noise and hull self-noise, artificial incorrect installation operation, and measurement errors of devices such as RTK-GPS and OCTANS. These factors have different effects on heading angle, pitch angle and roll angle, so the horizontal and heading installation errors are different.

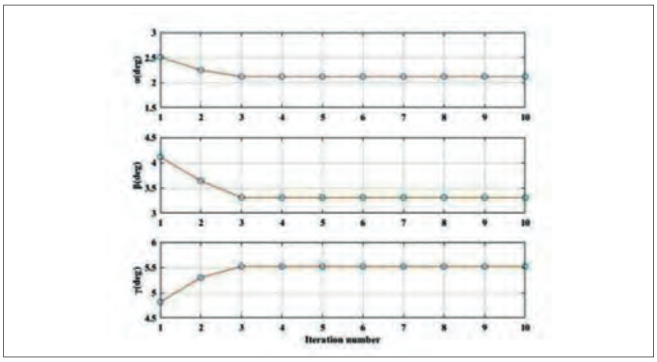

Fig. 10. Convergence of the estimates based on the observations collected along path l1.

Tab. 2. Estimates of alignment errors from different paths

Survey path	α	β	γ	Survey path	α	β	γ
11	2.12°	3.31°	5.52°	15	2.03°	3.15°	5.55°
12	2.05°	3.25°	5.45°	16	2.20°	3.40°	5.44°
13	2.25°	3.36°	5.61°	17	1.99°	3.33°	5.42°
14	2.18°	3.22°	5.37°	18	2.28°	3.13°	5.51°
Mean	2.14°	3.27°	5.48°	Std. dev.	0.10°	0.09°	0.07°

Based on the results in Table 2, the USBL observations were corrected, and the seabed transponder positioning before and after the error correction is shown in Fig. 11. The point distribution shows that there is a large horizontal positioning error before the correction and after the error correction, the positioning results are more concentrated at the true position of the transponder. The horizontal position error is reduced by nearly 75% after the error angle correction of the USBL installation, and the horizontal positioning accuracy of USBL has been significantly improved.

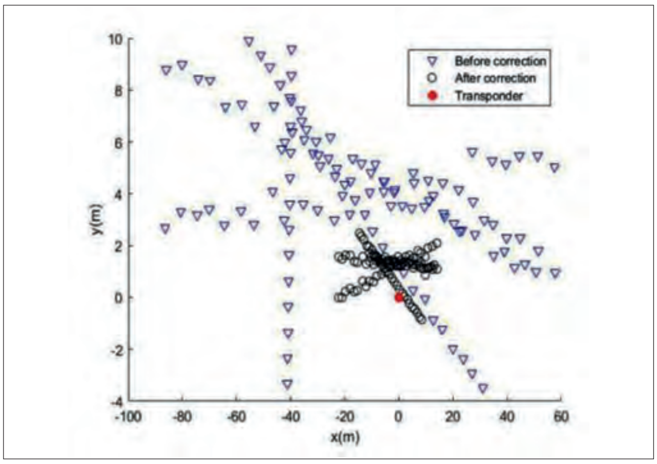

Fig. 11. The seabed transponder positioning before and after the error correction

CONCLUSION

This paper deduces and analyses the influencing laws of heading, pitch, and roll on positioning errors. Then, an iterative calibration algorithm of USBL installation angle error, based on dual transponders, is proposed, which first estimates the rolling alignment error, then estimates the pitch alignment error, and, finally, estimates the heading alignment error. Simulations and experience show that the convergence speed of the three alignment errors is very fast; only a few iterations are needed to accurately calculate the alignment errors, and the new algorithm has good effectiveness and stability, which can greatly improve the positioning accuracy of the USBL system.

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