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COMPARATIVE FATIGUE TESTING OF GEARS WITH INVOLUTE AND CONVEXO-CONCAVE TEETH PROFILES

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Summary

Paper presents preliminary comparative fatigue test methodology of convexo-concave Novikov and involute gears. Closed loop power circulation test stand used for experiment was introduced. Moreover the experiment process and adopted criteria of fatigue wear were described. The analysis of results for Novikov and involute gear pair were given.

Keywords: fatigue testing, Novikov gears, circular-arc gears, convexo-concave tooth profile

Porównawcze badania zmęczeniowe przekładni zębatych o ewolwentowym i wklęsło-wypukłym zarysie zębów

Streszczenie

W pracy przedstawiono metodykę prowadzenia wstępnych badań zmęczeniowych przekładni zębatych o uzębieniu typu Nowikowa i ewolwentowym. Opisano stanowisko mocy krążącej, którą badano. Przyjęto kryterium oceny zużycia zębów oraz scharakteryzowano przebieg eksperymentu. Przeprowadzono analizę wyników badań dla przekładni o zarysie Nowikowa i ewolwentowym.

Słowa kluczowe: badania zmęczeniowe, przekładnie Nowikowa, przekładnie kołowo-łukowe, wklęsło-wypukły zarys zębów

1. Introduction

Modern gearboxes should be characterized by high efficiency, low noise levels and high load capacity at as small dimensions as possible. A common teeth profile in gear transmissions is an involute one, where a convex pinion tooth flank works with a convex wheel tooth flank. The character of this work is particularly adverse in case of surface strength. If the increase of the surface load capacity is required it is necessary to increase the radius of tooth profiles and therefore pitch diameter or pressure angle. Pressure angle can be increased by means of addendum modification. Further increase of surface strength is connected with increasing the overall dimension of a gearbox.

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Due to the above-mentioned disadvantage of the involute teeth profile there were attempts to withdraw from it. The alternative could be gear transmissions with convexo-concave teeth profiles. The first gearbox of this kind was a gearbox designed by E. Wildhaber in 1926 [1]. The circular-arc appears in the plane normal to the teeth line. Another solution of gear with circular-arc teeth was proposed by M.L. Novikov in 1955 [2]. Unlike the Wildhaber's solution, here circular-arc teeth profile appears in transverse section. The essential difference between these two ideas is that Novikov proposed the mismatch of convex and concave teeth profile radius.

Gears with circular-arc teeth profiles were within the scope of interests of many researchers around the world. They were the subject of theoretical analysis [3-14] as well as tests aimed at evaluation of its load capacity [15-17]. Both the experiments and the theory seems to confirm that in case of quenched and tempered gears the Novikov ones are distinguished by higher surface load capacity than standard involute gears. Taking into account gears, which have been heat treated above the 35 HRC the higher surface load capacity in comparison with involute gears has not been clearly proved.

Among the many applications of circular-arc gears [3, 4, 18, 19] special attention deserves to the final reduction stage of Westland Lynx helicopter gearbox [3, 4]. The gearbox was a reducer of reduction ratio 20. The pinion has convex teeth whereas the wheel has the concave ones. The convexo-concave teeth profile allowed to decrease the number of gears to 40% of number of involute gears used previously [18].

Circular-arc gearing was also applied in the reduction gearbox of pumping unit [19]. The LS Petrochem company, which is the manufacturer of gearbox states that the prove of its reliability lies in the proper operation in thousands of pumping units around the world over the 50 years. What's more company ensures that applied double circular-arc gearing translates into higher pitting resistance, higher fracture strength and better lubrication conditions.

Although gears with convexo-concave tooth profile are reluctantly applied. It can be influenced by the fact that in contrast to involute gears there is no developed methodology of designing and analysing of this king of gears.

In this paper the description of preliminary comparative tests of quenched and tempered gears with convexo-concave Novikov and involute teeth profile was presented. The aim of this research is verification of the theoretically higher surface load capacity of Novikov gears in comparison with involute ones [12] and the methods of gears positions errors (assembly errors, deflection of shafts, bearings, etc.) compensation [11, 20]. Moreover shown tests are introduction to nitrided gears testing.

2. Closed Loop Power Circulation Test Stand

The used closed loop power circulation test stand is the standard FZG test rig. Figure 1 shows the view of this stand in the Transmission Laboratory in Machine Department of Rzeszów University of Technology.

Fig. 1. Closed loop power circulation test stand, based on [21]

Its scheme is shown on Figure 2.

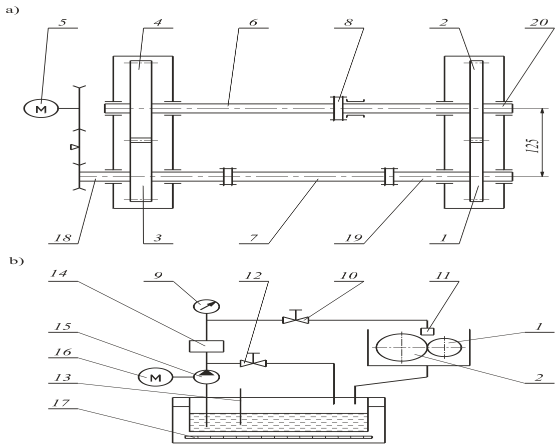

Fig. 2. Closed loop power circulation test stand scheme, based on [21]

The test stand consists of two gearbox with identical ratios. The stand gearbox (gears 3 and 4) and the tested gearbox (gears 1 and 2). Both of these gearboxes are connected to each other by means of shafts 6 and 20 with load clutch

8 and torsional shaft 7 and shafts 18 and 19. Torque and speed of rotation is brought through by electrical motor 5 with belt transmission. The load clutch 8 allows to change the mutual angular position of shafts 6 and 20. The loading of gearbox is achieved by loosening clutch 8 and twisting shaft 6. Then clutch is tightened and thanks to torsional deflection of shaft 7 preload of meshing is applied. Tested gear pair is damaged by forces acting in gear mesh and the motor covers only energy loses connected with friction. To prevent damage of stand gearbox the tooth width of its wheels should be increased. Moreover stand gearbox can also be manufactured in higher accuracy class and be subjected to extra heat treatment or thermochemical treatment. In presented test stand the stand gearbox was gas nitrided and width of their wheels was about two times greater than width of wheels of tested gearbox.

3. Test Methodology

The tests were carried out on the above-mentioned closed-loop power circulation test stand. Tested gear pairs include one involute and one Novikov gear pair quenched and tempered up to 28÷30 HRC. Tested gears were made from 42CrMo4 steel. The data of tested gears are presented in Tabl. 1 and 2.

Tested involute gears (Fig. 3) were made on Koepfer EMAG 200 CNC gear hobbing machine.

Measurements of gears were performed on Klingelnberg P40 coordinate measuring machine. Measured parameters were classified in eight accuracy class according to DIN standard. Moreover average teeth surface roughness didn't exceed 0,4 µm for pinion and 0,6µm for wheel.

Table 1. Data of tested Novikov gears

Parameters	**Pinion**	**Wheel**
Normal module, mm	$m_n = 3$	$m_n = 3$
Number of teeth	$z_1 = 30$	$z_2 = 47$
Overlap ratio	$\varepsilon_\beta = 1{,}21$	$\varepsilon_\beta = 1{,}21$
Tooth width, mm	$b = 30$	$b = 30$
Helix angle, deg	$\beta = 22{,}48$	$\beta = 22{,}48$
Normal pressure angle, deg	$\alpha_n = 20$	$\alpha_n = 20$
Translation of concave tooth profile, mm	$d_{CO'} = 0$	$d_{CO'} = 0$
Profile	convex	concave
Profile radius, mm	$\rho_1 = 6{,}33$	$\rho_2 = 6{,}55$
Pitch diameter, mm	$d_1 = 97{,}40$	$d_2 = 152{,}59$
Tip diameter, mm	$d_{a1} = 104{,}30$	$d_{a2} = 152{,}59$
Root diameter, mm	$d_{f1} = 95{,}30$	$d_{f2} = 143{,}59$

Table 2. Data of tested involute gears

Parameters	Pinion	Wheel
Normal module, mm	$m_n = 3$	$m_n = 3$
Number of teeth	$z_1 = 30$	$z_2 = 47$
Overlap ratio	$\varepsilon_\beta = 1{,}21$	$\varepsilon_\beta = 1{,}21$
Tooth width, mm	$b = 30$	$b = 30$
Helix angle, deg	$\beta = 22{,}48$	$\beta = 22{,}48$
Normal pressure angle, deg	$\alpha_n = 20$	$\alpha_n = 20$
Addendum modification coefficient, mm	$x = 0$	$x = 0$
Profile	convex	convex
Profile radius, mm	$\rho_1 = 17{,}84$	$\rho_2 = 27{,}96$
Pitch diameter, mm	$d_1 = 97{,}40$	$d_2 = 152{,}59$
Tip diameter, mm	$d_{a1} = 103{,}40$	$d_{a2} = 158{,}59$
Root diameter, mm	$d_{f1} = 89{,}90$	$d_{f2} = 145{,}09$

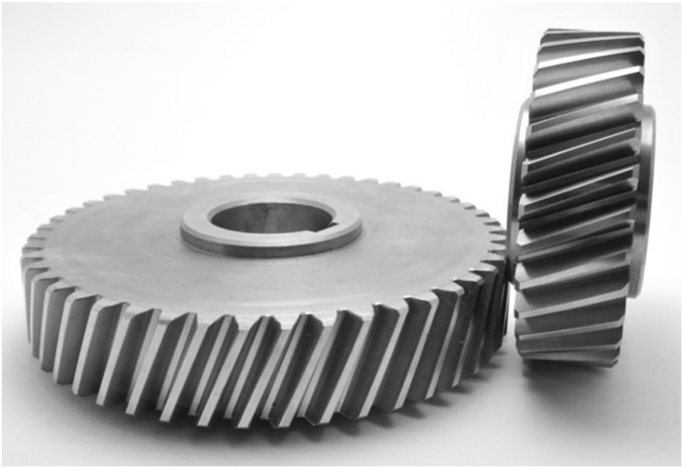

Fig. 3. Tested involute gears

Novikov gears (Fig. 4) were manufactured on Stama MC726/MT 5-axis milling machine with the aid of formed milling cutters specially designed for this purpose [22, 23].

Gears accuracy standards as well as all measuring parameters defined there refers to gear wheels with an involute tooth profile. From this reason it is hard to value the accuracy class of Novikov gears. P40 Klingelnberg measuring machine is equipped with software for measuring involute gears, which don't allow to measure gears with other tooth profiles. In case of measurements other than involute profiles this software enables only measuring the deviations of tooth line. Parameters defining the tooth line accuracy were classified in ninth (for pinion) and tenth (for wheel) accuracy class according to DIN standard. The measurements of Novikov gearing were carried out with the use of conventional methods. The radial runout of gearing was checked by mean of dial gauge with ball. Tooth thickness was measured by gear-tooth vernier. All deviations of measured parameters were in estimated tolerance ranges. What's more average surface roughness of teeth surfaces didn't exceed 0,6 µm for pinion and 0,8 µm for wheel.

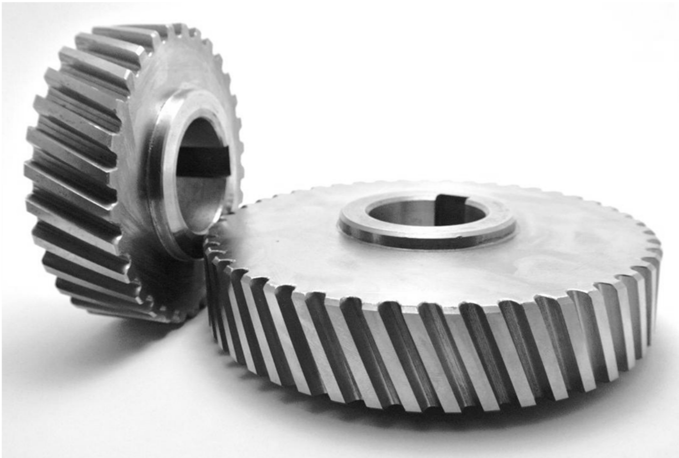

Fig. 4. Tested Novikov gears

Fatigue tests were carried out with the speed of rotation of pinion of 2500 rpm. Running in of gears was performed at pinion torque of 42 Nm. Moreover tests were performed under increasing load. Table 3 compares pinion torque, theoretical Hertz stresses [12], number of pinion load cycles and duration time of each load stage.

Table 3. Load stages of tested gears

Load stage	**Pinion torque, Nm**	**Hertz stresses, MPa**		**Number of pinion load cycles**	**Duration time of load stage**
		Novikov	**Involute**		
0	42	249	249	$1{,}5\cdot10^5$	1 h
1	138	370	450	$2{,}5\cdot10^6$	16 h 40 min
2	244	446	598	$2{,}5\cdot10^6$	16 h 40 min
3	342	500	708	$2{,}5\cdot10^6$	16 h 40 min
4	455	550	817	$2{,}5\cdot10^6$	16 h 40 min
5	455	550	817	$2{,}5\cdot10^6$	16 h 40 min

Tested gear pairs have been designed not to be damage by fracture just by pitting. Pitting is the phenomenon of occurrence of small pits on the surface of tooth resulting from detaching pieces of metal [24]. Pits are growing as the oil, which works as wedge is cyclically pressed into them under high pressure [25].

Pitting can occur in micro or macro scale. In case of macropitting the dimension of pits is about few millimeters [1]. In case of micropitting pits diameter is about a few micrometers (usually 10 um) [1]. Macropitting is usually observed in tempered and quenched gears while the micropitting appears in gears with hardened teeth surfaces (case hardened, nitrided).

Moreover in case of both types of wear the destructive and non-destructive pitting can be distinguished. The destructive macropitting can lead to tooth fracture which is caused by decreasing of its effective surface and weakening its root. Micropitting can lead to macropitting which results the above-mentioned tooth damage.

Non-destructive pitting can occur at gear running in stage and after quite short time of operation disappear. It is harmless unless it turns into destructive pitting.

There is a few methods of measuring the pitting wear of teeth. One of these methods is based on accurate measurements of mass of wheels after each load stage [26]. The wear of teeth is described as the loss of mass in relationship with load stage. Another method is the photographic method. It involves photographing of teeth surfaces after each load stage and calculating the ratio of pitting area to overall area of tooth surface. Nowadays this process can be automated with the use of digital image processing techniques [27]. The other method is measuring of teeth surface roughness which illustrates it state.

In presented paper the photographic method was used. Surfaces of teeth were photograph with constant light. Next they were subjected to image processing algorithm aiming at evaluation the percentage wear of tooth. The block diagram of algorithm is shown on Fig. 5.

Before starting the algorithm image was properly prepared. The contrast was increased and possible shadows were removed. Algorithm begins from reading the monochromatic tooth surface image. Next the edge detection based on Sobel operator is performed. Subsequently the noise is removed – small clusters of white pixels resulting from fine scratches and wear different than pitting. Such image is subjected to dilatation, hole filling and erosion. The last stage of algorithm consist of counting the white pixels and calculation the ratio of number of white pixels to overall area of image, which refers to percentage wear of tooth.

The state of the gear unit and level of its damage can be deduced based on the vibration of gear case [28] or the level of noise [29]. In this work the vibrations of gear case were measured. For this purpose the piezoelectric accelerometer mounted as it is shown on Fig. 6 was used.

Accelerations were measured over one second along the y axis at the end of each load stage.

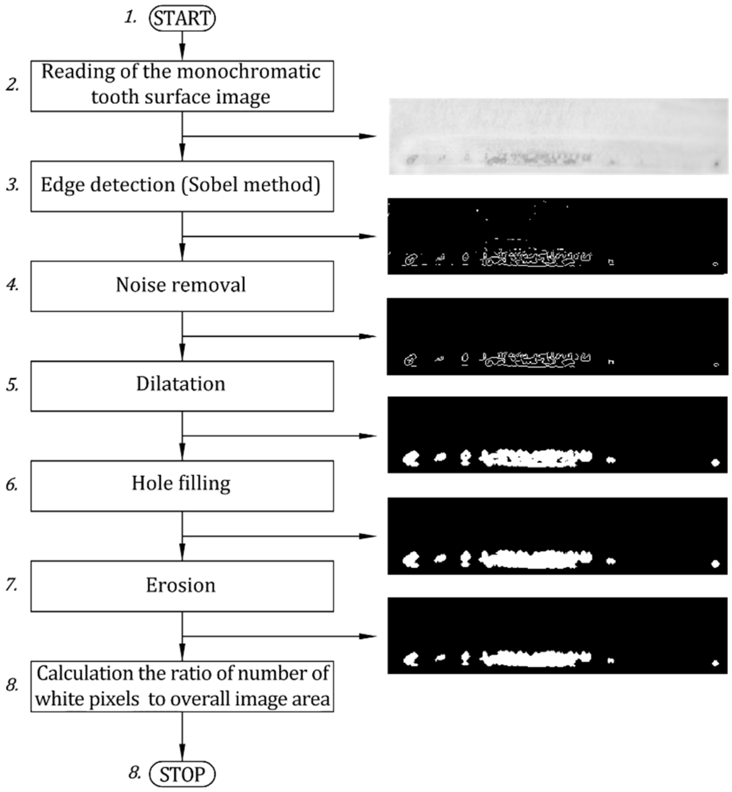

Fig. 5. Block diagram of algorithm of evaluation of percentage wear of teeth and example of its application

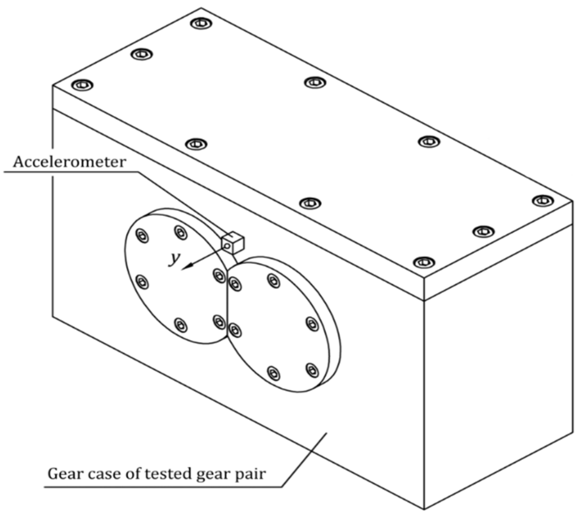

Fig. 6. Location of accelerometer

4. Wear of Involute Gear

After zero and first load stage, roughness of teeth surfaces decreases as results from polishing. As a first, pitting occurred on the pinion gearing below the pitch cylinder after second load stage (Fig. 7).

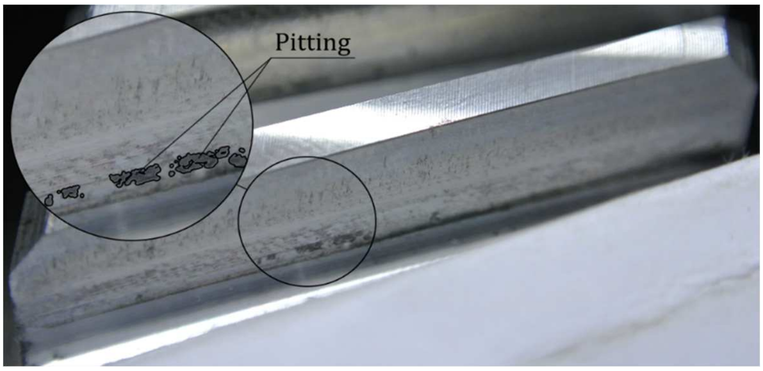

Fig. 7. Pitting on pinion of tested involute gear after second load stage

The type of wear in relation with tooth height can be clearly seen on Fig. 7. On the pitch cylinder tooth surfaces is not damaged because there the sliding velocity is zero. The wear near below the pitch cylinder reveals as a fine scratches along the tooth line and further below as a pitting. Above the pitch cylinder the fine scratches along the tooth height can be observed (normal wear). In case presented on Fig. 7 pitting wear is small and covers approximately 3% of tooth area. On the wheel tooth the wear was not observed.

After third load stage percentage pitting wear on the surface of pinion tooth increased to about 6%. Image of pinion tooth surface is presented on Fig. 8.

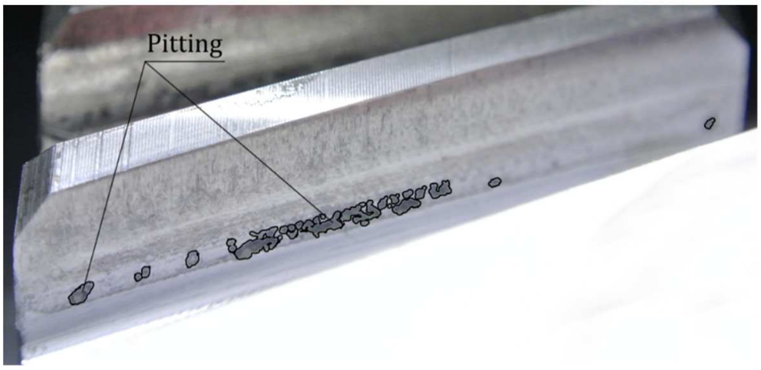

Fig. 8. Pitting on pinion of tested involute gear after third load stage

The pits extends along the tooth line. After third load stage the pitting wear also appeared on wheel tooth surface (Fig. 9).

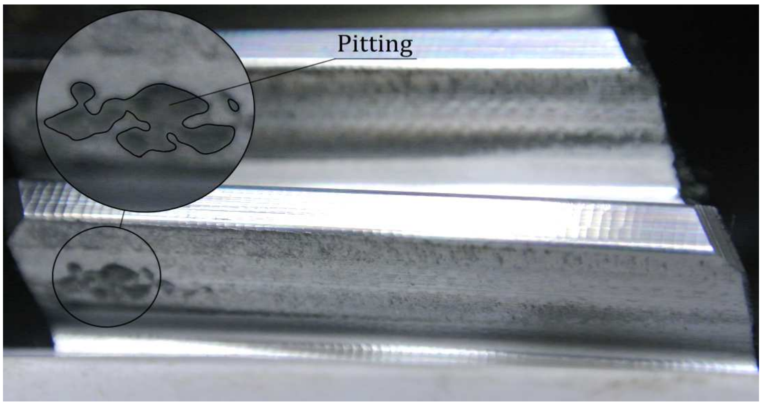

Fig. 9. Pitting on wheel of tested involute gear after third load stage

Large cluster of pits was localized near the edge of tooth which can indicate the nonparallelism of gear axis. In that case pits started to grow on the height of pitch cylinder. The percentage wear was about 4%.

After fourth load stage pitting propagation still proceeds along the tooth line as well as along his height. Damage of pinion tooth surface after fourth load stage is shown on Fig. 10.

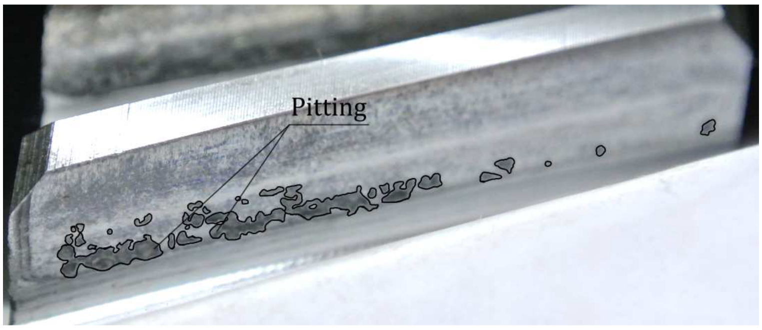

Fig. 10. Pitting on pinion of tested involute gear after fourth load stage

Fatigue wear started to appear on the pitch diameter and near below. Percentage damage of tooth was 10%. The propagation of pitting could also be seen on wheel tooth surface what was shown on Fig. 11.

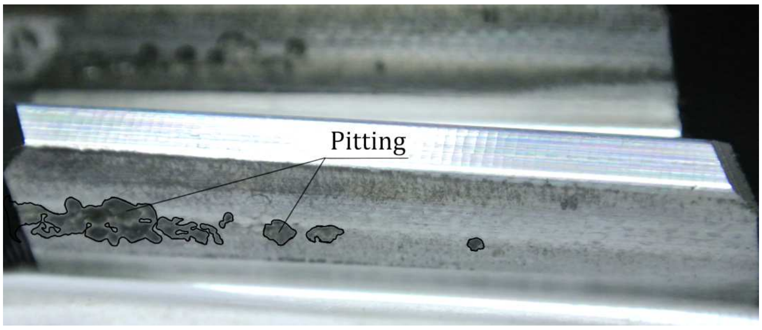

Fig. 11. Pitting on wheel of tested involute gear after fourth load stage

Here the percentage wear of tooth reached 6%. Moreover in case of pinion as well as in case of wheel abrasive wear appeared as matted surface.

Fifth load stage caused further propagation of pitting failure on the pinion and wheel teeth. Figure 12 presents wear of pinion tooth after fifth load stage.

Fig. 12. Pitting on pinion of tested involute gear after fifth load stage

Pitting wear covered 13% of tooth surface. Figure 13 presents wear of wheel tooth.

After fifth load stage wheel tooth was damaged by pitting in 11%.

The summary percentage wear of gear teeth defined as the average wear of pinion and wheel and the percentage wear of pinion and wheel tooth in relationship with load stage are shown on Fig. 14.

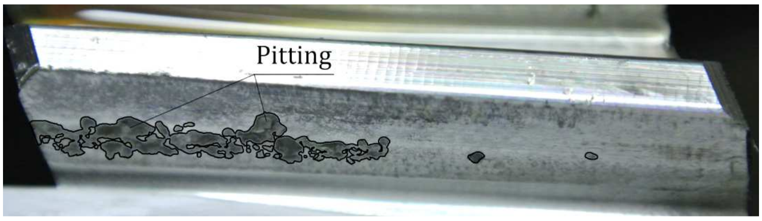

Fig. 13. Pitting on wheel of tested involute gear after fifth load stage

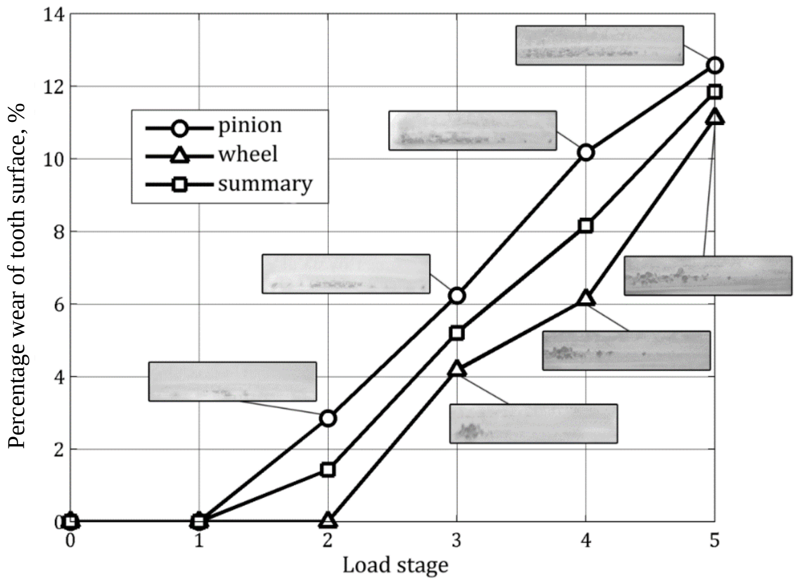

Fig. 14. Percentage wear of tooth surface and summary wear in relation with load stage

5. Wear of Novikov Gear

Similarly as in involute gears after zero and first load stage the roughness of teeth was decreased as results from polishing.

After second load stage the wear of pinion tooth could be observed on figure 15 as fine scratches along the tooth height (the sliding direction).

Fig. 15. Pinion tooth surface of tested Novikov gear after second load stage

Moreover the undercutting (interference wear) of wheel tooth root by the pinion tooth tip was observed (Fig. 15). In case of wheel wear also appeared as fine scratches along the tooth height (Fig. 16) although they were more densely arranged than in case of pinion.

Fig. 16. Wheel tooth surface of tested Novikov gear after second load stage

Any signs of pitting were not observed.

After third load stage still there were no signs of pitting and even propagation of wear which occurred after previous load stage on both of pinion tooth surface (Fig. 17) and wheel tooth surface (Fig. 18).

Fig. 17. Pinion tooth surface of tested Novikov gear after third load stage

Fig. 18. Wheel tooth surface of tested Novikov gear after third load stage

After fourth load stage the pitting on pinion tooth (Fig. 19) and wheel tooth (Fig. 20) was observed. Surface of pinion tooth was damaged by pitting in 2%.

In case of wheel pitting wear covered 3% of tooth. The pits started to growing along the contact line.

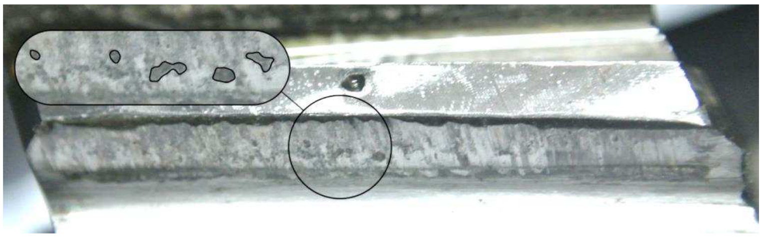

Fig. 19. Pitting on pinion tooth of tested Novikov gear after fourth load stage

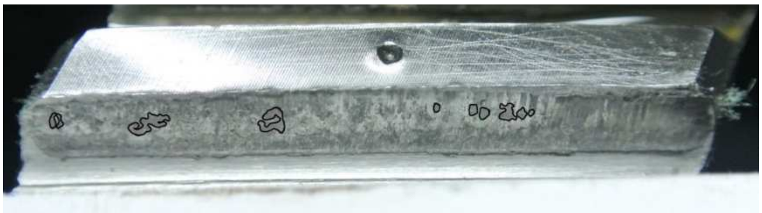

Fig. 20. Pitting on wheel tooth of tested Novikov gear after fourth load stage

What's more with the pitting the abrasive wear appeared. This kind of wear can result from affecting the detached pieces of metal on surfaces of teeth being in contact. It could be observed as wide scratches along teeth height.

After fifth load stage the propagation of pitting along the contact line occurred. The percentage wear of pinion tooth was 8% (Fig. 21).

Fig. 21. Pitting on pinion tooth of tested Novikov gear after fifth load stage

Pitting wear of wheel tooth covered 13% of its area (Fig. 22).

Fig. 22. Pitting on wheel tooth of tested Novikov gear after fourth load stage

The summary percent of wear of gear teeth defined as the average wear of pinion and wheel and the percentage wear of pinion and wheel tooth in relationship with load stage are shown on Fig. 23.

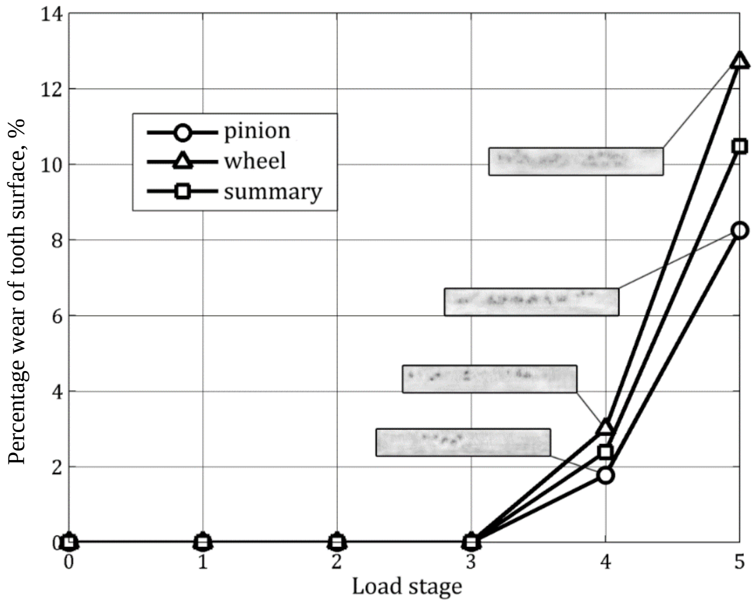

Fig. 23. Percentage wear of tooth surface and summary wear in relation with load stage

6. Comparison of Results

Figure 24 presents comparison of summary percentage wear of teeth for Novikov and involute gears.

The involute gear operated without any signs of pitting over one load cycle (without taking into account running in load stage) which gives $2{,}5\cdot 10^6$ load cycles of pinion. The pitting of Novikov gear appeared after fourth load stage which means running over $7{,}5\cdot 10^6$ load cycles of pinion (without taking into account running in load stage). Therefore the Novikov gear operated three times longer without any signs of fatigue wear. It follows that tested Novikov gear is distinguished by at least three times greater durability than tested involute gear and therefore greater load capacity.

The final summary percentage wear of involute gear teeth was 12%. After fifth load stage on the pinion (Fig. 25) and wheel (Fig. 26) teeth the deep pits with a large diameters (up to 5mm) could be observed.

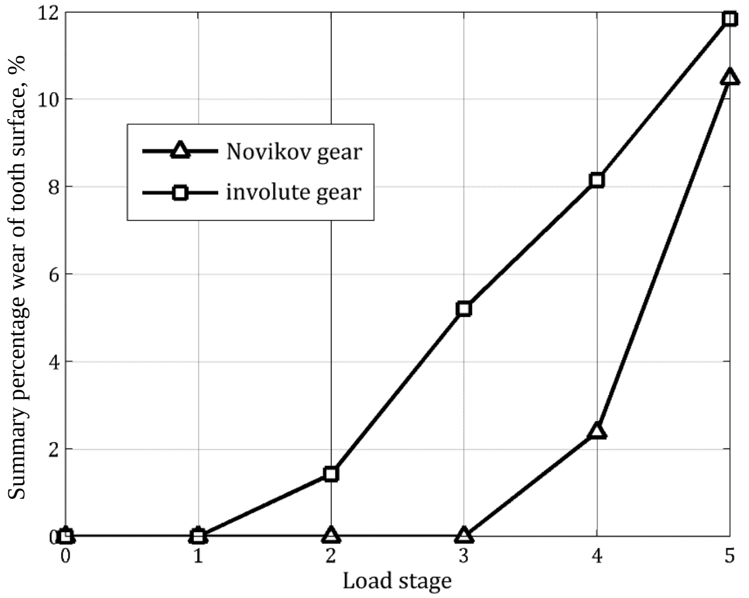

Fig. 24. Comparison of summary percentage wear of pinion and wheel teeth for Novikov and involute gears

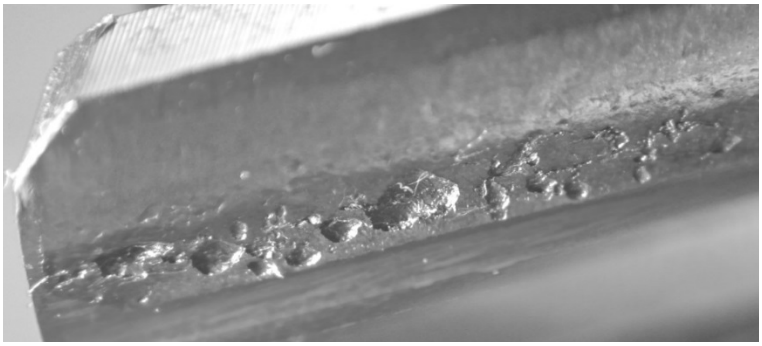

Fig. 25. Macrophotograph of pitting on pinion tooth surface of involute gear

The mechanism of creation of such large pits can be explained based on the wheel pitting (Fig. 26). At the initial stage of pitting the small pits are formed. These small pits becomes the origin of fatigue crack. Due to oil cyclically pressed into small gap the propagation of crack takes place, which can be observed as a beach marks. Next the rapid detachment of piece of metal at the crack growth location occurs. This kind of wear is dangerous because it can lead to fracture.

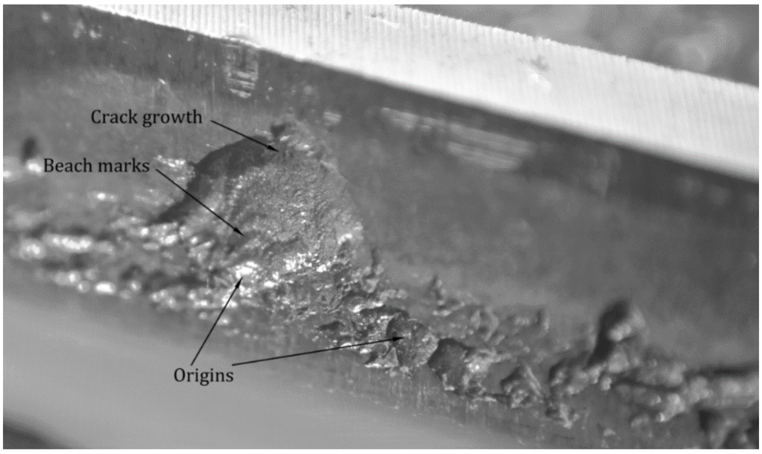

Fig. 26. Macrophotograph of pitting on wheel tooth surface of involute gear

Final summary percentage wear of Novikov teeth was 10%. It should be mentioned that Novikov gear teeth are lower than involute ones. In case of tested gears the tooth height was 4,5 mm for Novikov and 6,75 mm for involute. That's why the percentage wear of Novikov gear is similar to involute one. The real area of wear could be smaller. In case of Novikov gears pitting occurred as a shallow pits with small diameters up to 2 mm (Fig. 27).

Fig. 27. Macrophotograph of pitting on pinion tooth surface of Novikov gear

Along with pitting the abrasive wear could be observed (Fig. 28). This wear occurred simultaneously with pitting so pitting could lead to it due to rubbing the teeth surfaces by detached pieces of metal. Moreover because of smaller than in involute gear diameters of pits eventually tooth fracture could be delayed.

Fig. 28. Macrophotograph of pitting on wheel tooth surface of Novikov gear

Figure 29 shows comparison of harmonic components amplitudes of gear case acceleration signal with frequency of meshing for tested involute and Novikov gears.

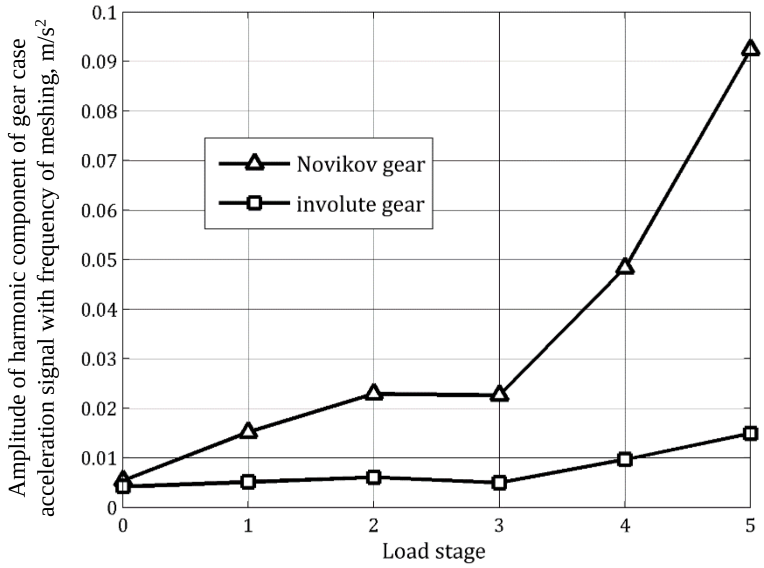

Fig. 29. Comparison of harmonic components amplitudes of gear case acceleration signal with frequency of meshing for tested involute and Novikov gears

Novikov gear generated vibration of greater amplitude. For third load stage the vibration amplitude generated by involute gear was 4.6 times lower. It can results from the fact that involute gearing is conjugated in transverse section and total tooth contact ratio is the sum of overlap ratio and transverse ratio (for tested gear pair $\varepsilon_\gamma = 2{,}73$), while the Novikov gearing is realizing the continuity of meshing only by overlap ratio (for tested gear pair $\varepsilon_\beta = 1{,}21$). After occurrence of pitting, amplitudes of vibration increased but for involute gear this increase is less sharp. It can results from that the Novikov gear was simultaneously damaged by scoring. After fifth load stage amplitude of Novikov gear vibration is 6.2 times greater than involute one. The increased vibration amplitude of Novikov gear in comparison with involute one is with no doubt the disadvantage of this kind of tooth profile which can disqualify it in some applications.

7. Conclusions

Presented results of comparative fatigue tests of gears with convexo-concave Novikov and involute teeth profiles shows that the Novikov gears are distinguished by higher surface load capacity than involute ones. Tested Novikov gear wheels were manufactured less accurately then tested involute ones. Similarly the teeth surface roughness of involute gears was a class lower than in case of Novikov gears. The above-mentioned parameters (accuracy class and surface roughness) should decrease the load capacity of Novikov gear. However, the Novikov gear was at least three times more durable than involute one.

The pitting wear of involute gear was first observed on pinion tooth. In case of Novikov gear pitting occurred simultaneously on pinion and wheel tooth surfaces.

The Novikov type gears have with no doubt a lot of disadvantages. The one as it was presented is the generation of vibration with greater amplitudes in comparison with involute type gears. In case of tested gears the amplitude of vibration generated by gears with convexo-concave teeth profile was about five times greater (for third load stage) than in gears with involute teeth profile. The increased vibration amplitude of Novikov gearing can lead to faster wearing of other gearbox elements (i.e. bearings). The possible cause of this phenomenon is that Novikov gears are realizing the continuity of meshing only by overlap ratio and that the tested Novikov gear wheels were manufactured less accurate.

Summarizing, Novikov gears are the alternative for involute gears which allows to increase the surface load capacity of gear pair or to decrease their dimensions.

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