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Assessment of the Fatigue Durability of Standard Smooth and Fancy Flame Cotton Yarns Using a Statistical Model

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Abstract
The fatigue durability of smooth and fancy flame cotton yarns was analysed with a Wöhler's curve. It was found out that the characteristic of fatigue durability of the curve mentioned determined for cotton fancy flame yarns enables the determination of the complete diagram of these curves. In turn, for smooth yarns a Wöhler's curve containing only the areas of quasi-static strength and those of low-cycle strength can be determined.

Key words: *static strength, fatigue durability, Wöhler's curve, smooth cotton yarns, fancy flame yarns, statistical model.*

Introduction

The issue of calculation of fatigue durability in conditions of variable loads is the subject of the wide interest of engineers in various fields of science [2, 9]. The assessment of fatigue durability is often carried out at the stage of designing or exploitation of machines and devices. Fatigue durability at the stage of exploitation concerns existing prototype units. In this case, phenomenological models of fatigue damage accumulation requiring a minimal number of experimental data are usually used [1]. Such information can be obtained from the analysis of a Wöhler's curve, and possible executive corrections can be applied at the design stage [6]. This approach to the assessment of the material studied is recommended as it leads to a reduction of costs connected with the elimination of defects which could be found in the ready product. During the analysis of fatigue durability, statistical data relating to distributions of the parameters of material fatigue are often used. Forecasting fatigue durability is an important aspect of the assessment of fatigue durability. Moreover it should have a statistical character and use probabilistic methods. However, such methods are very difficult and not very accessible in engineering practice. For those reasons, their pragmatic applicability is also slight. Because of the fact that no studies describing the methodology of measurements of the assessment of the fatigue durability of fancy flame yarns has been found so far, it was decided to fill the existing gap. Knowledge of the characteristic of fatigue durability creates the possibility of the breakage prognosis of yarns, both smooth and fancy flame, in the technological operations following spinning. In those operations, the conditions in which the existing temporary tensions of yarns can be close to the maximal values of their strength.

During the use of a ready textile product, forces impacting the article do not belong to the category of quick-changeable forces and are characterised by maximum values. In this regard, the process of destroying the ready textile product lasts much longer, with stresses being considerably lower than the strength of the given material. This finding implied the range of strains to be considerably smaller than the durability of the given material.

Material and methods of investigations

To perform the technological identification of smooth and decorative yarns, American cotton Strict Middling of average fibrous length 27.78 - 28.58 mm and thickness 150 - 162 mtex was chosen. Smooth and fancy flame cotton yarns of 30 tex and 40 tex linear density, typical for a spinning mill of fine thread in the range of yarns of medium thickness utilised for garments, were chosen for the investigation. Both kinds of yarns were produced on a spinning frame - Zinser From 351 equipped with Fancy-Draft modules, i.e. an option enabling the production of flame yarn by changing the main draft and number of twist [2]. The static strength of smooth and fancy flame yarns was determined according to PN-EN ISO 2062:1997. Measurements of the static strength of yarns were conducted with a Instron 5544 tensile tester. The samples of fancy flame yarns were fixed in the clamps of the tensile tester and the place of the effect was located at half the distance between the clamps of the Instron tensile tester, equalling

***Table 1.** Strength properties of smooth and fancy flame cotton yarns determined using an Instron 5544 tensile tester [2].*

Statistic parameter analysed	Symbol	Smooth cotton yarns, tex		Fancy flames cotton yarns, tex	
		30	40	30	40
Average breaking force, cN	$\overline{R}_{m_n}$	431,17	570,51	395,07	547,38
Coefficient of variation of breaking force, %	$V(R_{m_n})$	9,67	6,15	6,34	10,11
Average tenacity of yarn, cN/tex	$\overline{W}_{t_n}$	14,37	14,26	13,17	13,68
Average relative breaking elongation, %	$\overline{\varepsilon}_{r_n}$	7,41	6,49	7,44	7,50
Coefficient of variation of elongation at break, %	$V(\varepsilon_{r_n})$	9,94	6,05	6,09	11,90
Young's modulus (modulus of initial elasticity), cN/tex	$\overline{Mp}_n$	2,74	2,59	2,75	2,38
Coefficient of variation of modulus of initial elasticity, %	$V(Mp_n)$	11,33	18,22	25,68	16,76
Coefficient of static strength, %	η_p			91,63	95,95

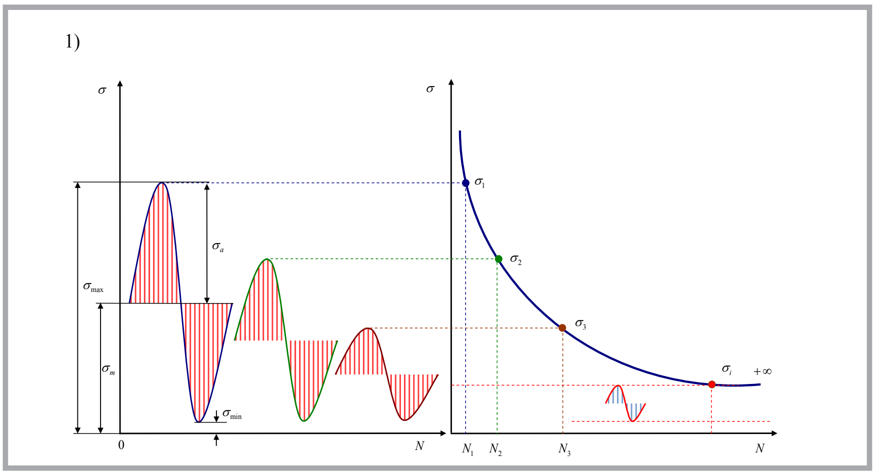

***Figure 1.** Scheme of determining the Wöhler's curve of the yarns investigated [2].*

500 mm, whereas the speed of testing equalled 150 mm/min.

In order to characterise changes in the static strength of fancy flame yarns in relation to smooth ones of similar linear density, the coefficient of static durability was introduced:

$$\eta_p = \frac{R_{m_p}}{R_{m_n}} \cdot 100, \% . \quad (1)$$

These yarns were subjected to changeable cyclic fatigue loads maintained till the moment of the total break of the yarns mentioned. The investigations of fatigue durability were conducted at nine levels of strain, making twenty positive repetitions of tests for each of the levels defined. Tests in which the breaking of the yarn was confirmed in the place of the effect of the flame were accepted as the criterion of fancy flame destruction. In the case of smooth yarns, tests in which the break occurred at approximately half of the distance between the clamps of the tensile tester were recognised as positive.

Because of the stochastic character of strength proprieties of the yarns analysed, the number of series was increased. The strength properties of smooth and fancy flame cotton yarns were determined using an Instron 5544 tensile tester and presented in ***Table 1***.

Investigations of the fatigue strength were conducted at eight levels of strain, carrying out 20 positive repetitions of tests for each of the levels defined. Tests in which the crack was detected were accepted as the criterion of destruction. The investigations of the fatigue strength of yarns were conducted using an Instron 5544 tensile tester equipped with Merlin software and Test Profiler. The distance between the clamps of the tensile tester amounted to 500 mm. The Wöhler's diagram is a classic diagram enabling the determination of the fatigue of a material. The procedure of obtaining the diagram mentioned was performed by subjecting the smooth and fancy flame yarns produced to changeable cycles σ_m till the moment of the total destruction of the sample. The investigation was interrupted after achieving the quantity defined. The graphic way of drawing the Wöhler's curve is presented in ***Figure 1***.

Changing the values of the amplitudes of the cycle load σ_a in consecutive measuring series, where σ_{max} was a fixed value, a search for a causal relationship between the amplitude of the cycle load and the number of cycles N leading the yarns investigated to fatigue failure was conducted. The investigations of fatigue durability were carried out and assessed at a constant amplitude of loads during each cycle (loading-unloading) and stable frequency of cycle f_c = 4 Hz, which approximately reflects existing conditions in the majority of technological processes [10, 11]. In literature of the subject, it was found that investigations dedicated to the fatigue strength of textile articles had been conducted in the range of frequency f_c = 1 - 5 Hz [1, 9] and higher [7, 14]. During fatigue investigations, the following constant quantities were accepted:

- σ_{min} - minimal load in each cycle,
- σ_{max} - maximal load in each cycle,
- σ_m - average load of cycle,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} . \quad (2)$$

Before the beginning of the investigations, the range of amplitude of the load of the cycle - σ_a at which investigations of fatigue durability were performed was determined (***Figure 1***)

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} . \quad (3)$$

The minimal σ_{min} and maximal σ_{max} cycle load was programmed with an Instron 5544 tensile tester and Test Profiler software. The average value of Young's modulus for smooth yarns $\overline{Mp}_n$ as well as fancy flame yarns $\overline{Mp}_p$ was arbitrarily accepted as the minimal value of the cycle load σ_{min}:

$$\sigma_{min} = \overline{Mp}_n = const ,$$
$$\sigma_{min} = \overline{Mp}_p = const . \quad (4)$$

Taking into consideration traditional designations applied in the textile industry, the linear limit of elasticity R_e was defined as the modulus of the initial elasticity Mp in the next part of the article. The modulus of initial elasticity of smooth yarns $\overline{Mp}_n$ and fancy flame yarns $\overline{Mp}_p$ was determined with Merlin software of the Instron 5544 tensile tester during the static strength investigations. The maximum stress of the cycle σ_{max} was calculated on the basis of the average stress breaking the yarn $\overline{W}_t$. Because of the material investigated, the coefficient of the real strain of the cycle σ_s dependent on the level of stresses in the given cycle was introduced to characterise the maximum stress of the cycle. It was arbitrarily assumed that the fatigue durability would be determined at the eight levels of maximal stress [2]:

$$\sigma_s = [0.98 \cdot \overline{W}_t \cdot 0.95 \cdot \overline{W}_t\, 0.85 \cdot \overline{W}_t\, 0.75 \cdot \overline{W}_t \cdot 0.65 \cdot \overline{W}_t \cdot 0.55 \cdot \overline{W}_t \cdot 0.50 \cdot \overline{W}_t \cdot 0.40 \cdot \overline{W}_t]^T, \text{ cN/tex.}$$

The coefficient of the real stress of the cycle σ_s was accepted arbitrarily on the basis of earlier experiments [3]. The average breaking stress of yarns $\overline{W}_t$ was determined from 50 measurements carried out for each yarn.

The acceptance of the maximal upper value of the cycle load $\sigma_{max} = \sigma_s = 0.98 \cdot \overline{W}_t$ was caused by the fact that in the case of fatigue investigations at the stress $\sigma_{max} > 0.9 \cdot \overline{W}_t$ over 99% samples of yarns underwent failure at a number of fatigue cycles $N_p \leq 1$ and $N_n \leq 1$ (where: N_p - number of fatigue cycles determined for fancy yarn, N_n - number of fatigue cycles determined for smooth yarn). Thus this range was close to that of stresses causing the failure of yarns. In turn, the acceptation of the bottom value of the cycle load $\sigma_{max} = \sigma_s = 0.40 \cdot \overline{W}_t$ resulted from the fact that the fatigue durability was close to the range of fatigue limit strength considered as the fatigue strength of the material causing its unrestricted work at periodically changeable loads.

Aiming at the determination of a Wöhler's curve for the yarns analysed in conditions of the definite cycle of the loads and estimation of the results of fatigue cycles influencing the magnitude of coefficients of the real stress of the cycle of the yarns analyzed, non-linear regression was applied to approximate dependences obtained with logarithmic functions [13]:

- for cotton smooth yarns:

$$\hat{\sigma}_{sn} = B_0 + B_1 \cdot \ln(N_n), \qquad (5)$$

- for cotton fancy flame yarns:

$$\hat{\sigma}_{sp} = B_0 + B_1 \cdot \ln(N_p), \qquad (6)$$

where:

$[B_0; B_1]^T$ – vector of the coefficient of the logarithmic function,

$\hat{\sigma}_{sn}$ and $\hat{\sigma}_{sp}$ – value of the regression function of the coefficient of the real stress of the cycle for cotton smooth yarn and cotton fancy flame yarn, respectively,

N_n and N_p – number of fatigue cycles determined for cotton smooth yarn and cotton fancy flame yarn, respectively.

For calculation of the coefficients of regression functions B_0 and B_1 the method of the smallest sums of the squares of deviations was applied, solving the following system of Equations [8]:

$$\left.\begin{aligned} &B_1 \cdot \sum_{i=1}^{n} \ln N_i + B_0 \cdot n = \sum_{i=1}^{n} \sigma_{si} \\ &B_1 \cdot \sum_{i=1}^{n} (\ln N_i)^2 + B_0 \cdot \sum_{i=1}^{n} \ln N_i = \sum_{i=1}^{n} \ln N_i \cdot \sigma_{si} \end{aligned}\right\} \qquad (7)$$

After transformations, the values of coefficients B_0 and B_1 were obtained and presented in ***Equations 8*** and ***9***:

$$B_0 = \frac{\sum_{i=1}^{n} \ln(N_i) \cdot \sum_{i=1}^{n} \sigma_{si} - \sum_{i=1}^{n} \ln(N_i) \cdot \sum_{i=1}^{n} \ln(N_i) \cdot \sigma_{si}}{n \cdot \sum_{i=1}^{n} \ln(N_i^2) - (\sum_{i=1}^{n} \ln(N_i))^2} \qquad (8)$$

$$B_1 = \frac{n \cdot \sum_{i=1}^{n} \ln(N_i) \cdot \sigma_{si} - \sum_{i=1}^{n} \ln(N_i) \cdot \sum_{i=1}^{n} \sigma_{si}}{n \cdot \sum_{i=1}^{n} \ln(N_i^2) - (\sum_{i=1}^{n} \ln(N_i))^2} \qquad (9)$$

The agreement of the output of the model with that of the object was estimated on the basis of the coefficient of the multiple correlation R, expressed by the following formula:

$$R = \sqrt{\frac{\sum_{i=1}^{n} (\hat{\sigma}_{si} - \overline{\sigma}_s)^2}{\sum_{i=1}^{n} (\sigma_{si} - \overline{\sigma}_s)^2}} \qquad (10)$$

where:

σ_{si} - output of the object in i-th experiment,

$\hat{\sigma}_{si}$ - output of the model in i-th experiment,

n - number of tests in the experiment,

$\overline{\sigma}_s$ - average value of output of object and model, where:

$$\overline{\sigma}_s = \frac{1}{n} \cdot \sum_{i=1}^{n} \sigma_{si} = \frac{1}{n} \cdot \sum_{i=1}^{n} \hat{\sigma}_{si} \qquad (11)$$

The significance of the regression function determined was checked on the basis of the dependence occurring between the statistics F and coefficient of the multidimensional correlation 'R':

$$F_{n-k-1}^{k} = \frac{n-k-1}{k} \cdot \frac{R^2}{1-R^2} \qquad (12)$$

In order to obtain a regression equation useful for steering the process of the production of the yarns examined and simultaneously fulfilling requirements at the specified level of significance ($\alpha = 0.05$), the initial condition must be fulfilled:

$$F_{n-k-1}^{k} \geq F_{crit}, \qquad (13)$$

where: F_{calc} – critical value of F – Snedecor's statistics determined from tables of Fisher's and Snedecor's distribution at the level of significance $\alpha = 0.05$ and at the number of degrees of freedom k and $n - k - 1$.

To determine the limits of the confidence interval of the feature studied (the real strain of the cycle for cotton smooth yarn), the following dependence was used [4]:

$$P\left\{\overline{\sigma}_s - t_\alpha \cdot \frac{s}{\sqrt{n-1}} < m < \overline{\sigma}_s + t_\alpha \cdot \frac{s}{\sqrt{n-1}}\right\} = 1 - \alpha \qquad (14)$$

where: s – standard deviation calculated from formula:

$$s = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (\sigma_{si} - \overline{\sigma}_s)^2} \qquad (15)$$

α – level of significance,

t_α– t-Student's variable read from tables of the Student's distribution for $n - 1$ degrees of freedom in a way that allows the following inequality to be fulfilled by the probability $1 - \alpha$, assumed in advance:

$$P\{-t_\alpha < t < t_\alpha\} = 1 - \alpha \qquad (16)$$

The relationship (7 – 16) was also applied for cotton fancy flame yarns [2]. The Wöhler's diagrams are presented in ***Figures 2*** and ***3***.

Analysing the course of variation of the number of cycles to failure at consecutive fatigue cycles presented in ***Figure 2***, it can be stated that cotton fancy flame yarns of 30 tex linear density are characterised by higher fatigue durability in the whole range of the load. The highest coefficients of fatigue durability of the

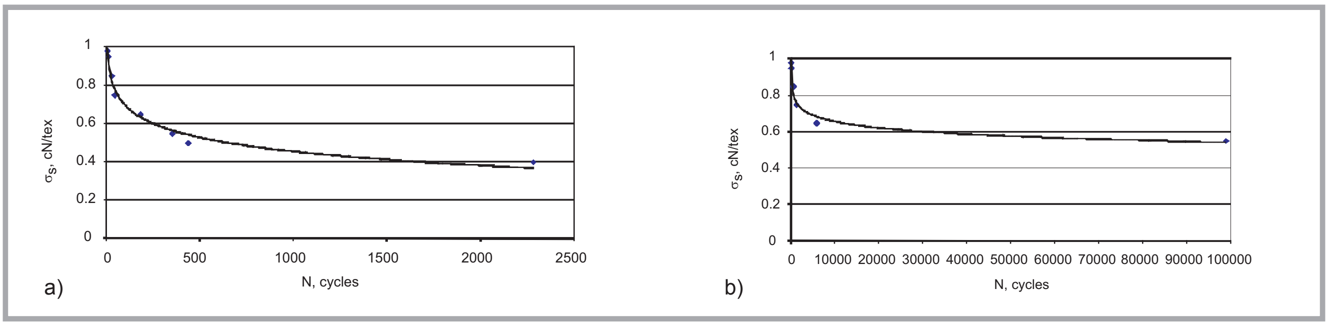

***Figure 2.** a) Diagram of Wöhler's curve for smooth cotton yarns of 30 tex linear density, approximating the logarithmic function of the equation $\hat{\sigma}_{sn} = -0.105\ln(N_n) + 1.1816$, where $R^2 = 0.98$ and $F_{calc} = 335.85 > F_{crit} = 5.99$; b) full diagram of the Wöhler's curve for cotton fancy flame yarns of linear pulp 40 tex, approximating the logarithmic function of equation $\hat{\sigma}_{sp} = -0.047\ln(N_p) + 1.1029$, where $R^2 = 0.986$ and $F_{calc} = 109.01 > F_{crit} = 7.71$.*

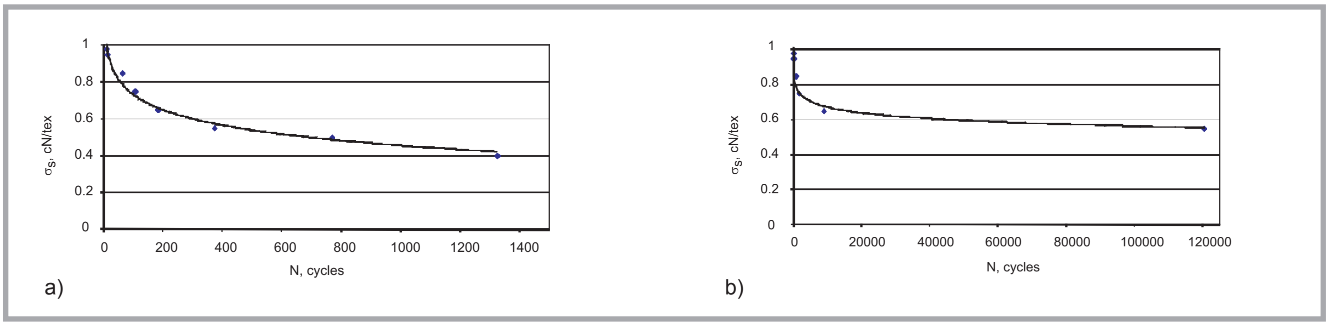

***Figure 3.** a) Diagram of Wöhler's curve for smooth cotton yarns of 40 tex linear density, approximating the logarithmic function of the equation $\hat{\sigma}_{sn} = -0.12\ln(N_n) + 1.2878$, where $R^2 = 0.98$ and $F_{calc} = 270.49 > F_{crit} = 5.99$; b) full diagram of the Wöhler's curve for cotton fancy flame yarns of linear pulp 30 tex, approximating the logarithmic function of equation $\hat{\sigma}_{sp} = -0.05\ln(N_p) + 1.1158$, where $R^2 = 0.987$ and $F_{calc} = 148.09 > F_{crit} = 7.79$.*

yarns can be observed in the range of low-cycle loads, i.e. σ_s = [0.55, 0.75], which means that fancy flame yarns are characterised by over 95% higher fatigue durability than cotton smooth yarns of the same or similar linear density. In the range of low-cycle loads, considerable differences in the fatigue durability of fancy flame yarns can be noticed. In the case of the area of high-cycle loads, i.e. at σ_S = [0.50, 0.40], smooth yarns, in contrast to fancy flame yarns, do not achieve limit values from this range.

The assessment of fatigue durability below the coefficient of the real stress $\sigma_S = 40\% \overline{W_{t_n}}$ is baseless for cotton smooth yarns of 30 and 40 tex linear density. The unlimited fatigue strength of fancy flame yarns of 30 tex linear density equals $N_{un\ \mathrm{lim}}$ (at a stress of ≈ 6.58 cN/tex). In turn, for smooth yarns of this linear density we can only speak of the average fatigue durability, which for the maximum load of the cycle $\sigma_{\max}$ = = 5.75 cN/tex equaled N_n = 2283 cycles. For yarns of 40 tex linear density (***Figure 3***), the unlimited fatigue strength of fancy flame yarns equalled $N_{un\ \mathrm{lim}}$ (at a stress of ≈ 6.84 cN/tex). In turn, for smooth yarns of the same linear density, only the average fatigue durability, which for the maximum load of the cycle $\sigma_{\max}$ = 5.71 cN/tex equalled N_n = 1325 cycles, can be estimated.

In summary, it can be stated that diagrams of curves presented in ***Figures 2*** and ***3*** represent a traditional way of recording and analysis of results of the assessment of fatigue durability [3]. The results of fatigue durability assessment presented earlier are subject to large dispersions. The dispersion of results around the average value grows together with the growth of the level of the stress σ_m. The feature mentioned is perceptible for all linear densities of smooth cotton yarns and fancy flame considered, the main causes of which are differences resulting from the characteristics of the material investigated. The variability of the structure of the material investigated also can result from the technological process and conditions of the investigation. Thus a traditional curve should be designed with statistical investigations in order to obtain the full characteristic of the fatigue durability of the material.

The aim of the planning of statistical investigations is to determine the minimal number of tests necessary to confirm the accuracy of Wöhler's curves obtained. The diagrams of fatigue durability in semi-logarithmic or the logarithmic system of co-ordinates should be straight lines, thanks to which the results of investigations can be processed using linear dependences between the parameters considered. Simultaneously making an assumption that stresses are magnitudes determined while planning the experiment, then the logarithms of fatigue durability are random magnitudes characterised by the normal distribution of the probability. That is why Wöhler's curves describing the fatigue durability of the given material can be processed on the basis of linear dependences in the semi-logarithmic or logarithmic system of co-ordinates [5, 6].

The theoretical line of the regression can be applied in the following form:

$$\eta = \delta + \beta \cdot (x - \overline{x}) \qquad (17)$$

where: $\eta = E(Y/x)$ is the value of the magnitude of the logarithm of durability expected lg $N = Y$, where the logarithm of the stress established is lg $\sigma = X = x$.

Estimation of the theoretical line of regression is the experimental line of regression in the form of equation:

$$\hat{Y} = B_0 + B_1 \cdot (x - \bar{x}) \tag{18}$$

In the opinion of Kocańda and Szala [6], Wöhler's curve diagrams match the probability - P of the failure of samples, equalling 50%. This fact allows for the use of the linear correlation on the basis of the method of smallest squares. Accepting corresponding pairs of values from the investigations of fatigue durability conducted

$$(x_1, y_1); (x_2, y_2); \ldots (x_n, y_n) \tag{19}$$

the dependences considered can be described by a linear equation in the form [4]:

$$\hat{Y} = B_0 + B_1 \cdot x \tag{20}$$

where: B_0 and B_1 are coefficients of the function of regression.

The value of parameters B_0 and B_1 is determined by the method of the smallest squares, according to which the sum of squares of deviations among values σ_s determined from ***Equation 20*** for $N = N_i$ and the values obtained during the experiment should reach the minimum:

$$\sum_{i=1}^{m} (\sigma_{si} - B_0 - B_1 \cdot N_i)^2 = \min \tag{21}$$

A suitable equation is obtained when partial derivatives of the left side of ***Equation 21*** in relation to B_0 and B_1 are equated to zero. Differentiating with respect to B_0 gives:

$$\frac{\partial}{\partial B_0}[\sum_{i=1}^{m} (\sigma_{s} - B_0 - B_1 \cdot N_i)^2] = \\ = -2\sum_{i=1}^{m} (\sigma_{s} - B_0 - B_1 \cdot N_i) = 0 \tag{22}$$

Dividing both sides of this relationship by the general number n of pairs of corresponding values obtained during the experiment, equation (22) can be expressed in the following form:

$$B_0 = \bar{\sigma}_s - B_1 \cdot \bar{N} \tag{23}$$

Substituting the value calculated into ***Equation 20***, the following equation is obtained:

$$\hat{y} = \sigma_s + B_1 \cdot (N - \bar{N}) \tag{24}$$

After differentiation expression (21) with respect to B_1, the following equation is obtained:

$$\frac{\partial}{\partial B_1}[\sum_{i=1}^{m} (\sigma_{si} - B_0 - B_1 \cdot N_i)^2] = \\ = -2\sum_{i=1}^{m} (\sigma_{si} - B_0 - B_1 \cdot N_i) = 0 \tag{25}$$

Suitably transforming formulae (23) and (25) and also substituting values from these formulae, after reduction, the B_1 value is obtained in the form:

$$B_1 = \frac{\sum_{i=1}^{m} N \cdot \sigma_s}{\sum_{i=1}^{m} N^2} \tag{26}$$

where:

$$\sum_{i=1}^{m} N \cdot \sigma_s = \sum_{i=1}^{m} (N_j \cdot \sigma_{sij} - \bar{N} \cdot \bar{\sigma}_s) = \\ = \sum_{i=1}^{m}\sum_{j=1}^{k} (N_j \cdot \sigma_{sj} - \bar{N} \cdot \bar{\sigma}_s) + \\ - \frac{1}{n}\left(\sum_{i=1}^{m} \sum_{j=1}^{k} N_j \sum_{i=1}^{m} \sum_{j=1}^{k} \sigma_{sj}\right) \tag{27}$$

$$\sum_{i=1}^{m} N^2 = \sum_{i=1}^{m} (N_j - \bar{N}) = \\ = \sum_{i=1}^{m}\sum_{j=1}^{k} N_j^2 - \frac{1}{n}\left(\sum_{i=1}^{m}\sum_{j=1}^{k} N_j\right)^2 \tag{28}$$

$$n = \sum_{i=1}^{m} k_i \tag{29}$$

where:

i - the level of the maximal stress expressed by the coefficient of the real stress of the cycle σ_s,

k_i - number tests at i-th level of stresses $i = 1, 2, \ldots, \sigma_s$,

m - number of levels of stresses.

After substitution of the value calculated B_1 into ***Equation 24***, an equation of the straight regression is obtained. As was earlier mentioned, fatigue durability is characterised by considerable dispersion of the results. As a measure of dispersion, quotient B of the sum of the squares of differences among values σ_s calculated from the equation of regression and average value σ_s from measurements for the sum of the squares of differences among values σ_{si} obtained from measurements of σ_s was introduced. According to this definition, the following relationship is obtained:

$$R = \frac{\sum_{i=1}^{n} (\sigma_s - \bar{\sigma}_s)^2}{\sum_{i=1}^{n} (\sigma_{s} - \bar{\sigma}_s)^2} \tag{30}$$

Theoretically, if all experimental points lie on the line of regression, the value of coefficient R is equal to 1. In practice, coefficient R characterises what percent of results between the limits of the stress evaluated for a given N can be described by a linear relationship. Instead of the coefficient of multiple correlation R, the coefficient of correlation r is often used, as in the formula below:

$$r = \frac{\sum_{i=1}^{m}{}' N\sigma_s}{\sqrt{\sum_{i=1}^{m}{}' N^2 \sum_{i=1}^{m}{}' \sigma_s^2}} \tag{31}$$

where:

$$\sum_{i=1}^{m}{}' \sigma_s^2 = \sum_{i=1}^{m} \left(\sigma_{sij} - \bar{\sigma}_s\right)^2 = \\ \cdot \bar{\sigma}_s)^2 = \sum_{i=1}^{m}\sum_{j=k}^{k} \sigma_j^2 - \frac{1}{n}\left(\sum_{i=1}^{m}\sum_{j-k}^{k} \sigma_{sij}\right)^2 \tag{32}$$

In the literature, ***Equation 20*** often has the form:

$$\log N = B_0 + B_1 \cdot \sigma \tag{33}$$

The significance of the model was checked using formula (12). Diagrams of the fatigue durability of smooth and fancy flame cotton yarns, taking into account a confidence interval at the level $(1 - \alpha) = 0.95$ for the average fatigue durability N_n and N_p (cycles), are presented in ***Figures 4*** and ***5*** [2]. The functions proposed, on the basis of the statistical analysis, are very useful for assessment of the fatigue durability of the yarns analysed.

The equation of regression for smooth yarns (30 tex) is described by the logarithmic function in the form: $\hat{\sigma}_{sn} = 1.181 - 0.105 \cdot N_n$. The coefficient of the multiple correlation $R = 0.991$; the F-Snedecor's statistics $F_6^1 = 336.75 >$ $> F_{crit} = 5.99$, whereas the confidence interval is $2.855 < N_n < 6.226$. In turn, the equation of regression for fancy flame yarns (30 tex) has the form: $\hat{\sigma}_{sp} = 1.115 +$ $- 0.050 \cdot N_p$. The coefficient of multiple correlation R = 0,986; the F-Snedecor's statistics $F_4^1 = 148.09 > F_{crit} = 7.71$; whereas the confidence interval is $3.038 < N_p < 10.08$.

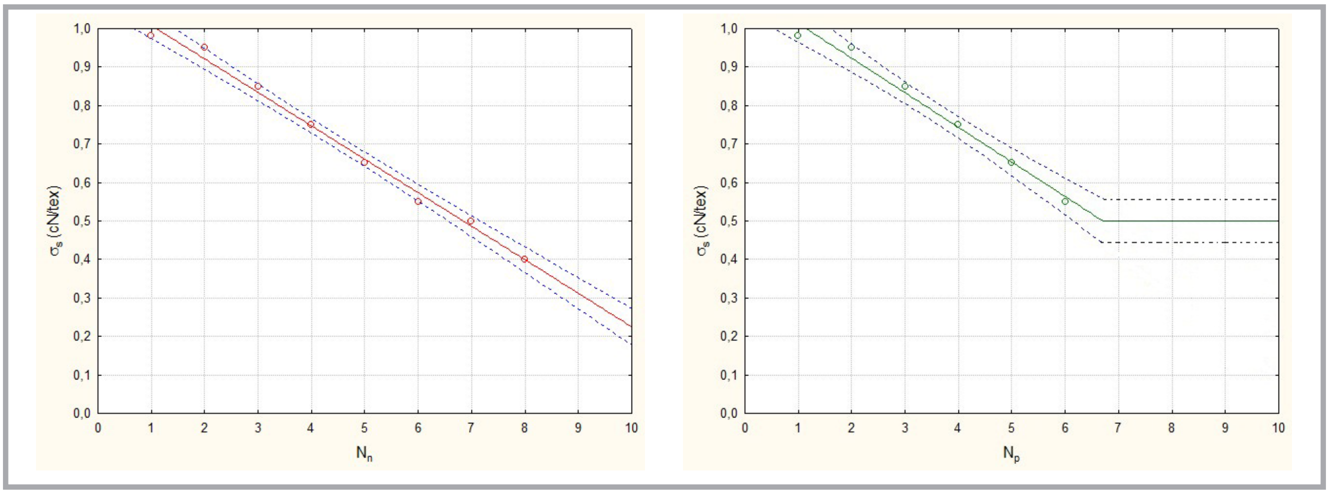

Figure 4. *Statistically processed diagrams of the fatigue durability of smooth (N_n) and fancy flame (N_p) cotton yarns (linear density of yarns – 30 tex).*

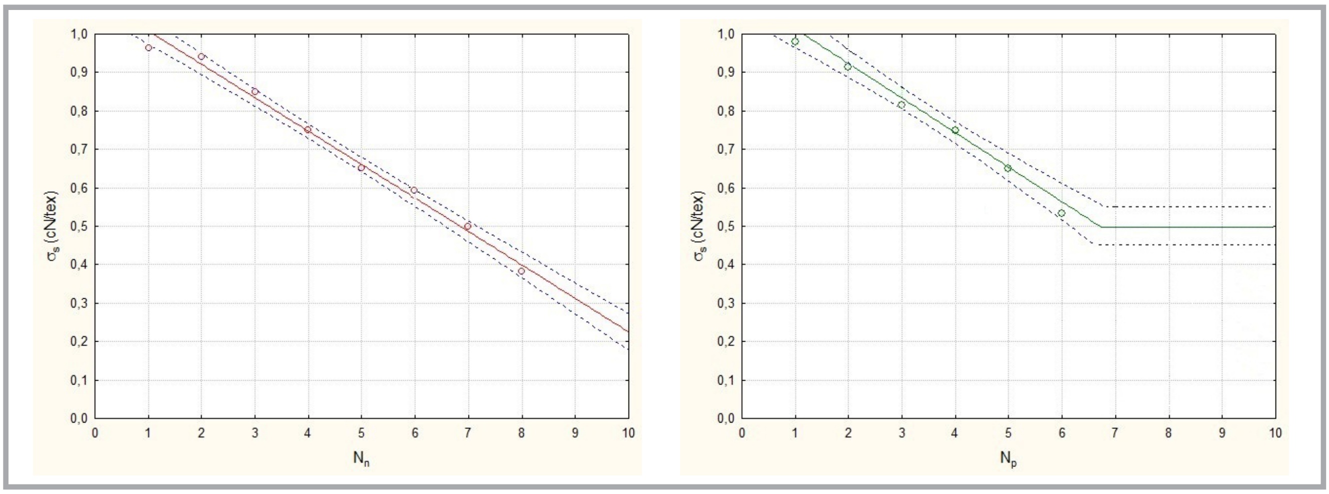

Figure 5. *Statistically processed diagrams of the fatigue durability of smooth (N_n) and fancy flame (N_p) cotton yarns (linear density of yarns – 40 tex).*

The equation of regression for smooth yarns (40 tex) is described by a logarithmic function in the form: $\hat{\sigma}_{sn}$ = 1.252 + - 0.114·N_n. The coefficient of multiple correlation R = 0.991; the F-Snedecor's statistics $F_6^{\,1}$ = 175.85 > F_{crit} = 5.99, whereas the confidence interval is 3.259 < N_n < 6.340. In turn, the equation of regression for fancy flame yarns (40 tex) has the form $\hat{\sigma}_{sp}$ = 1.116 + - 0.048·N_p. The coefficient of multiple correlation R = 0.985; the F-Snedecor's statistics $F_4^{\,1}$ = 138.40 > F_{crit} = 7.71; whereas thr confidence interval is 3.155 < N_p <10.40.

Summary

The assessment of the fatigue durability of yarns using the statistical approach represents the failure of yarns in the area of low-cycle loads. In the course of one-sidedly positive tensile straining, tension has the character of plastic cracking. Theoretically the failure of this type of yarns proceeds at limiting plastic strains comparable with strains at static loads. The fatigue durability of cotton fancy flame yarns is higher. However, in comparison with the assessment of static durability, the relations are converse and the course of failure of the material is significantly different from the typical course for static loads. The value of the absolute increase in the rate of elongation of the sample equal to 150 mm/min had a direct influence on the results obtained while performing investigations of static strength. In the case of the assessment of fatigue durability, the absolute increase in the rate of elongation was considerably higher, which approximately equalled over 1000 mm/min and was dependent on the constant (for all yarns analysed) frequency of stresses programmed f_c = 4 Hz. In the case of yarns of 30 tex and 40 tex linear density the coefficient of static strength η_p for fancy flame yarns equalled 91.63% and 95.95%, respectively. Changeable loads acting on the yarns and causing their failure had an influence on the fatigue durability of smooth and fancy flame cotton yarns. Fatigue strength significantly differs from typical static strength at constant static elongations.

Conclusions

1. Decorative effects designed in the form of flames for yarns of all linear densities considered had a direct influence on the variability of the physical proprieties of these yarns.
2. The characteristics of fatigue durability described with Wöhler's curves and determined for cotton fancy flame yarns made it possible to determine the full diagram of these curves, including

the area of quasi-static strength, the area of low-cycle strength and also the area of high-cycle strength. In contrast to fancy flame yarns, the Wöhler's curve for smooth yarns included only the areas of quasi-static durability and those of low-cycle strength. It can be stated that assuming static strength as the criterion of usefulness is not completely legitimate.

References

1. Asayesh A, Ali Jeddi AA, Ghadimi P. Modeling the Fatigue Behavior of Plain Woven Fabrics Constructed from Textured Polyester Yarn, *Textile Res. J.*, 2009; 79(13): 1213-1222.
2. Drobina R. P*robabilistyczny model trwałości zmęczeniowej przędz bawełnianych gładkich i płomykowych,* Scientific Ed. of Technical University of Bielsko-Biała 2012, Scientific Disartations No. 40.
3. Drobina R, Włochowicz A, Machnio M, Drobina E, Lewandowski S. Fatigue curves elaborated for selected worsted wool yarns, *Fibers & Text. in East. Eur.*, 2007; 15, 5-6(64-65): 54-58.
4. Greń J. *Statystyka matematyczna - modele i zadania*, ed. PWN, Warsaw 1982.
5. Jakubowicz A, Orłoś Z. Wytrzymałość materiałów, Wyd. VI, ed. WNT Warsaw 1984, s. 580-613.
6. Kocańda S, Szala J. *Podstawy obliczeń zmęczeniowych*, ed. PWN - Warsaw, 1997.
7. Kowalski K, Włodarczyk B, Kowalski TM. Probabilistic Model of Dynamic Forces in Thread in the Knitting Zone of Weft Knitting Machines, Allowing for the Heterogeneity of Visco-Elasticity Yarn Properties, *Fibres & Text. in East. Eur.*, 2010; 18, 4(81): 61-67.
8. Kryński HE. *Matematyka dla ekonomistów*, ed. PWN, Warsaw, 1975.
9. Nosraty H, Ali Jeddi AA. Jamshidi Avanaki M. Fatigue Behavior of Filament Warp Yarns under Cyclic Loads during Weaving Process, *Textile Res. J.*, 2009; 79(2): 154-165.
10. Owczarz R. Analisis of abrasive resistance of wooven fabrics produced from classical and fancy threads (In Polish), *Przegląd Włókienniczy* 1991; 45(6): 160- 163.
11. Owczarz R. Programming tensile strength of woven fabrics produced from fancy braid yarns (In Polish), *Przegląd Włókienniczy* 1991; 45(10): 247-251.
12. PN-EN ISO 2062:1997 „The textiles. The thread in packages".
13. Stanisz A. Przystępny kurs statystyki na przykładach z medycyny z wykorzystaniem programu STATISTICA", v. II, ed. II, Statsoft, Krakow, 2006.
14. Tumajer P, Ursíny P, Bílek M, Moučková E. Research Methods for the Dynamic Properties of Textiles, *Fibers & Text. in East. Eur.*, 2011; 19, 5(88): 33-39.

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