

## Global non-probabilistic reliability sensitivity analysis based on surrogate model

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### Highlights

- Global non-probabilistic sensitivity analysis method is proposed.
- The proposed method considers main effect of interval variables.
- The proposed method is easy to use and does not require probability distribution.

### Abstract

Sensitivity analysis is used to find the key variables which have significant effect on system reliability. For a product in early design stage, it is impossible to collect sufficient samples. Thus, the probabilistic-based reliability sensitivity analysis methods are difficult to use due to the requirement of probability distribution. As an alternative, interval can be used because it only requires few samples. In this study, an effective global non-probabilistic sensitivity analysis based on adaptive Kriging model is proposed. The global accuracy Kriging model is constructed to reduce overall computational cost. Subsequently, the global non-probabilistic sensitivity analysis method is developed. Compared to existing non-probabilistic sensitivity analysis methods, the proposed method is a global non-probabilistic reliability sensitivity analysis method. The proposed method is easy to use and does not require probability distribution of the input variables. The applicability of proposed method is demonstrated via two examples.

### Keywords

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structural reliability; non-probabilistic reliability sensitivity; adaptive Kriging; surrogate models; interval variables.

### 1. Introduction

In reliability engineering, sensitivity analysis is widely used to find the key variables that have significant effect on system reliability [1, 13]. In general, reliability sensitivity can be classified into two types [11, 17]: local reliability sensitivity and global reliability sensitivity.

Nowadays, probability distribution is often used to represent random parameter, and most of probabilistic-based reliability sensitivity analysis methods have been reported. For example, Proppe [14] presented a local reliability-based sensitivity analysis based on moving particles method. Cadini et al. [1] proposed an adaptive Kriging importance sampling-based method for global sensitivity analysis. Dubourg and Sudret [2] proposed Kriging model-based importance sampling for reliability sensitivity analysis, which can be used in reliability-based design optimization. It should be noted that many samples are required to accurately determine a probability distribution. However, it is impossible to collect enough samples for a product in early design stage. As an alternative, interval or convex model can be used because they only require a few samples, which are useful for a product in early design state. If input variable is represented

using interval or convex model, the sensitivity problem is called as non-probabilistic sensitivity in this study. Until now, a few research efforts for non-probabilistic sensitivity analysis have been reported. For example, Li et al. [9] presented the definition of non-probabilistic sensitivity analysis, and optimization-based method is suggested to solve complex non-probabilistic sensitivity problems. Xiao et al. [19] proposed a non-probabilistic sensitivity analysis method under considering correlations among interval variables. Qiao et al. [15] proposed a non-probabilistic reliability sensitivity analysis method based on convex model. Wang et al. [18] used non-probabilistic sensitivity analysis for optimization of aeronautical hydraulic pipelines.

It should be noted that existing non-probabilistic reliability sensitivity methods are, generally, local reliability sensitivity methods. Moreover, performance functions in real applications are typically implicit functions involving time-consuming simulations. Subsequently, non-probabilistic reliability sensitivity analysis involving simulation is extremely computationally expensive. To address these issues, a global non-probabilistic reliability sensitivity analysis based on adaptive Kriging model is proposed in this study. Kriging models have been widely used in reliability engineering in recent years [3, 12, 20]

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[21, 22]. Therefore, to significantly reduce computational burden for systems with time-consuming simulations, the global accuracy Kriging model is constructed adaptively that can be used to replace simulations. Subsequently, a global non-probabilistic reliability sensitivity analysis method is developed based on Sobol's sensitivity indices [16] for systems with interval variables.

This paper is structured as follows. Section 2 reviews of existing non-probabilistic reliability analysis. Section 3 presents a global non-probabilistic sensitivity analysis method in detail. Two numerical examples are used to demonstrate the applicability of proposed method in section 4. Section 5 is the conclusion.

## 2. Review of existing non-probabilistic reliability analysis

### 2.1. Non-probabilistic reliability analysis

Interval variable can be used to represent random parameter. An interval variable is expressed as:

$$X^I = [X^l, X^u] = \{X | X^l \leq X \leq X^u\} \quad (1)$$

where  $X^l$  and  $X^u$  are, respectively, the lower and upper bounds of interval variable. The midpoint and radius of  $X^I$  can be, respectively, calculated as follows:

$$\bar{X} = \frac{X^u + X^l}{2}, X^r = \frac{X^u - X^l}{2} \quad (2)$$

Suppose that the performance function of a system is  $Z = g(\mathbf{X}^I)$  with interval variables  $\mathbf{X}^I = (X_1^I, X_2^I, \dots, X_n^I)$ , then the system response  $Z$  is also an interval variable. The non-probabilistic reliability index is defined as follows [8]:

$$\eta = \frac{\bar{Z}}{Z^r} \quad (3)$$

Based on Eq. (3), it shows that if  $\eta > 1$ , the system is safe; if  $\eta < -1$ , the system is failure;  $-1 \leq \eta \leq 1$  means that system is in uncertain state. Eq. (3) can be rewritten as:

$$\eta = \frac{(Z^u + Z^l)}{(Z^u - Z^l)} \quad (4)$$

where  $Z^u, Z^l$  are, respectively, the lower and upper bounds of system response. To accurately calculate  $Z^u, Z^l$ , the following optimization model can be solved:

$$\begin{cases} Z^l/Z^u = \min/\max g(\mathbf{X}) \\ s.t. \\ \mathbf{X}^l \leq \mathbf{X} \leq \mathbf{X}^u \end{cases} \quad (5)$$

In general, for a system with multiple components, the system non-probabilistic reliability index can be computed as :

$$\eta_{sys} = \begin{cases} \min(\eta_1, \eta_2, \dots, \eta_m), \text{ for a series system} \\ \max(\eta_1, \eta_2, \dots, \eta_m), \text{ for a parallel system} \end{cases} \quad (6)$$

where  $\eta_i$  is non-probabilistic reliability index of the  $i$ th component.

### 2.2. Non-probabilistic sensitivity reliability analysis

Reliability sensitivity analysis is useful because it can be used to find the key variable that has significant effect on system reliability. Traditionally, non-probabilistic reliability sensitivity is defined as follows [9]:

$$\left( \frac{\partial \eta}{\partial \bar{X}}, \frac{\partial \eta}{\partial X^r} \right) \quad (7)$$

From Eq. (7), it is easy to know that existing non-probabilistic reliability sensitivity is a local sensitivity measure. Note that there is no direct relationship between non-probability reliability index  $\eta$  and interval parameters  $\bar{X}$  and  $X^r$ . Thus, analytical solution for the non-probabilistic reliability sensitivity is, generally, impossible, except for some special cases such as performance function  $Z = g(\mathbf{X}^I)$  is a linear function. To approximately estimate non-probabilistic reliability sensitivity in Eq. (7), the finite difference technique can be used as:

$$\begin{cases} \frac{\partial \eta}{\partial \bar{X}} = \frac{\eta(\bar{X} + \Delta \bar{X}) - \eta(\bar{X})}{\Delta \bar{X}} \\ \frac{\partial \eta}{\partial X^r} = \frac{\eta(X^r + \Delta X^r) - \eta(X^r)}{\Delta X^r} \end{cases} \quad (8)$$

where  $\Delta \bar{X}$  and  $\Delta X^r$  are very small variations of interval midpoint and radius, respectively. Note that for system non-probabilistic reliability sensitivity problem, Eq. (7) can be revised as follows:

$$\left( \frac{\partial \eta_{sys}}{\partial \bar{X}}, \frac{\partial \eta_{sys}}{\partial X^r} \right) \quad (9)$$

Using the finite difference technique, Eq. (9) can be calculated as follows:

$$\begin{cases} \frac{\partial \eta_{sys}}{\partial \bar{X}} = \frac{\eta_{sys}(\bar{X} + \Delta \bar{X}) - \eta_{sys}(\bar{X})}{\Delta \bar{X}} \\ \frac{\partial \eta_{sys}}{\partial X^r} = \frac{\eta_{sys}(X^r + \Delta X^r) - \eta_{sys}(X^r)}{\Delta X^r} \end{cases} \quad (10)$$

## 3. Proposed global non-probabilistic sensitivity analysis method

### 3.1. Global non-probabilistic sensitivity analysis

Since non-probabilistic reliability sensitivity in Eq. (7) is a local sensitivity measure, a global non-probabilistic reliability sensitivity analysis method is proposed in this study. Based on Sobol's indices [16], the proposed global non-probabilistic reliability sensitivity measure for a component is defined as:

$$S_{X_j^I} = \frac{\text{Var}_{X_j^I}(\eta|_{x_j})}{\sum_{i=1}^n \text{Var}_{X_i^I}(\eta|_{x_i})} \quad (11)$$

where  $\text{Var}_{X_j^I}(\eta|_{x_j})$  is computed as follows:

$$\text{Var}_{X_j^I}(\eta|_{x_j}) = \frac{1}{N} \sum_{k=1}^N (\eta_{x_j^k} - \mu)^2 \quad (12)$$

where  $\eta_{x_j^k}$  is a non-probabilistic index under interval variable  $X_j^I = x_j^k$ , and  $\mu = \frac{1}{N} \sum_{k=1}^N \eta_{x_j^k}$ . In this study, the range of interval variable  $X_j^I$  is evenly divided into  $N-1$  sub-intervals. Subsequently, the  $N$  samples can be determined, i.e.,  $x_j^1 = x_j^1 \leq x_j^2 \leq \dots \leq x_j^{N-1} \leq x_j^N = x_j^u$ , where  $x_j^k$  is the  $k$ th sample. To ensure the accuracy, the  $N$  is suggested as  $N \geq 50$ . From Eq. (11), it is easy to know that the value of  $S_{X_j^I}$  belongs to interval  $[0,1]$ . It should be noted that the non-probabilistic reliability index under configuration of interval variable, i.e.,  $\eta|_{x_j^k}$ , is not a number in some special cases. Subsequently,  $\eta|_{x_j^k}$  should be ignored to calculate global component non-probabilistic reliability sensitivity.

Based on Eqs. (6) and (11), global non-probabilistic reliability sensitivity of a system can be calculated as follows:

$$S_{X_j^I} = \frac{\text{Var}_{X_j^I}(\eta_{\text{sys}}|_{x_j})}{\sum_{i=1}^n \text{Var}(\eta_{\text{sys}}|_{x_i})} \quad (13)$$

### 3.2. Construct global accuracy surrogate model based on Kriging

In real applications, the system performance functions  $\{g_1, g_2, \dots, g_m\}$  may implicit functions involving time-consuming simulations. Using Eqs. (11) and (13) for calculating global non-probabilistic reliability sensitivity is extremely computationally expensive. For example, using Eq. (11) for calculating component global non-probabilistic reliability sensitivity. Suppose that the average number of simulations for calculating  $\eta_{x_j^k}$  is  $C$ , then the total number of simulations for global non-probabilistic reliability sensitivity analysis of all input variables is  $C \times N \times n$ . It is easy to know that the computational burden is extremely huge which is impossible in real applications. To reduce computational burden, a global accuracy Kriging model in whole uncertainty space is constructed to replace time-consuming simulation. Kriging is a Gaussian process[10], for an unobserved point  $\mathbf{x}$ , the kriging prediction is a normal random variable with mean value  $\mu_{\tilde{g}}(\mathbf{x})$  and Kriging variance  $\sigma_{\tilde{g}}^2(\mathbf{x})$  as follows:

$$\tilde{g}(\mathbf{x}) \sim \mathbb{N} \left[ \mu_{\tilde{g}}(\mathbf{x}), \sigma_{\tilde{g}}^2(\mathbf{x}) \right] \quad (14)$$

For more detailed information of Kriging, please see Ref. [4]. To effectively construct a Kriging model with global accuracy, the best added training sample can be determined as follows[7]:

$$\mathbf{x}^* = \arg \max_{X^I \leq \mathbf{x} \leq X^u} \sigma_{\tilde{g}}^2(\mathbf{x}) \quad (15)$$

From Eq. (15), finding  $\mathbf{x}^*$  is an optimization problem within the whole uncertainty space. Existing intelligent optimization algorithms such as genetic algorithm can be used to solve the problem, which is complex. In this study, a large number of candidate

samples are randomly generated in whole uncertainty space, i.e.,  $\{\mathbf{x}_c\} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N_c}\}$ . Subsequently,  $\sigma_{\tilde{g}}^2(\{\mathbf{x}_c\})$  are available based on current Kriging model. The  $\mathbf{x}^*$  can be approximately determined as follows:

$$\mathbf{x}^* = \arg \max \sigma_{\tilde{g}}^2(\{\mathbf{x}_c\}) \quad (16)$$

To terminate the process of selecting training samples, the following stopping strategy is adopted:

$$\text{Rmse}(\mathbf{x}_{\text{test}}) \leq \Delta \xi \quad (17)$$

where  $\text{Rmse}(\mathbf{x}_{\text{test}})$  is the root mean square error,  $\mathbf{x}_{\text{test}}$  are randomly generated test samples with the number of  $10 \times n$ , and  $\Delta \xi$  is defined as:

$$\Delta \xi = \lambda \times |E[g(\mathbf{x}_{\text{test}})]| \quad (18)$$

where  $|\cdot|$  denotes absolute operator,  $E(\cdot)$  denotes expectation, and  $\lambda$  is a small positive number such as  $\lambda = 0.0001$ .  $\text{Rmse}(\mathbf{x}_{\text{test}})$  is calculated as:

$$\text{Rmse}(\mathbf{x}_{\text{test}}) = \sqrt{\frac{\sum_{i=1}^{n_{\text{test}}} [g(\mathbf{x}_{\text{test}}^i) - \tilde{g}(\mathbf{x}_{\text{test}}^i)]^2}{n_{\text{test}}}} \quad (19)$$

where  $\mathbf{x}_{\text{test}}^i$  is the  $i$ th test sample,  $n_{\text{test}} = 10 \times n$  is the number of test samples. When the stopping strategy is met, the final surrogate models  $\{\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_m\}$  are obtained for system performance functions  $\{g_1, g_2, \dots, g_m\}$ .

### 3.3. Summary of proposed method

Compared to existing non-probabilistic sensitivity analysis methods, the proposed method provides a new way for global non-probabilistic sensitivity analysis. The details of proposed method for system global non-probabilistic reliability sensitivity analysis can be summarized in Table 1.

Table 1. Algorithm of proposed method

1. Generate a small number of training samples  $\{\mathbf{x}^s, \mathbf{z}^s\}$ , where  $\mathbf{x}^s = (\mathbf{x}_1^s, \mathbf{x}_1^s, \dots, \mathbf{x}_n^s)$  and  $\mathbf{z}^s = (\mathbf{z}_1^s, \mathbf{z}_2^s, \dots, \mathbf{z}_m^s)$ , and build initial surrogate models.
2. Select training sample using Eq. (16) to refine each surrogate model until stopping strategy in Eq. (17) is met.
3. Global accuracy surrogate models are denoted as  $\tilde{\mathbf{g}} = (\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_m)$ .
4. **for**  $j=1$  to  $n$ , **do**

$X_j^I$  is evenly divided into  $N-1$  sub-intervals, and  $N$  samples can be determined as  $x_j^1 = x_j^1 \leq x_j^2 \leq \dots \leq x_j^{N-1} \leq x_j^N = x_j^u$ .

**for**  $k=1$  to  $N$ , **do**

Calculate non-probabilistic index  $\eta_{\text{sys}, x_j^k}$  under interval variable  $X_j^I = x_j^k$

**end for**

Calculate global non-probabilistic reliability sensitivity  $S_{X_j^I}$  using Eq. (13).

**end for**

5. Sort global non-probabilistic reliability sensitivities

$$\{S_{X_1^I}, S_{X_2^I}, \dots, S_{X_n^I}\} \text{ in descending order.}$$

#### 4. Numerical examples

In this section, two numerical examples are investigated to show the proposed method. The first is a beam with a single failure mode and five interval variables. The second is a parallel system with two highly nonlinear performance functions. To demonstrate the proposed method, all performance functions are viewed as implicit functions to construct surrogate models.

##### Example 1—a cantilever with single failure model

A cantilever, as shown in Fig. 1., is considered. The performance function is defined as [9]:

$$g(m_{cr}, p_1, p_2, b_1, b_2) = m_{cr} - p_1 b_1 - p_2 b_2$$

where  $m_{cr}$  is critical limit bending moment,  $p_1$  and  $p_2$  are two applied loads,  $b_1$  and  $b_2$  are the length between applied loads and end point. All parameters are interval variables and the detailed information is shown in Table 2.

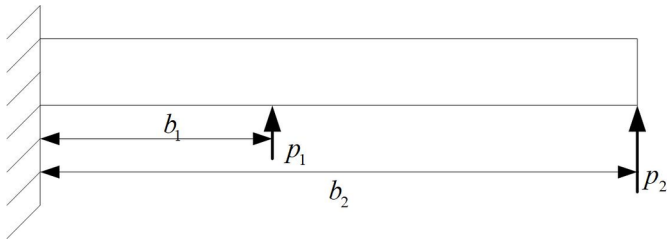


Fig. 1 A cantilever

In this example, 10 samples are used to build initial surrogate model,  $\lambda = 0.0001$ , and the number of test samples is 50. Moreover, 57 training samples are used to construct a global accuracy Kriging model. For global non-probabilistic reliability sensitivity analysis, each interval is evenly divided into 100 subintervals, respectively. The results of global non-probabilistic reliability sensitivity from proposed method are shown in Table 3.

In Table 3, the results with “\*” are from the proposed method with true performance function. Based on Table 3, the results are very accurate compared to reference values based on real performance function. The proposed method provides a new way to measure global non-probabilistic reliability sensitivity. It is easy to know that the vari-

Table 2. Detailed information of interval variables

Interval variables	$p_1$ (kN)	$p_2$ (kN)	$b_1$ (m)	$b_2$ (m)	$m_{cr}$ (kN.m)
Lower bound	4.4	1.7	1.8	4.5	32
Upper bound	5.6	2.3	2.2	5.5	40

Table 3. Global non-probabilistic sensitivities of example 1

Sensitivity	$S_{m_{cr}}$	$S_{p_1}$	$S_{b_1}$	$S_{p_2}$	$S_{b_2}$
Proposed method	0.8220	0.0423	0.0300	0.0732	0.0325
	0.8214*	0.0426*	0.0298*	0.0734*	0.0328*

Table 4. Global non-probabilistic sensitivities of example 2

Sensitivity	$S_{X_1}$	$S_{X_2}$	$S_{X_1}$	$S_{X_2}$
Proposed method	0.9537	0.0463	0.9541*	0.0459*

able  $m_{cr}$  has a significant effect on system reliability, which should pay more attention in design stage. Note that the proposed method is quite different to local non-probabilistic reliability sensitivity method. The local method provides local sensitivity of midpoint and radius of interval variable, whereas the proposed method provides a global non-probabilistic reliability sensitivity of interval variable in whole uncertainty space. In this example, the total number of original function call is  $57+50=107$ .

##### Example 2—a parallel system with two failure modes

Suppose that a parallel system with two failure modes, and the corresponding performance functions are defined as follows:

$$\begin{cases} g_1(X_1, X_2) = (X_1 + 2)^2 - X_2 - 2 \\ g_2(X_1, X_2) = (X_1 - 3)^2 - 2X_1X_2 + 4 \end{cases}$$

$X_1, X_2$  are two independent interval variables,  $X_1 \in [0.5, 1]$ , and  $X_2 \in [1, 2]$ .

In this example, 10 samples are used to build initial surrogate models for  $g_1$  and  $g_2$ , respectively.  $\lambda = 0.0001$ , and the number of test samples for both is 20. To construct global accuracy Kriging models for  $g_1$  and  $g_2$ , the number of training samples are 14 and 13, respectively. For system global non-probabilistic reliability sensitivity analysis, each interval is evenly divided into 100 subintervals, respectively. The results from the proposed method are shown in Table 4.

In Table 4, the results with “\*” are from the proposed method with true performance functions. Based on Table 4, the results are very accurate compared to reference values based on real performance functions. The proposed method provides a new way to measure system global non-probabilistic reliability sensitivity. It is easy to know that the variable  $X_1$  has a significant effect on system reliability, which should pay more attention to control it in design stage. In this example, the total number of original function calls are 34 (14+20) and 33 (13+20), respectively.

#### 5. Conclusions

Sensitivity analysis is used to find the key variables which have significant effect on system reliability. For a product in early design stage, it is impossible to collect enough samples due to the limitations of time and resources. Thus, the probabilistic-based reliability sensitivity analysis methods are difficult to use because many samples are required to accurately determine a probability distribution. Existing non-probabilistic reliability sensitivity methods are local sensitivity methods. To address the issue, a new global non-probabilistic reliability sensitivity method is proposed in this study. Surrogate models with

global accuracy are constructed with adaptive manner to reduce overall computational burden. Subsequently, time-consuming simulations can be replaced by constructed surrogate models, which are much cheaper than simulations. Numerical examples have demonstrated the applicability of proposed method, which provides a new way for global non-probabilistic reliability sensitivity analysis.

Compared to probabilistic-based global reliability sensitivity methods, the major advantage of proposed method is that the distribution type of input variables is not required. Note that the proposed method is different to local non-probabilistic reliability sensitivity methods. The local method provides local sensitivity of midpoint and radius of

interval variables, whereas the proposed method provides global non-probabilistic reliability sensitivity of interval variables in whole uncertainty space. Moreover, the proposed method only considers main effect of input interval variables.

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