Stefan KUBISA, **Zygmunt L. WARSZA** [1]
[1] INDUSTRIAL RESEARCH INSTITUTE of AUTOMATION and MEASUREMENTS, PIAP
202 Jerozolimskie Ave., 02-486 Warszawa Pl

The frequency analysis of real resistors in relative values

Abstract

Three-element impedance including the resistance R as the main parameter and the residual parameters: serial inductance L and a parallel capacitance C is used as an equivalent circuit of resistor. It is the basic model of resistor for AC current. The real component of this impedance is not equal to the resistance R for the DC current, and there is the imaginary component. Both components depend on the frequency. Variants of the model with four different connections are also analyzed. Patterns of the relative resistor frequency errors in the generalized form, i.e. as a function of the relative values of resistance and frequency of the circuit are determined and their curves are given. Considerations are illustrated by few numerical examples.

Keywords: resistor, AC models, equivalent circuits, impedance, characteristic parameters, frequency errors.

1. Introduction

The flow of electrical current in the real objects is described by the electromagnetic field equations as functions of space and time. If current is direct in time (DC) the ratio of voltage and current of any two-terminal object is determinated as its resistance R. If geometric sizes of real physical object are much smaller than the shortest wavelength of AC current waveform, then the simpler description by theory of the electrical circuits containing ideal dimensionless elements can be used. In models of AC circuits of the such low frequency f beside the ideal resistance R, the ideal reactive elements: capacitance C, inductance L, mutual inductance M and also an ideal stationary or controlled sources of voltage E and current I are applied. If for the permissible ranges of voltages and currents the values of these parameters are constant, then modeled circuit is linear. The method of analysis frequency characteristics of the practical two-terminal element with constant parameters, described in relative values is presented and analyzed below on the example of resistors used in measurements and electronics as metal foil and bulk resistors [1], [9].

For the alternating current (AC) of frequency f, the complex impedance describes any passive two-terminal device

$$\underline{Z}(\mathrm{j}\omega) = \mathrm{Re}\,\underline{Z} + \mathrm{j}\mathrm{Im}\,\underline{Z} = Z e^{\mathrm{j}\varphi}$$

with both rectangular or polar components as functions of pulsation $\omega = 2\pi f$. Impedance $\underline{Z}$ [1] in the circuit equivalent schemes is presented as ideal resistance $R_S \equiv Re\,\underline{Z}$ and ideal reactance $\mathrm{j}X_S = \mathrm{j}\mathrm{Im}\,\underline{Z}$ connected in series. For very high frequencies, in which wavelengths of AC current and geometrical sizes of the real object are comparable, description with distributed parameters and the wave impedance Z as for transmission lines, have to be used.

As the simple example of the two-terminal object in AC circuits is the real resistor. In addition to the its basic parameter R the impact of the residual parameters, the inductance L and capacitance C, called also as parasitic parameters, must be taken into account. These parameters are distributed in a volume of analyzed device and depend on its structure, shape and sizes. An impedance $\underline{Z}$ on the terminals depends on combination of all R L C values of its equivalent scheme.

[1] For simplicity in the following text as frequency are called both f and pulsation $\omega=2\pi f$ also known as the circular frequency. Often in text a parameter dependence on the frequency is not marked as is written shorter, e.g. Z instead of $Z(\mathrm{j}\omega)$, R_s instead of $R_s(\omega)$, etc.

Properties of real resistors used in AC circuits usually are described by equivalent schemes with values of R L C parameters independent on frequency ω. The basic there-element model has the structure given in Figure 1a. In the following text it is referred as model type Γ (gamma). The similar model, but with a different values of parameters can be used also for coils without a ferromagnetic core. The equivalent to the three-element model type Γ are two-element models (Fig. 1b) as serial R_s L_s and parallel R_r C_r circuits. In the general case their parameters depend on the frequency ω. The serial model is preferred for $\mathrm{Im}(\underline{Z}) > 0$ as then $L_s > 0$. If $\mathrm{Im}(\underline{Z}) < 0$ then $C_r > 0$ and resistor is modeled more simply by the parallel circuit of admittance $\underline{Y}(\mathrm{j}\omega) = 1/R_r + \mathrm{j}\omega C_r$ [3].

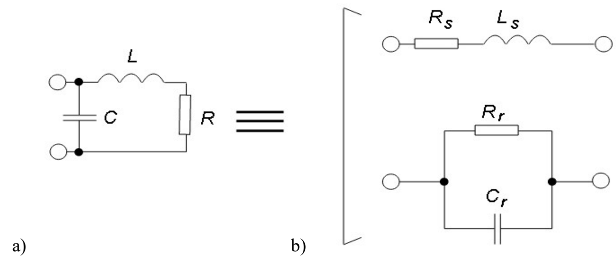

Fig. 1. The basic models of a real two terminal objects: a) type Γ of three constant elements R L C, b) its two-parameter equivalents: serial R_s L_s or parallel R_r C_r with both parameters dependent on frequency ω

2. Assumptions

The real resistor can fulfill properly the function of an ideal single resistance R in AC circuits only when $R_s(\omega) = R$ and $X_s(\omega) = 0$. If not, then the difference $\Delta R = R_s - R$ is the resistive component of its frequency error, and X_s is the reactive component of this error. A resistor, for which $\Delta R \approx 0$ and $X_s \approx 0$ may be called as *good* resistor.

In considerations below it is assumed that:

- the fixed values of R L C and $\underline{Z}$ parameters can be used throughout the full range of permissible values of currents and voltages. This applies to such electrical devices which structures do not include the non-linear ferromagnetic and dielectrics materials and semiconductor junctions.
- Resistor is treated as a individual element, i.e. it is not directly or electromagnetically coupled to other components of the circuit,
- L and C parameters of the resistor do not depend directly on the value of R, but only on its geometry including connections,
- R L C parameters also do not depend on the frequency, i.e. in thin film resistors and resistors made of materials with high resistivity the skin effect is negligible.

Numerical examples and resulting conclusions are mainly given for a resistors of flat thin film plotted on the insulating substrate. At a given geometrical shape and sizes, their resistance R depends only on the thickness of the conductive layer. Parameters L, C of such resistors with identical shape and size, including the leads, are the same for the various R. This condition does not meet the resistors with a complex form of resistance track as meander or helix and wire wounded ones.

3. The relative values or numbers of similarities

The analysis of frequency properties of the equivalent schemes of real objects can be more general if all their terminal parameters are expressed in relative values. In particular of two-terminal object shown in Fig. 1a, their impedance $\underline{Z}$ and its components R_s, X_s, can be functions of its resistance R for DC current or functions of the non damped vibration frequency ω_0 and of the characteristic resistance R_0 of this circuit [2]:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \tag{1}$$

$$R_0 = \sqrt{L/C}. \tag{2}$$

It is seen that ω_0 and R_0 are linked themselves by L and C. The frequency ω_0 is when $R \to 0$. And the value of resistance R_0 results from the equality of capacitive and inductive time constants $\tau_C = R_0 C$ and $\tau_L = L/R_0$, respectively.

The frequency $f_0 = \omega_0/2\pi$ inversely depends to a geometric sizes of resistor, e.g. for flat resistors of the centimeter sizes f_0 is of the GHz order, and for micrometer sizes - of the THz order. If the resistor is made of materials with the magnetic and dielectric relative permeability equal to 1 and is located in the environment of such both permeabilities, then the characteristic resistance R_0 is very close to the wave resistance of vacuum, approx. 377 Ω. In the presence of materials with greater than vacuum permeabilities the value of R_0 is reduced, usually to the range (50 -300) Ω - see the numerical example.

Generalization of the impedance model description may be achieved by the use of so-called similarity numbers. These dimensionless parameters include, for example ratios of two quantities of the same dimensions. They are also referred often as the relative values. In describing the frequency characteristics of resistors the similarity numbers given below are used:

- Relative frequency η as ratio of the considered frequency f and the characteristic ones f_0 of the model:

$$\eta \overset{\text{def}}{=} \omega/\omega_0 = f/f_0, \tag{3}$$

where: $\omega = \eta\,\omega_0$ and $f = \eta\, f_0$.

- relative resistance ρ as the ratio of resistance R for DC current and characteristic resistance R_0:

$$\rho \overset{\text{def}}{=} R/R_0 \quad \text{and hence} \quad R = \rho R_0. \tag{4}$$

Similarity numbers of the impedance components related to R of the serial circuit model shown in Figure 1b, i.e.:

- relative serial resistance:

$$r_s \overset{\text{def}}{=} \frac{R_s}{R} \quad \text{and from it} \quad R_s = r_s R, \tag{5}$$

- relative serial reactance:

$$x_s \overset{\text{def}}{=} X_s/R, \quad \text{and from it} \quad X_s = x_s R \tag{6}$$

Figure 2 shows on the Gaussian plane the vector diagram of impedance Z components of the serial model of Figure 1b and components ΔR and jX_s of an vector error $\underline{\Delta Z}$ of the resistor. Directions of arrowheads indicate the signs of components.

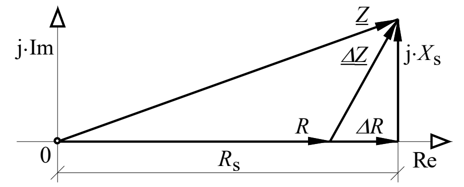

Fig. 2. Graph vector of serial impedance components of the resistor

From this graph, you can also determine the relationship between the relative errors frequency resources δR, δX, δZ resistor and components R_s, X_s its impedance Z, i.e.:

- relative resistance error:

$$\delta R \overset{\text{def}}{=} \frac{\operatorname{Re}\underline{Z} - R}{R} = \frac{R_S - R}{R} = r_S - 1 \tag{7}$$

- relative reactance error:

$$\delta X \overset{\text{def}}{=} \frac{\operatorname{Im}\underline{Z}}{R} = \frac{X_S}{R} = x_S \tag{8}$$

- relative impedance error:

$$\delta Z \overset{\text{def}}{=} \left|\underline{\Delta Z}\right|/R = \sqrt{\delta R^2 + \delta X^2} \tag{9}$$

4. Frequency properties of resistor model Γ

The Laplace operator impedance Z(s) of the model of Figure 1a can be described by following patterns:

$$Z(\mathrm{s}) = \frac{(R+\mathrm{s}L)/\mathrm{s}C}{R+\mathrm{s}L+1/\mathrm{s}C} = R\frac{1+\mathrm{s}L/R}{\mathrm{s}^2 LC+\mathrm{s}RC+1} = R\frac{1+\mathrm{s}\tau_L}{\frac{\mathrm{s}^2}{\omega_0^2}+\frac{R}{R_0}\frac{\mathrm{s}}{\omega_0}+1}. \tag{10}$$

The impedance Z(s) formulas (10) are products of two transfer functions: the non-ideal first order differentiator (in nominator) with time constant $\tau_L = L/R$ and the second order inertia element with transfer function reciprocal to the denominator of (10). For $R < 2R_0$ (i.e. $\rho < 2$) second one is a attenuated oscillating element of the damping coefficient $\zeta = R/2R_0$. Then for $R \geq 2R_0$ its characteristic equation has two real roots:

$$\mathrm{s}_{1,2} = \omega_0\left[\frac{-R}{2R_0} \pm \sqrt{\left(\frac{R}{R_0}\right)^2 - 4}\right] = \omega_0\left[-\frac{\rho}{2} \pm \sqrt{\rho^2 - 4}\right] \tag{10a}$$

and transfer function is the product of two first degree members.

From (10), if s→jω the following complex impedance $Z(\mathrm{j}\omega)$ is obtained

$$\underline{Z}(\mathrm{j}\omega) = R\frac{1+\mathrm{j}\omega L/R}{(1-\omega^2 LC)+\mathrm{j}\omega RC} = R_s(\omega) + \mathrm{j}X_s(\omega) = Z(\omega)\cdot e^{\mathrm{j}\varphi(\omega)}, \tag{11}$$

where: Z - magnitude of impedance $\underline{Z}$, φ - the phase angle of $\underline{Z}$.

Magnitude of impedance

$$Z \equiv \left|\underline{Z}(\mathrm{j}\omega)\right| = R\frac{\sqrt{1+(\omega\tau_L)^2}}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}. \tag{12}$$

The phase angle

$$\varphi = \operatorname{arctg}\frac{\omega L_s}{R_s} = \operatorname{arctg}\tau \;, \qquad (13)$$

Where: time constant

$$\tau = L_s / R_s = \tau_L (1 - \omega^2/\omega_0^2) - \tau_C \,. \qquad (14)$$

The time constant τ can be positive or negative, depending on whether the predominant is its component with τ_L or τ_C. For $\omega^2/\omega_0^2 << 1$

$$\tau \approx \tau_L - \tau_C = (L/R) - RC \,. \qquad (14a)$$

For relative values of frequency $\eta \equiv \omega/\omega_0 = \omega\sqrt{CL}$ and related to R_0 the relative resistance $\rho \equiv R/R_0 = R\sqrt{C/L}$, from (10) it follows

$$\underline{Z}(\mathrm{j}\eta) = R\frac{1 + \mathrm{j}\eta/\rho}{1 - \eta^2 + \mathrm{j}\eta\rho} \,. \qquad (15)$$

Then after elementary transformations, the relative impedance $\underline{z}(\mathrm{j}\eta) \equiv \underline{Z}(\mathrm{j}\eta)/R$ and its rectangular components r_s x_s are [2]:

$$\underline{z}(\mathrm{j}\eta) = \frac{1 + \mathrm{j}\eta/\rho}{1 - \eta^2 + \mathrm{j}\eta\rho} \equiv r_s + \mathrm{j}x_s \,, \qquad (16)$$

$$r_s = \frac{1}{\left(1-\eta^2\right)^2 + \eta^2\rho^2} \,; \qquad x_s = \frac{\eta\left(\frac{1-\eta^2}{\rho} - \rho\right)}{\left(1-\eta^2\right)^2 + \eta^2\rho^2} \,. \qquad (17a,b)$$

Functions of $r_s(\eta)$ and $x_s(\eta)$ for three values of the relative resistance ρ are given in Figure 3.

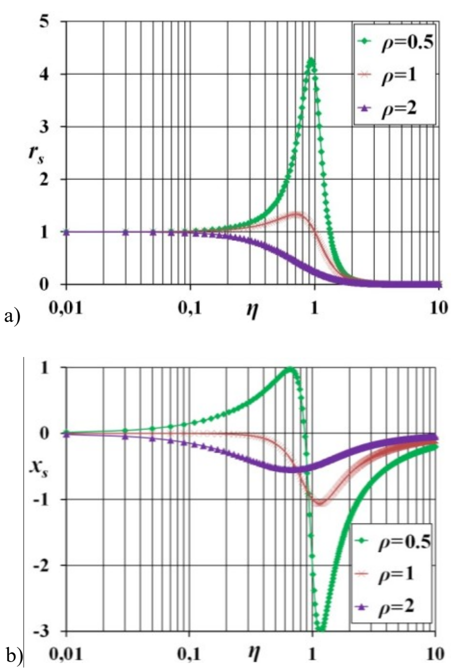

Fig. 3. Serial relative components of equations (17a,b) as function of relative frequency η for three values of $\rho = R/R_0$: a) -relative resistance r_s, b) -relative reactance x_s

As noticed before, the resistor is considered to be *good* if $r_s \approx 1$ and $x_s \approx 0$. Figure 3 shows that this can occur only for not too high relative frequency, i.e. $\eta < 0.2$ and when the relative resistance $\rho \approx 1$. For example if $\eta = 0.1$ and $\rho = 1$ from equations (17a,b) is $r_s = 1$ and $x_s = 0.0001$.

Resistor of $\rho = 1$, i.e. of the resistance $R = R_0$, is proposed to call as *compensated*.

The relative frequency errors of the circuit model of Figure 1a are resulting from definitions (7), (8) and described by patterns:

$$\delta R = r_s - 1 = -\frac{\eta^2\left(\rho^2 - 2\right) + \eta^4}{1 + \eta^2\left(\rho^2 - 2\right) + \eta^4} \,; \quad \delta X = x_s \,. \qquad (18a,b)$$

For $\eta^4 << 1$ i $\eta^4\rho^4 << 1$ simplified formulas can be used [3], i.e.:

$$\delta R \approx -\eta^2(2 - \rho^2) \,, \; x_s \approx \omega(\tau_L - \tau_C) = \eta(\rho^{-1} - \rho) \approx \delta Z \,, \qquad (19)$$

where: $\tau_L = L/R$, $\tau_C = RC$, $\eta^2 = \omega^2 LC = \omega^2\,\tau_L\,\tau_C$, $\rho^2 = R^2\,(\tau_C/\tau_L) = C/L$.

Further considerations concern on the scope of relatively low frequency, i.e. $\eta < 0.1$. Formulas (18a,b) of frequency error plots to η in the range 0.1 = 0.001 ... 0.1 and several values ρ are given in Figure 4 [2]. For the frequency range η = 0.001 ... 0.01 and relative resistance ρ = 0.1 ... 1 the resistance error δR (Fig. 4a) is very near to zero. When $\eta = 0.1$ the error δR is achieved approx. 2% for $\rho = 0.1$; +1%, for $\rho = 1$;–2% for $\rho = 2$; –20% for $\rho = 5$ and–50% for $\rho = 10$.

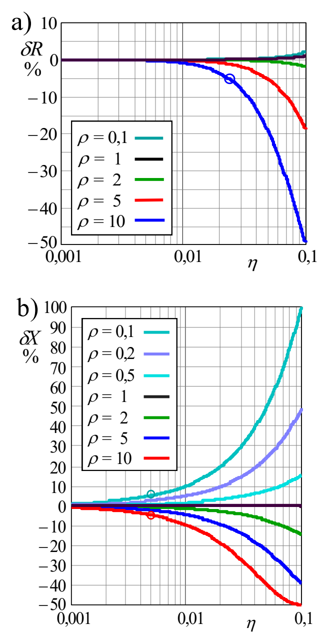

Fig. 4. Frequency errors of the resistor model Γ as function of relative frequency η for several resistance values relative ρ: a) relative resistive error δR, b) relative reactance error δX

From the Figure 4a, for a given value of ρ the upper limit of frequency η_{gr} can be estimated. It is the relative frequency, at which the relative error δR does not exceed the permissible value. For example, if relative resistance $\rho = 10$ (i.e. the resistance R is $10R_0$) and the limited error is 5%, then the upper relative frequency η_{gr} is approx. 0.023 – see point with the blue circular outline in Figure 4a.

Graphs in Figure 4b also allows you by the same way to assess the upper frequency η_{gx} for which the reactance error δX does not exceed its limited value. So for the permissible value 5% of the δX error, the upper relative frequency amounts to approx. 0.005 for $\rho = 0.1$ (point with a blue border), and for $\rho = 10$ (point with a red border). For intermediate values of ρ between 0.1 and 10 a relative frequency limits are higher. These graphs also show that for $x_s = 0$, a good approximation can be taken by equation(14a) rule that resistors with a resistance R smaller than the characteristic resistance R_0 (i. e. $\rho < 1$) are inductive type ($\delta X > 0$), and at the resistance R greater from R_0 ($\rho > 1$) - capacitive type ($\delta X < 0$). This can be exactly recognized from pattern (14) for $\tau = 0$ and was specified in [2].

Figures 4a and 4b also show that the requirement of upper frequency due to the permissible reactance error is higher than the for the maximum value of resistive error. Therefore, to apply in practice as a more convenient the relative impedance δZ is proposed – see the formula (9). This error is only slightly larger than the larger of errors δR or δX. Charts of δZ error in double logarithmic scale are given in Figure 5 [2].

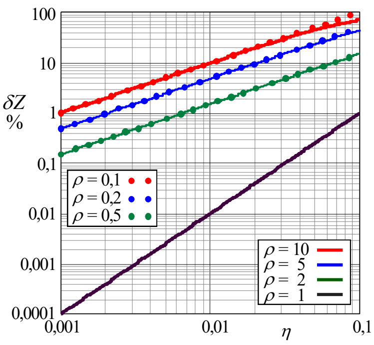

Fig. 5. Dependence of the relative impedance error δZ of resistor on the relative frequency η for several relative resistance values ρ

5. Assessment of residual parameters

A significant influence on the frequency characteristic of the resistor has also geometry of its connection with other elements of the electric circuit. It is assumed that these connections are two conductors of the diameter d, with the negligible resistance arranged parallely, as shown in Figure 6.

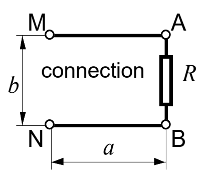

Fig. 6. Resistor with connections

The length of the connection is a, and the distance between axes of wires is b.

Inductance L_p and the capacity C_p of connections can be evaluated with the approximate formulas for a symmetric line of two diameter d wires of a length a and the interval between the wire's centers b. The appropriate formulas, reprinted from [4], are given in [2]. These formulas are valid for b and $d >> a >> b$. When the second condition is not fulfilled the obtained values L_p and C_p are undercounting, because they do not take into account the increase in inductance and capacitance dependent on the shape of the magnetic and electric field on the ends of the connection line. This increase takes into account by entering the additional coefficient $\psi > 1$ and then is obtained:

$$L_p = \psi \cdot \frac{\mu_0}{\pi} \cdot a \cdot \ln\frac{2 \cdot b}{d}; \quad C_p = \psi \cdot \pi \cdot \varepsilon_0 \cdot a \cdot (\ln\frac{2 \cdot b}{d})^{-1}. \quad (20)$$

We assume here that the connection is in the environment of magnetic and dielectric permeabilities of vacuum.

5.1. Example of calculation

Let us calculate parameters L_p and C_p of the connection with sizes of a=b=10 mm and d=1.0mm. Assuming for this so short line the relatively large $\psi \approx 1.5$ after [4], [5], obtained is: $L_p \approx 0.018$ µH and $C_p \approx 0.14$ pF. If the values of L and C of the model Γ of real resistor mainly depend on the inductance L_p and capacitance C_p of this connection, then from relation (1) the frequency $f_0 \approx 3.2$ GHz and from (2) – the resistance $R_0 \approx 360\ \Omega$ are obtained.

From formulas (12) it is clear that the m-time change of the geometric size of the connection give the m-time increase of the inductance and capacitance values. Thus, from the equation (1), that will be the m-fold decrease in frequency of ω_0. E.g. if leads sizes are $a = b = 100$ mm and $d = 10$ mm then the two terminal circuit would have a natural frequency $f_0 \approx 20$ MHz. For connections with sizes of $a = b = 1.0$ mm and $d = 0.10$ mm if $f_0 \approx 32$ GHz. This qualitatively explains why electronic systems due to the miniaturization of sizes achieved the higher operating frequencies. Additionally, from formula (2) it is seen that m-fold change of the resistor sizes does not changes the characteristic resistance R_0.

6. Other models of resistor with connections

The two wire connection of resistor (Fig. 7) has an inductance L_p and the capacitance C_p and can be considered as a lossless long line. If the frequency is so small that the electromagnetic wave length is far greater than the length of a connection, this can be modeled by two-port type **Π** or type **T** [2] as shown in Figure 7.

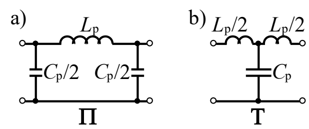

Fig. 7. Models of connections: a) type **Π**, b) type **T**

Connection of leads models type **Π** or **T** given in Figure 7 and model Γ of resistor from Figure 1a with residual parameters L_r, C_r gives models with 5-five parameters L_p, C_p, L_r, C_r, R. Separate determination the values of L_p, L_r and C_p, C_r is very difficult in practice, and such models would not convenient to use. Four simpler models given in Figure 8 with the resultant parameters L, C as sums $L = L_p + L_r$; $C = C_p + C_r$ are therefore applied. We shall show that for the relative frequency $\eta < 0.1$, these models are roughly equivalent each other.

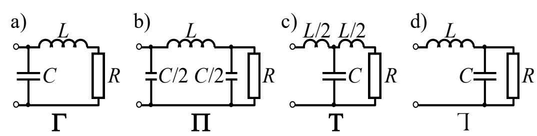

Fig. 8. The equivalent 3-parameter circuits as models of the resistor with connection leads: a) the type **Γ** (gamma as in Fig. 1a); b) **Π** type, c) **T** type, d) ꓶ type (reversed gamma)

Frequency characteristics of the relative resistance $r_s(\eta)$ and the relative reactance $x_s(\eta)$ of two terminal models of Figure 8 differ from each other, but significant differences exist only for higher values of the relative frequency approaching $\eta = 1$.

The relative values of resistance r_s and reactance x_s of type Γ model are represented by the formulas (17a,b). For a model ꓶ these formulas are different [2]:

$$r_s = \frac{1}{1+\eta^2 \cdot \rho^2}; \qquad x_s = \frac{\eta}{\rho} \cdot \frac{1-\rho^2 \cdot \left(1-\eta^2\right)}{1+\eta^2 \cdot \rho^2}. \qquad (21a,b)$$

To obtain the similar formulas for **Π** and **T** models is quite cumbersome. More easy is find the pattern of relative complex impedance $\underline{z} = \underline{Z}/R$ and by the application of appropriate mathematical formulas from $\underline{z}$ numerically calculate $r_s = \mathrm{Re}\,\underline{z}$ and $x_s = \mathrm{Im}\,\underline{z}$. For relative frequencies $\eta < 0.1$, and the relative resistances of $0.1 < \rho < 10$ models **Γ** and **⅂** have very similar values of impedance error δZ. They are slightly larger than for the models **Π** and **T** - see Figures 9 and 10 [2].

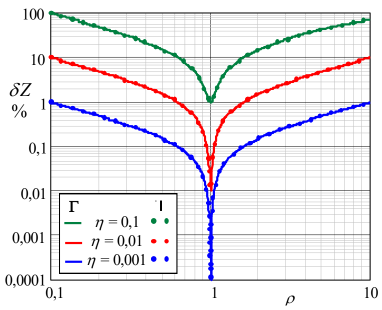

Fig. 9. The impedance error δZ of the resistor models **Γ** and **⅂** dependence on η and ρ

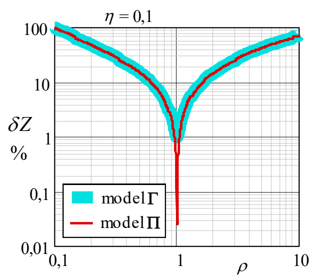

Fig. 10. The impedance error δZ of the resistor models **Γ** and **Π** dependence on η and on ρ

The small differences in the impedance error δZ between different models for the relative resistance value ρ around 1 is demonstrated by drawings in Figure 11.

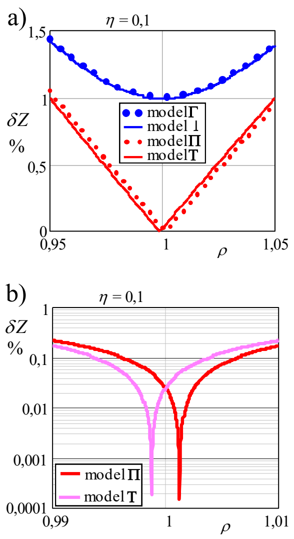

Fig. 11. The impedance error δZ of the resistor dependence on the relative resistance ρ (around ρ =1) for models shown in Figure 8: a) r = 0.95 … 1.05, b) r = 0.99 … 1.01

Such small differences of δZ however, have no significance in practice. It is rather a theoretical curiosity. Possible to keep the ρ $=R/R_0$, even to the extent of ±1% is very difficult or impossible, since the value R_0 depends on changes of L and C of the resistor with connection and leakages.

There is no possibility of deciding in practice which model of Figure 8 is the most appropriate in particular situation. This leads to choice the model of the simplest formulas to calculations. For the estimation of the limited values of the frequency error δZ of the resistor impedance, the model **⅂** seems to be the most convenient. The formula for this model has simple form:

$$\delta Z(\eta,\rho) = \eta\,\frac{\sqrt{\eta^2\rho^4 + \left[\rho^{-1} - \rho\left(1-\eta^2\right)\right]^2}}{1+\eta^2\rho^2}. \qquad (22)$$

The possibility of correction the frequency characteristics of resistors used in practice are discussed in the part 2 of work [2]. The correction of the residual reactance $X_r > 0$ by small C can be only successfully implemented, e.g. for standard resistors or resistance sensors used in AC circuits.

7. Accuracy of resistor in frequency band

The foregoing considerations, including the drawings 3 - 5 show that for the relative frequency $\eta < 0.1$ (i.e. at a frequency f to approx. 0.1 of f_0), the equivalent resistance R_s of the real resistor (Fig. 1b) is equal to the resistance value R for direct current. If the relative resistance $\rho \approx 1$, i.e. when the resistor is compensated, in a wide frequency band is $R_s \approx R = R_0$.

Dependence of resistance R_0 characteristic on resultant residual parameters - the capacity C and the inductance L on the ends of the resistor together with connections are defined by the formula (1). Even if the values of L and C resulting from the construction of resistor are constant, other may be parameters of connection in each of the system. Furthermore, the L, C and R_0 values are known with the low accuracy. Only the value R can be measured accurately.

Specific values of L and C can be taken as the nominal inductance and the nominal capacity of the resistor. Calculated according to the formula (1) of the value of the frequency ω_0 - is the nominal own frequency of the resistor. Resistor with resistance $R = R_0$ defined by the formula (2) is the nominally compensated resistor. To describe changes of parameters of the model of resistor with a connection the concept of multiplier $\boldsymbol{\kappa}$ as a number greater than 1 ($\boldsymbol{\kappa} > 1$) is proposed. To describe changes of parameters of the model of resistor with a connection, the concept of multiplier $\boldsymbol{\kappa} > 1$ can be introduced. The values of C and L then will be in ranges:

- capacity: between the bottom value of C_d κ-fold less than the nominal value C, up to a upper value C_g κ-fold greater than C;
- inductance: between the lower value L_d κ times less than the nominal value L up to a upper value L_g which is κ L. It is described as follows:

$$C_d = \frac{C}{\kappa};\; C_g = C \cdot \kappa;\; L_d = \frac{L}{\kappa};\; L_g = L \cdot \kappa;\, \text{if } \kappa > 1. \quad (23)$$

Table 1 summarizes the four cases when the capacitance and inductance take the extreme (lower or upper) values.

In cases given in the Table 1, values of relative frequency and relative resistance should be rescaled respectively as given below.

Tab. 1. Relative frequency and relative resistance in the four situations of extreme values of inductance and capacitance

Case	Relative frequency η	Relative resistance ρ
I. C_d, L_d	$\eta_I = \omega \cdot \sqrt{C_d \cdot L_d} = \frac{\eta}{\kappa}$	$\rho_I = R \cdot \sqrt{\frac{C_d}{L_d}} = \rho$
II. C_d, L_g	$\eta_{II} = \omega \cdot \sqrt{C_d \cdot L_g} = \eta$	$\rho_{II} = R \cdot \sqrt{\frac{C_d}{L_g}} = \frac{\rho}{\kappa}$
III. C_g, L_d	$\eta_{III} = \omega \cdot \sqrt{C_g \cdot L_d} = \eta$	$\rho_{III} = R \cdot \sqrt{\frac{C_g}{L_d}} = \rho \cdot \kappa$
IV. C_g, L_g	$\eta_{IV} = \omega \cdot \sqrt{C_g \cdot L_g} = \eta \cdot \kappa$	$\rho_{IV} = R \cdot \sqrt{\frac{C_g}{L_g}} = \rho$

- Case I: Relative frequency η_I decreases κ-fold, and the relative resistance ρ_I remains unchanged. Frequency characteristics shown in Figures 3 – 5 require only sharing relative frequency by $\kappa\eta$. In a logarithmic scale on the abscissa, it means the shift of graphs by logκ to the left.
- Case II: frequency relative η_{II} does not change, and the resistance decreases relative ρ_{II} κ times. That mean that characteristics run for the κ times smaller values of resistance ρ.
- Case III: Relative frequency η_{III} remains unchanged, and the relative resistance ρ_{III} growing κ times. This means that the frequency response must be calculated for κ times larger values of resistance ρ.
- Case IV: The relative frequency η_{IV} increases κ-fold and the relative resistance ρ remains unchanged. This means that the frequency characteristics shown in Fig. 3 - 5 require multiplying the relative frequency η by κ. In a logarithmic scale on the abscissa, this corresponds to a shift of plots to right on logκ.

Measures of the resistance R inaccuracy for alternating current are its relative frequency errors, i.e. an resistance error δR - formulas (5) and (22), the reactance error δX - formulas (6) and (22) and the impedance error δZ - patterns (7) and (22).

In cases II and III the frequency axis rescaling is not required. As the result of derogations of capacitance C and inductance L from their nominal values is the inaccuracy of resistance R, which depends on changes of the relative resistance ρ value. According to Table 1, the resistance ρ of the nominally compensated resistor can take the extreme values of $1/\kappa$ and κ. Figure 12 shows the changes of the impedance error δZ of resistor as function of relative frequency η for four values of multiplier κ.

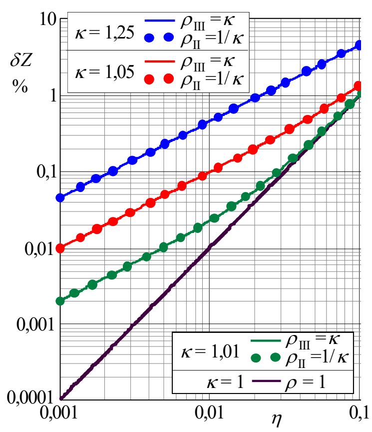

Fig. 12. The impedance error δZ of nominally compensated resistor (ρ=1) as function of the relative frequency η for four values of multiplier κ

When changing η from 0.001 to 0.1 then error δZ varies in four given below ranges.

- From approx.1×10^{-4}% to approx. 1% at κ = 1. That is the perfectly compensated real resistor;
- From approx. 2×10^{-3}% to approx. 1% at κ = 1.01. That is resistor of L and C deviations from their nominal values within the range from –1% to 1%;
- From approx. 0.01% to approx. 1.4% at κ = 1.05. That is resistor of deviations of L and C are in the range of approx. 4.8% to 5%,
- From approx. 0.045% to approx. 4.5% at κ = 1.25, that is resistor of L and C the deviations in the range of –20% to 25%.

From these considerations is resulted that the best accuracy of the pure resistance R at higher frequencies has the compensated resistor, i.e. satisfying the relationship (16). However, even a small deviation determined by multiplier κ, causes a considerable error. For example, for κ = 1.05, i.e. for deviations of the capacity C and the inductance L from their nominal values of approx. 5% the impedance error δZ increases from approx. 0.0001% to approx. 0.01% with relative frequency η = 0.001, and from approx. 1% to approx. 1.4% at η = 0.1. Maintaining the C and L values in the range of 5% is not easy in practice. More realistic is the range of –20% to +25% (κ = 1.25). Then the impedance error δZ may be in the range from 0.05% for η = 0.001 up to ~ 5% for η = 0.1.

Anticipating the distribution $p(\delta Z)$ in a given range of the relative frequency η, the uncertainty component B-type of the resistor with connection can be also estimated. Randomness is mainly coming from random changes of parasitic parameters C and L, and frequency.

8. Summary

In this work, the generalized description of the frequency characteristics of resistors not encountered in the literature is presented. These characteristics are expressed in numbers of similarity that is in relative terms, which is not meet in literature. Such description can be used for the frequency analysis of the equivalent schemes of different electrical devices. As examples from measurement instrumentation are resistors, including wounded [3] and performed by different technologies [1], [6], [9], and also models of other passive objects of the dominant parameter L or C etc.

Results include as a special case the frequency characteristics of real resistors currently manufactured [1], [9].

Generalized description in relative terms can be applied also for standard platinum temperature sensors (Eng. acronym SPRT) in the analysis of the accuracy of temperature measurements between control points with application of the highest precision AC bridges [7].

If the model Γ is used for impedances at higher frequencies, an additional parallel R C branch extends it. Analysis and application of such model in the impedance spectrometry was presented at the last IMEKO 2015 Congress in Prague [8].

9. References

[1] www.vishay.com/company/press/releases/2016/160216WSK1206/
[2] Kubisa S., Warsza Z. L.: Analiza błędów częstotliwościowych rezystorów. Część 1 i 2. (Analysis of frequency errors of resistors. Part 1 and 2). Przegląd Elektrotechniczny - ISSN 0033-2097 vol. 92 (2016) Part 1 no 6 pp. 211-216, Part 2 no.7 pp.211 -216, (in Polish.
[3] Dudziewicz J. (editor) and others: Etalony i precyzyjne pomiary wielkości elektrycznych (Etalons and precise measurement of electrical quantities). Chapter 8.2: Standards of resistance. WKiŁ, Warsaw (1982), pp. 207 - 227, (in Polish).
[4] Kurdziel R.: Elektrotechnika (Electro-technology) PWN Warsaw, ed. 2 (1973), (in Polish).

[5] Wikipedia: https://pl.wikipedia.org/wiki/Indukcyjność (in Polish), and pages in English about inductance.

[6] Korytkowski Jacek: Układy przetworników cyfrowo-analogowych napięcia, prądu i rezystancji oraz metoda ich analizy (Digital-to-analog voltage, current and resistance converter circuits and method of their analysis). Monographs, Studies and Thesis. PIAP (Industrial Research Institute for Automation and Measurements) Warsaw (2004), Chapter 5.3 Accurate resistors. pp.108 -119 (in Polish).

[7] Mikhal A. A., Warsza Z. L.: Electromagnetic Protection in High Precision Tri-axial AC Thermometric Bridges. Progress in Automation Robotics and Measuring Techniques. (Editors: R. Szewczyk, C. Zielinski, M. Kaliczyńska), vol. 3 "Measuring Techniques and Systems". (ISBN 978-3-319-15834-1), vol. 352 of series "Advances in Intelligent Systems and Computing" (ISSN 2194-5357), Springer (2015) pp.147 -156.

[8] Baltianski S.: Impedance Spectroscopy: Separation and Asymptotic Model Interpretation. Proc. of XXI IMEKO World Congress "Measurement in Research and Industry" 30 August - 4 Sept., (2015), Prague, Czech Republic, 493 - 497.

[9] Pieńkowski D.: Impact of parasitic reactance on the application parameters of resistors and capacitors. http://elektronika.2b.pl/technika /1236 (Edition on line) 01. 15. (2007).

Received: 02.12.2015 *Paper reviewed* *Accepted: 03.02.2016*

Prof. Stefan KUBISA, DSc, eng.

Graduated on the Faculty of Electrical Engineering at Technical University of Szczecin. Since 1963 teaching at the Faculty; since 2001 full professor; in 2012 retired. In 1990 he became a member of the Committee of Metrology and Scientific Instrumentation of Polish Academy of Sciences, and in the years 2001-11 as chair of the Section organized a series of seminars and symposia on the measurement uncertainty. Research interests: processing algorithms in the power, energy and in measurement of mechanical quantities.

e-mail: dskubisa@hotmail.com

Zygmunt L.WARSZA, Assoc. Prof., PhD, EE

Graduated: at Warsaw Technical University: MSc of EE 1959; PhD on Eectrical Measurements 1967; and Assoc. Prof. (docent) from 1970. Work: at 3 Technical Universities: 1960–70 Warsaw TU; 1970-78 Kielce-Radom TU- organizer and Dean of Transport Eng. Faculty; 1983-2002 Radom TU; - at 4 industrial research Institutes: 1958-63, 1978 -92, and adviser to Ministry of Education 1992 -94. Now: at Industrial Research Institute of Automation and Measurement PIAP Warsaw. President of Polish Society of Metrology (PTM).

e-mail: zlw@op.pl