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UNSTEADY MHD PLANE COUETTE-POISEUILLE FLOW OF FOURTH-GRADE FLUID WITH THERMAL RADIATION, CHEMICAL REACTION AND SUCTION EFFECTS

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This study investigates the unsteady MHD flow of a fourth-grade fluid in a horizontal parallel plates channel. The upper plate is oscillating and moving while the bottom plate is stationary. Solutions for momentum, energy and concentration equations are obtained by the He-Laplace scheme. This method was also used by Idowu and Sani [12] and there is agreement with our results. The effect of various flow parameters controlling the physical situation is discussed with the aid of graphs. Significant results from this study show that velocity and temperature fields increase with the increase in the thermal radiation parameter, while velocity and concentric fields decrease with an increase in the chemical reaction parameter. Furthermore, velocity, temperature and concentric fields decrease with an increase in the suction parameter. It is also interesting to note that when $S_4 = 0$, our results will be in complete agreement with Idowu and Sani [12] results. The results of this work are applicable to industrial processes such as polymer extrusion of dye, draining of plastic films etc.

Keywords: thermal radiation, MHD, chemical reaction, fourth-grade fluid, suction, He-Laplace

1. Introduction

A fourth-grade fluid is an important subclass of differential type that is capable of describing shear thinning and shear thickening effects (examples are ketchup, blood, paint, cream, nail polish, etc.). This sort of model is used to explain the flow behaviour of non-Newtonian fluids which are considered vital and applicable in many industrial production processes such in the drilling of oil and gas wells, polymer extrusion from dye, glass fibre, paper production and draining of plastics films, etc.

Many empirical and semi-empirical non-Newtonian models or constitutive equations have been proposed. Rehan *et al.* [1] considered the steady flow of a fourth grade fluid between two parallel plates. They analyzed four types of flows: Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear differential equation describing the velocity field was solved by the optimal homotopy asymptotic method (OHAM). They observed that the OHAM was more efficient and flexible than the perturbation and homotopy analyses method. Islam *et al.* [2] considered the steady flow of a non-Newtonian fluid with slippage between the plate and the fluid. The constitutive equations of the fluids were modelled for a fourth-grade non-Newtonian fluid with partial slip. They employed homotopy perturbation and optimal homotopy asymptotic methods to solve the non-linear differential equation. Shehzad *et al.* [3] studied the electro-osmotic Couette-Poiseuille flow of a power law Al2O3- PVC nanofluid through a channel, in which the upper wall is moving with constant velocity. The influences of the magnetic field, mixed convection, Joule heating, and viscous dissipation were also examined. The flow was generated because of a constant pressure gradient in the axial direction. The resulting flow problem was described by coupled nonlinear ordinary differential equations,

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which were at first modeled and then transform into a dimensionless form through appropriate transformations. An analytical solution of the governing equation was given.

Fenuga *et al.* [4] investigated the mathematical model and solution for an unsteady MHD fourth grade fluid flow over a vertical plate in a porous medium with the effects of the magnetic field and suction/injection parameters using the Homotopy Perturbation Method. They displayed graphically and discussed the impact of dimensionless second, third and fourth grade parameters with the effects of the magnetic field and suction/injection parameters on the velocity field. They found out that on increase in the suction parameter decreases the momentum boundary layer thickness while the injection parameter enhances velocity distribution in the boundary layer. Magnetic field reduces velocity throughout the boundary layer because the Lorentz force which acts as a retarding force reduces the boundary layer thickness. Khan *et al.* [5] discussed the unsteady flow of a non- Newtonian fluid with the properties of heat/sink in the presence of thermal radiation. Santhosha *et al.* [6] studied the radiation and chemical combined effects on an MHD free convective heat and mass transfer flow of a viscous, incompressible, conducting elastic fluid through a porous medium limited by a porous plate within the presence of heat generation. The momentum, energy and mass diffusion equations were coupled non-linear partial differential equations. They employed the two term perturbation method.

Yurusoy [7] investigated the time dependent boundary layer flow of a modified power-law fluid of fourth grade on a stretched surface with an injection or suction boundary condition. The fluid model is a mixture of fourth grade and power-law fluids in which the fluid may display shear thickening, shear thinning or normal stress textures. He used the scaling and translation transformations which is a type of Lie Group transformation. Time dependent boundary layer equations were reduced into two alternative ordinary differential equations systems (ODEs) with boundary conditions. He found out that the boundary layer thickness decreases as the power-law index value increases. And also, as the fourth-grade fluid parameter, increases, the boundary layer thickness decreases while the velocity in the y direction increases.

Taza *et al.* [8] studied the unsteady thin film flow of a fourth grade fluid over a moving and oscillating vertical belt. They employed the adomian decomposition method (ADM) and optimal homotopy asymptotic method (OHAM) to find the solution of the non- linear differential equations that governed the flow. Hayat *et al.* [9] presented the exact solution four four types of flows between two parallel plates, viz. Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear second-order differential equation for the velocity field was solved exactly in each case. The nonlinear differential equation describing the velocity field was solved by the optimal homotopy asymptotic method (OHAM). They observed that the OHAM is more efficient and flexible than the perturbation and homotopy analyses method. Arifuzzaman *et al.* [10] analysed heat and mass transfer characteristics of naturally corrective hydro-magnetic flows of a fourth grade radiative fluid flow throush a vertical porous plate. They considered a non-linear order chemical reaction and heat generation with thermal diffusion. The complete fundamental equations were transformed into dimensionless equations by implementing the finite difference scheme explicitly.

Priyadarsan and Panda [11] carried out a numerical investigation to study the unsteady flow of an incompressible and electrically conducting fourth-grade fluid through a porous medium between two infinite parallel plates under a transverse magnetic field with time-dependent suction. The lower plate is at rest and the upper plate is moving and oscillating in its own plane at about a constant mean velocity with time-dependent suction. The basic equations governing the flow and heat transfer are reduced to a set of non-linear partial differential equations. The governing equations are simplified using the perturbation method with respect to time and the resulting sixth-order non-linear differential equations are solved numerically using the Runge-Kutta method in association with the multi-shooting technique. Their investigation revealed that the higher-grade fluid parameters influence significantly the fluid temperature.

Idowu and Sani [12] carried out an analysis for an unsteady magnetohydrodynamic (MHD) flow of a generalized third grade fluid between two parallel plates. The fluid flow was a result of the plate oscillation, movingment and pressure gradient. Three flow problems were investigated, namely: Couette, Poiseuille and Couette-Poiseuille flows and a number of nonlinear partial differential equations were obtained which were solved using the He-Laplace method. Expressions for the velocity field, temperature and concentration fields were given for each case and finally, effects of physical parameters on the fluid motion, temperature and concentration were plotted and discussed. They found that an increase in the thermal radiation parameter increases the temperature of the fluid and hence reduces the viscosity of the fluid while the concentration of the fluid reduces as the chemical reaction parameter increases.

In the above aforementioned investigations however, the effects of suction on the unsteady MHD flow of fourth-grade fluid with thermal radiation and chemical reaction have not been studied. Suction is the act or process of sucking. A force that causes a fluid or solid to be drawn into an interior space or to adhere to the surface because of the difference between the external and internal pressure. This is considered to be due to the porosity of the channel plates.

2. Formulation of the problem

We consider the unsteady flow of an electrically conducting incompressible fourth grade fluid between two horizontal parallel plates channel as shown in Fig.1. below. The fluid is subjected to a uniform transverse magnetic field. We assumed the bottom plate is fixed (stationary) and the top plate is moving with constant velocity.

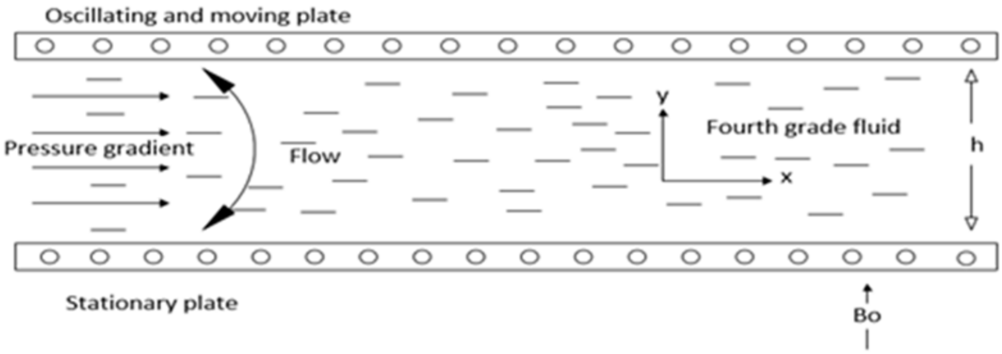

Fig.1. Physical configuration of the flow.

The state of this fluid is determined by the history of the deformation gradient without a preferred reference configuration. Its constitute equation can be written as

$$T(x,t) = -PI + f_{s=0}^{\infty}\left(F_t^t(s)\right) \tag{2.1}$$

where PI is the undetermined part of the stress tensor, F is the deformation gradient and f is the functional. Coleman and Noll [13] studied a different sort of incompressible fluid grade n as viscous fluid described in Hayat *et al.* [14]. An incompressible fluid of differential type of grade n is a simple fluid obeying the constitutive equation

$$T = -PI + \sum_{j=1}^{n} S_j , \tag{2.2}$$

obtained by asymptotic expansion of the functional in Eq.(2.1) through a retardation parameter α. For $n = 4$ as in Hayat *et al.* ([14], [15]]) and Arifuzzaman *et al.* [10], the first four (4) tensors S_j are given by

$$S_1 = \mu A_1 , \tag{2.3}$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2 , \tag{2.4}$$

$$S_3 = \beta_1 A_3 + \beta_2 \left(A_1 A_2 + A_2 A_1 \right) + \beta_3 \left(tr A_1^2 \right) A_1 , \tag{2.5}$$

$$\begin{aligned} S_4 = {} & \gamma_1 A_4 + \gamma_2 \left(A_3 A_1 + A_1 A_3 \right) + \gamma_3 A_2^2 + \gamma_4 \left(A_2 A_1^2 + A_1^2 A_2 \right) + \\ & + \gamma_5 \left(tr A_2 \right) A_2 + \gamma_6 \left(tr A_2 \right) A_1^2 + \left[\gamma_7 tr A_3 + \gamma_8 tr \left(A_2 A_1 \right) \right] A_1 \end{aligned} \tag{2.6}$$

where, μ is the coefficient of shear viscosity, $\alpha_i \left(i = 1,2 \right)$, $\beta_i \left(i = 1,2,3 \right)$ and $\gamma_i \left(i = 1(1)8 \right)$ are material constants. The A_n are the Rivlin-Ericksen tensors defined by the recursion relation

$$A_n = \frac{d}{dt} A_{n-1} + A_{n-1} L + L^T A_{n-1} , \quad n>1 , \tag{2.7}$$

$$A_1 = L + L^T \tag{2.8}$$

where $L = \nabla V$, $\frac{d}{dt}$ is the material time derivative and V is the velocity.

We note that when $\gamma_i = 0$, the fourth grade model reduces to the third grade model. When $\beta_i = 0$, the third grade model reduces to a second grade model. When $\alpha_i = 0$, $\beta_i = 0$ and $\gamma_i = 0$ then the model reduces to classical Navier-Stoke fluid.

The thermally radiative and chemically reactive flow is heading towards the x -direction along infinite porous plate with heat generation. Here, U_0 is the uniform velocity, T_∞ and C_∞ are the fluid temperature and concentration.

Under the above assumption, the equations that described the physical circumstances are

$$\frac{\partial v}{\partial y} = 0 , \tag{2.9}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = {} & - \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1 \nu}{\rho} \frac{\partial^3 u}{\partial y^2 \partial t} + \frac{\beta_1 \nu^2}{\rho} \frac{\partial^4 u}{\partial y^2 \partial t^2} + \frac{6 \left(\beta_2 + \beta_3 \right)}{\rho} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \\ & + \frac{\gamma_1 \nu^3}{\rho} \frac{\partial^5 u}{\partial y^2 \partial t^3} + \frac{2 \nu \left(3\gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_7 + \gamma_8 \right)}{\rho C_p} \left[2 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^3 u}{\partial y^2 \partial t} \right] + \\ & - \frac{\sigma B_0^2}{\rho C_p} u + g \beta_T \left(T - T_\infty \right) + g \beta_C \left(C - C_\infty \right) - \frac{\nu}{k} u , \end{aligned} \tag{2.10}$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} \left(T_w - T_\infty \right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} , \tag{2.11}$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_c \left(C - C_\infty \right) . \tag{2.12}$$

From Eq.(2.11), q_r is the radiative heat flux defined as

$$\frac{\partial q_r}{\partial y} = 4\alpha^2 \left(T_w - T_\infty\right). \tag{2.12a}$$

The initial and boundary conditions are

$$\left.\begin{aligned} &u = U_0 e^{-yh}, \quad T = T_0 + \left(T_w - T_\infty\right)e^{-yh}, \quad C = C_0 + \left(C_w - C_\infty\right)e^{-yh} \quad \text{at} \quad t = 0 \quad \text{for} \quad 0 \le y \ge h, \\ &u(y,t) = U, \quad T(y,t) = T_w, \quad C(y,t) = C_w \quad \text{at} \quad y = h \quad \text{for} \quad t \ge 0, \\ &u(y,t) \to \infty, \quad T(y,t) \to \infty, \quad C(y,t) \to \infty \quad \text{as} \quad y \to \infty \quad \text{for} \quad t > 0. \end{aligned}\right\} \tag{2.13}$$

where u is the fluid velocity, T is the temperature and C is the species concentration equation, q_r is the radiative heat flux, ρ is the density of the fluid, C_p is the heat capacity, B_0 is the external magnetic field.
In order to transform Eqs (2.10)-(2.13), we use the following dimensionless parameters

$$u^* = \frac{u}{U_0}, \quad p^* = \frac{p}{\mu U_0^2}, \quad t^* = \frac{tU_0^2}{\nu}, \quad Ha^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \quad G_r = \frac{g\beta_T \left(T_w - T_\infty\right)\nu}{U_0^3},$$

$$G_c = \frac{g\beta_C \left(C_w - C_\infty\right)\nu}{U_0^3}, \quad Da = \frac{KU_0^2}{h^2}, \quad S_c = \frac{D}{\nu}, \quad y^* = \frac{yU_0}{\nu}, \quad x^* = \frac{x}{h},$$

$$h = \frac{U_0}{\nu}, \quad P_r = \frac{kU_0^2}{\nu^2}, \quad S = \frac{\mathrm{v}_0}{U_0}, \quad \mathrm{v} = \frac{\mathrm{v}}{U_0}, \quad \theta = \frac{T - T_0}{T_w - T_\infty}, \quad C^* = \frac{C - C_0}{C_w - C_\infty}, \tag{2.14}$$

$$\delta = \frac{4\alpha^2 U_0^2}{\rho c_p \nu}, \quad \alpha = \frac{\alpha_1 U_0^2}{\rho \nu^2}, \quad \beta_a = \frac{\beta_1 U_0^4}{\rho \nu^3}, \quad \beta_b = \frac{\left(\beta_2 + \beta_3\right)U_0^4}{\rho \nu^3}, \quad \gamma_a = \frac{\gamma_1 U_0^6}{\rho \nu^3},$$

$$\gamma_b = \frac{2\left(3\gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + 3\gamma_7 + \gamma_8\right)U_0^6}{\rho \nu^4}, \quad Da = \frac{kU_0^2}{\nu^2}, \quad K_r = \frac{K_c \nu}{U_0^2}.$$

Substituting Eq.(2.14) into Eq.(2.10)-(2.13) and by dropping the asterisks, we have the following:

$$\frac{\partial v}{\partial y} = 0 \quad \Rightarrow \quad \mathrm{v} = -\mathrm{v}_0, \tag{2.15}$$

$$\begin{aligned} &\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} + \beta_a \frac{\partial^4 u}{\partial y^2 \partial t^2} + \beta_b \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_a \frac{\partial^5 u}{\partial y^2 \partial t^3} + \\ &+\gamma_b \left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - \left(Ha + \frac{1}{Da}\right)u + G_r\theta + G_c C, \end{aligned} \tag{2.16}$$

$$\frac{\partial\theta}{\partial t}-S\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+(Q_0+\delta)\theta\,, \tag{2.17}$$

$$\frac{\partial C}{\partial t}-S\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-K_rC\,. \tag{2.18}$$

And the initial and boundary conditions become

$$\left.\begin{array}{llllll} u(y,t)=e^{-y}, & \theta(y,t)=e^{-y}, & C(y,t)=e^{-y} & \text{at} \quad t=0 & \text{for} & 0\le y\ge 1 \\ u(y,t)=1, & \theta(y,t)=1, & C(y,t)=1 & \text{at} \quad y=1 & \text{for} & t\ge 0, \\ u(y,t)\to\infty, & T(y,t)\to\infty, & C(y,t)\to\infty & \text{as} \quad y\to\infty & \text{for} & t>0. \end{array}\right\} \tag{2.19}$$

3. Method of solution of the problem

In this section we employed the He-Laplace scheme to solve Eq.(2.16) to (2.18) subjected to the initial and boundary conditions (2.19).

Since Eq.(2.16) is a coupled non-linear partial differential equation, we have to solve Eq.(2.17) and (2.18) first.

Now applying Laplace transform on Eq.(2.18), we have

$$L\left\{\frac{\partial C}{\partial t}\right\}-SL\left\{\frac{\partial C}{\partial y}\right\}=\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\}-L\{K_rC\}\,. \tag{3.1}$$

Applying the initial condition and dividing through by s and rearranging, we obtain

$$L\{C(y,t)\}=\frac{e^{-y}}{s}+\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\}+SL\left\{\frac{\partial C}{\partial y}\right\}-L\{K_rC\}\right\}. \tag{3.2}$$

Taking the inverse Laplace transform of both sides of Eq.(3.2), gives

$$C(y,t)=e^{-y}+L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\}+SL\left\{\frac{\partial C}{\partial y}\right\}-L\{K_rC\}\right\}\right]. \tag{3.3}$$

Applying the homotopy perturbation technique, Eq.(3.3) yields

$$\sum_{n=0}^{\infty}P^nC_n(y,t)=e^{-y}+P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\}-L\{K_rC\}\right\}\right\}\right]. \tag{3.4}$$

Comparing the coefficients of the like powers of 'P', the following approximations were obtained

$$P^0: C_0(y,t)=e^{-y}, \tag{3.5}$$

$$P^1 : C_1(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C_0}{\partial y^2}\right\} + SL\left\{\frac{\partial C_0}{\partial y}\right\} - L\{K_r C_0\}\right\}\right] =$$
$$= L^{-1}\left\{\frac{1}{S_c}\left(\frac{e^{-y}}{s^2}\right) - S\left(\frac{e^{-y}}{s^2}\right) - K_r\left(\frac{e^{-y}}{s^2}\right)\right\} = \left(\frac{e^{-y}}{S_c} - Se^{-y} - K_r e^{-y}\right)t, \tag{3.6}$$

$$P^2 : C_2(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C_1}{\partial y^2}\right\} + SL\left\{\frac{\partial C_1}{\partial y}\right\} - L\{K_r C_1\}\right\}\right] =$$
$$= L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\left(\frac{e^{-y}}{S_c} - Se^{-y} - K_r e^{-y}\right)t\right\} - SL\left\{\left(Se^{-y} - \frac{e^{-y}}{S_c} + K_r e^{-y}\right)t\right\}\left[\left\{+\right\}\right]\right.\right. =$$
$$\left.\left. - L\left\{K_r\left(\frac{e^{-y}}{S_c} - Se^{-y} - K_r e^{-y}\right)t\right\}\right\}\right] = \left(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + \right.$$
$$\left. + 2K_r Se^{-y} + S^2 e^{-y} + K_r^2 e^{-y}\right)\frac{t^2}{2!}, \tag{3.7}$$

$$P^3 : C_3(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C_2}{\partial y^2}\right\} + SL\left\{\frac{\partial C_2}{\partial y}\right\} - L\{K_r C_2\}\right\}\right] =$$
$$= L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\left(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + +2K_r Se^{-y} + S^2 e^{-y} + K_r^2 e^{-y}\right)\frac{t^2}{2!}\right\} + \right.\right.$$
$$+ SL\left\{\left(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + 2K_r Se^{-y} + S^2 e^{-y} + K_r^2 e^{-y}\right)\frac{t^2}{2!}\right\} +$$
$$\left.\left. - L\left\{K_r\left(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + +2K_r Se^{-y} + S^2 e^{-y} + K_r^2 e^{-y}\right)\frac{t^2}{2!}\right\}\right\}\right] =$$
$$= \left(\frac{e^{-y}}{S_c^3} - \frac{3Se^{-y}}{S_c^2} - \frac{2K_r e^{-y}}{S_c^2} + \frac{6K_r Se^{-y}}{S_c} + \frac{3S^2 e^{-y}}{S_c} + \frac{3K_r^2 e^{-y}}{S_c} - \frac{K_r e^{-y}}{S_c} + \right.$$
$$\left. - K_r S^2 e^{-y} - 3K_r^2 Se^{-y} - S^3 e^{-y} - K_r^3 e^{-y}\right)\frac{t^3}{3!}. \tag{3.8}$$

Therefore, in view of equations (3.5), (3.6), (3.7) and (3.8), the solution to equation (2.18) is

$$C(y,t) = C_0(y,t) + C_1(y,t) + C_2(y,t) + C_3(y,t)\cdots \tag{3.9}$$

$$C(y,t) = e^{-y} + \left(\frac{e^{-y}}{S_c} - Se^{-y} - K_r e^{-y}\right)t + \left(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + 2K_r Se^{-y} + \right.$$
$$\left. + S^2 e^{-y} + K_r^2 e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{S_c^3} - \frac{3Se^{-y}}{S_c^2} - \frac{2K_r e^{-y}}{S_c^2} + \frac{6K_r Se^{-y}}{S_c} + \frac{3S^2 e^{-y}}{S_c} + \right. \tag{3.10}$$

$$+\frac{3K_r^2e^{-y}}{S_c}-\frac{K_re^{-y}}{S_c}-K_rS^2e^{-y}-3K_r^2Se^{-y}-S^3e^{-y}-K_r^3e^{-y}\Bigg)\frac{t^3}{3!}+\cdots \tag{cont.3.10}$$

Next, we consider Eq.(2.17), which is rearranged to give

$$\frac{\partial\theta}{\partial t}-S\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+(l_1)\theta \quad \text{where} \quad l_1=Q_0+\delta. \tag{3.10a}$$

Now applying the Laplace transform to Eq.(2.16)

$$L\left\{\frac{\partial\theta}{\partial t}\right\}-L\left\{S\frac{\partial\theta}{\partial y}\right\}=\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\}+L\{l_1\theta\}. \tag{3.11}$$

Applying the initial condition and dividing by s and rearranging we obtain

$$L\{\theta(y,t)\}=\frac{e^{-y}}{s}+\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\}+SL\left\{\frac{\partial\theta}{\partial y}\right\}+L\{l_1\theta\}\right\}. \tag{3.12}$$

Taking the inverse Laplace transform of both sides of Eq.(31) gives

$$\theta(y,t)=e^{-y}+L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\}+SL\left\{\frac{\partial\theta}{\partial y}\right\}+L\{l_1\theta\}\right\}\right]. \tag{3.13}$$

Applying the homotopy perturbation technique to Eq.(3.13), yields

$$\sum_{n=0}^{\infty}P^n\theta_n(y,t)=e^{-y}+P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\}+L\{\delta\theta\}\right\}\right\}\right]. \tag{3.14}$$

Comparing the coefficients of the like powers of 'P' in Eq.(3.14), the following approximations are obtained

$$P^0:\theta_0(y,t)=e^{-y}, \tag{3.15}$$

$$P^1:\theta_1(y,t)=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_0}{\partial y^2}\right\}+SL\left\{\frac{\partial\theta_0}{\partial y}\right\}+L\{l_1\theta_0\}\right\}\right]=$$
$$=L^{-1}\left\{\frac{1}{P_r}\left(\frac{e^{-y}}{s^2}\right)-S\left(\frac{e^{-y}}{s^2}\right)+l_1\left(\frac{e^{-y}}{s^2}\right)\right\}=\left(\frac{e^{-y}}{P_r}-Se^{-y}+l_1e^{-y}\right)t, \tag{3.16}$$

$$P^2:\theta_2(y,t)=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_1}{\partial y^2}\right\}+SL\left\{\frac{\partial\theta_1}{\partial y}\right\}+L\{l_1\theta_1\}\right\}\right]=$$
$$=L^{-1}\left[\left\{\frac{1}{s}\frac{1}{P_r}L\left\{\left(\frac{e^{-y}}{P_r}-Se^{-y}+l_1e^{-y}\right)t\right\}-SL\left\{\left(Se^{-y}-\frac{e^{-y}}{P_r}-l_1e^{-y}\right)t\right\}+\right.\right.$$
$$\left.\left.+L\left\{l_1\left(\frac{e^{-y}}{P_r}-Se^{-y}+l_1e^{-y}\right)t\right\}\right\}\right]=\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}-S^2e^{-y}+l_1^2e^{-y}\right)\frac{t^2}{2!}, \tag{3.17}$$

$$
\begin{aligned}
&P^3:\theta_3(y,t)=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_2}{\partial y^2}\right\}+SL\left\{\frac{\partial\theta_2}{\partial y}\right\}+L\{l_1\theta_2\}\right\}\right]=\\
&=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}-S^2e^{-y}+l_1^2e^{-y}\right)\frac{t^2}{2!}\right\}-SL\left\{\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}+\right.\right.\right.\right.\\
&\left.\left.\left.\left.-S^2e^{-y}+l_1^2e^{-y}\right)\frac{t^2}{2!}\right\}++\left(S^2e^{-y}-\frac{e^{-y}}{P_r^2}-\frac{2l_1e^{-y}}{P_r}-l_1^2e^{-y}\right)\frac{t^2}{2!}\right\}\right]=\\
&=\left(\frac{e^{-y}}{P_r^3}+\frac{3l_1e^{-y}}{P_r^2}-\frac{S^2e^{-y}}{P_r}+\frac{3l_1^2e^{-y}}{P_r}-\frac{Se^{-y}}{P_r}-\frac{2l_1Se^{-y}}{P_r}+\right.\\
&\left.-l_1^2Se^{-y}-l_1S^2e^{-y}+l_1^3e^{-y}+S^3e^{-y}\right)\frac{t^3}{3!}.
\end{aligned}
\tag{3.18}
$$

Therefore, in view of Eqs (3.15)-(3.18), the solution to (2.17) is,

$$
\theta(y,t)=\theta_0(y,t)+\theta_1(y,t)+\theta_2(y,t)+\theta_3(y,t)\cdots,
$$

$$
\begin{aligned}
&\theta(y,t)=e^{-y}+\left(\frac{e^{-y}}{P_r}-Se^{-y}+l_1e^{-y}\right)t+\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}-S^2e^{-y}+l_1^2e^{-y}\right)\frac{t^2}{2!}+\\
&+\left(\frac{e^{-y}}{P_r^3}+\frac{3l_1e^{-y}}{P_r^2}-\frac{S^2e^{-y}}{P_r}+\frac{3l_1^2e^{-y}}{P_r}-\frac{Se^{-y}}{P_r}-\frac{2l_1Se^{-y}}{P_r}+\right.\\
&\left.-l_1^2Se^{-y}-l_1S^2e^{-y}+l_1^3e^{-y}+S^3e^{-y}\right)\frac{t^3}{3!}+\cdots.
\end{aligned}
\tag{3.19}
$$

Finally, we now solve Eq.(2.16), which is rearranged to give

$$
\begin{aligned}
&\frac{\partial u}{\partial t}-S\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2u}{\partial y^2}+\alpha\frac{\partial^3u}{\partial y^2\partial t}+\beta_a\frac{\partial^4u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2u}{\partial y^2}+\\
&+\gamma_a\frac{\partial^5u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2u}{\partial y^2}\frac{\partial^2u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3u}{\partial y^2\partial t}\right]-l_2u+G_r\theta+G_cC
\end{aligned}
\tag{3.19a}
$$

where

$$
Ha+\frac{1}{Da}=l_2.
$$

Applying the Laplace transform to both sides of Eq.(3.19a) gives

$$
\begin{aligned}
&L\left\{\frac{\partial u}{\partial t}\right\}-L\left\{S\frac{\partial u}{\partial y}\right\}=L\left\{-\frac{\partial p}{\partial x}+\frac{\partial^2u}{\partial y^2}+\alpha\frac{\partial^3u}{\partial y^2\partial t}+\beta_a\frac{\partial^4u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2u}{\partial y^2}+\right.\\
&\left.+\gamma_a\frac{\partial^5u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2u}{\partial y^2}\frac{\partial^2u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3u}{\partial y^2\partial t}\right]-l_2u+G_r\theta+G_cC\right\}.
\end{aligned}
\tag{3.20}
$$

but

$$L\left\{\frac{\partial u}{\partial t}\right\}=sL\left\{u(y,t)\right\}-u(y,0). \tag{3.21}$$

Hence

$$L\left\{u(y,t)\right\}=\frac{u(y,0)}{s}+\frac{1}{s}L\left\{-\frac{\partial p}{\partial x}+S\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\right.$$
$$\left.+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_2u+G_r\theta+G_cC\right\}. \tag{3.22}$$

Taking the inverse Laplace transform of both sides of Eq.(3.22), we have

$$L^{-1}\left\{L\left\{u(y,t)\right\}\right\}=L^{-1}\left\{\frac{u(y,0)}{s}-\frac{\partial p}{\partial x}+\frac{1}{s}L\left\{S\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\right.\right.$$
$$+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_2u+$$
$$+\frac{G_r}{s}\left(e^{-y}+\left(\frac{e^{-y}}{P_r}-Se^{-y}+l_1e^{-y}\right)t+\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}-S^2e^{-y}+l_1^2e^{-y}\right)\frac{t^2}{2!}\left(\frac{t^3}{3!}\right)+\right.$$
$$+\left(\frac{e^{-y}}{P_r^3}+\frac{3l_1e^{-y}}{P_r^2}-\frac{S^2e^{-y}}{P_r}+\frac{3l_1^2e^{-y}}{P_r}-\frac{Se^{-y}}{P_r}-\frac{2l_1Se^{-y}}{P_r}-l_1^2Se^{-y}-l_1S^2e^{-y}+\right.$$
$$\left.\left.+l_1^3e^{-y}+S^3e^{-y}\right)\right)+\frac{G_c}{s}\left(e^{-y}+\left(\frac{e^{-y}}{S_c}-Se^{-y}-K_re^{-y}\right)t+\left(\frac{e^{-y}}{S_c^2}-\frac{2Se^{-y}}{S_c}+\right.\right. \tag{3.23}$$
$$\left.-\frac{2K_re^{-y}}{S_c}++2K_rSe^{-y}+S^2e^{-y}+K_r^2e^{-y}\right)\frac{t^2}{2!}+\left(\frac{e^{-y}}{S_c^3}-\frac{3Se^{-y}}{S_c^2}-\frac{2K_re^{-y}}{S_c^2}+\right.$$
$$+\frac{6K_rSe^{-y}}{S_c}+\frac{3S^2e^{-y}}{S_c}+\frac{3K_r^2e^{-y}}{S_c}+-\frac{K_re^{-y}}{S_c}-K_rS^2e^{-y}-3K_r^2Se^{-y}+$$
$$\left.\left.\left.\left.-S^3e^{-y}-K_r^3e^{-y}\right)\frac{t^3}{3!}\right)\right\}\right\},$$

or

$$u(y,t)=\lambda+e^{-y}+\left(G_ce^{-y}+G_re^{-y}\right)t+\left(\frac{e^{-y}}{P_r}-2Se^{-y}+l_1e^{-y}+\frac{e^{-y}}{S_c}-K_re^{-y}\right)\frac{t^2}{2!}+$$
$$+\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}+l_1^2e^{-y}+\frac{e^{-y}}{S_c^2}-\frac{2Se^{-y}}{S_c}-\frac{2K_re^{-y}}{S_c}+2K_rSe^{-y}+K_r^2e^{-y}\right)\frac{t^3}{3!}+ \tag{3.24}$$
$$+\left(\frac{e^{-y}}{P_r^3}+\frac{3l_1e^{-y}}{P_r^2}-\frac{S^2e^{-y}}{P_r}+\frac{3l_1^2e^{-y}}{P_r}-\frac{Se^{-y}}{P_r}-\frac{2l_1Se^{-y}}{P_r}-l_1^2Se^{-y}-l_1S^2e^{-y}+l_1^3e^{-y}+\right.$$

$$+\frac{e^{-y}}{S_c^3}-\frac{3Se^{-y}}{S_c^2}-\frac{2K_re^{-y}}{S_c^2}+\frac{6K_rSe^{-y}}{S_c}+\frac{3S^2e^{-y}}{S_c}+\frac{3K_r^2e^{-y}}{S_c}-\frac{K_re^{-y}}{S_c}+$$
$$\left.-K_rS^2e^{-y}-3K_r^2Se^{-y}-K_r^3e^{-y}\right)\frac{t^4}{4!}+L^{-1}\left\{\frac{1}{s}L\left\{S\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\right.\right.$$
$$+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+ \qquad \text{(cont.3.24)}$$
$$\left.\left.+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_2u\right\}\right\}.$$

Applying the homotopy perturbation method to Eq.(3.24), gives

$$\sum_{n=0}^{\infty}P^nu_n(y,t)=\lambda+e^{-y}+\left(G_ce^{-y}+G_re^{-y}\right)t+\left(\frac{e^{-y}}{P_r}-2Se^{-y}+l_1e^{-y}+\frac{e^{-y}}{S_c}+\right.$$
$$\left.-K_re^{-y}\right)\frac{t^2}{2!}+\left(\frac{e^{-y}}{P_r^2}+\frac{2l_1e^{-y}}{P_r}+l_1^2e^{-y}+\frac{e^{-y}}{S_c^2}-\frac{2Se^{-y}}{S_c}-\frac{2K_re^{-y}}{S_c}+2K_rSe^{-y}+\right.$$
$$\left.+K_r^2e^{-y}\right)\frac{t^3}{3!}+\left(\frac{e^{-y}}{P_r^3}+\frac{3l_1e^{-y}}{P_r^2}-\frac{S^2e^{-y}}{P_r}+\frac{3l_1^2e^{-y}}{P_r}-\frac{Se^{-y}}{P_r}-\frac{2l_1Se^{-y}}{P_r}-l_1^2Se^{-y}+\right.$$
$$-l_1S^2e^{-y}+l_1^3e^{-y}+\frac{e^{-y}}{S_c^3}-\frac{3Se^{-y}}{S_c^2}-\frac{2K_re^{-y}}{S_c^2}+\frac{6K_rSe^{-y}}{S_c}+\frac{3S^2e^{-y}}{S_c}+\frac{3K_r^2e^{-y}}{S_c}+ \qquad (3.25)$$
$$\left.-\frac{K_re^{-y}}{S_c}-K_rS^2e^{-y}-3K_r^2Se^{-y}-K_r^3e^{-y}\right)\frac{t^4}{4!}+P\left(L^{-1}\left\{\frac{1}{s}L\left\{S\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\right.\right.\right.$$
$$+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_bH_a(u_n)+\gamma_1\frac{\partial^5 u}{\partial y^2\partial t^3}+$$
$$\left.\left.\left.+\gamma_b\left[2H_b(u_n)+H_c(u_n)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_2u\right\}\right\}\right)$$

where, $H_a(u_n), H_b(u_n)$ and $H_c(u_n)$ are the He's polynomials for

$$\left(\frac{\partial u}{\partial y}\right)^2,\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} \quad \text{and} \quad \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t},$$

respectively.

The He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2$ are as follows

$$\begin{cases} H_0(u) = \left(u_0'\right)^2, \\ H_1(u) = 2u_0'u_1', \\ H_2(u) = 2u_0'u_2' + \left(u_1'\right)^2, \\ H_3(u) = 2u_1'u_2', \\ \vdots \end{cases} \tag{3.26}$$

The He's polynomials for $\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y}$ are as follows

$$\begin{cases} H_0(u) = u_0'''u'_{0t}, \\ H_1(u) = u_0'''u'_{1t} + u_1'''u'_{1t}, \\ H_2(u) = u_0'''u'_{2t} + u_1'''u'_{1t} + u_2'''u'_{0t}, \\ H_3(u) = u_1'''u''_{2t} + u_2'''u''_{1t}, \\ \vdots \end{cases} \tag{3.27}$$

The He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}$ are as follows

$$\begin{cases} H_0(u) = \left(u_0'\right)^2\left(u_0''u_{0t}'\right), \\ H_1(u) = \left(u_0'\right)^2\left(u_0''u_{1t}'\right) + \left(u_0'\right)^2\left(u_1''u_{0t}'\right) + 2u_0'u_1'\left(u_0''u_{0t}'\right), \\ H_2(u) = \left(u_0'\right)^2\left(u_0''u_{2t}'\right) + \left(u_0'\right)^2\left(u_1''u_{1t}'\right) + \left(u_0'\right)^2\left(u_2''u_{0t}'\right) + \\ +2u_0'u_1'\left(u_0''u_{1t}'\right) + 2u_0'u_1'\left(u_1''u_{0t}'\right) + 2u_0'u_2'\left(u_0''u_{0t}'\right), \\ \vdots \end{cases} \tag{3.28}$$

Now, comparing the like powers of "P" in Eq.(3.25) and equating their coefficients gives

$$P^0; u_0(y,t) = \lambda + e^{-y} + \left(G_c e^{-y} + G_r e^{-y}\right)t + \left(\frac{e^{-y}}{P_r} - 2Se^{-y} + l_1 e^{-y} + \frac{e^{-y}}{S_c} - K_r e^{-y}\right)\frac{t^2}{2!} +$$
$$+\left(\frac{e^{-y}}{P_r^2} + \frac{2l_1 e^{-y}}{P_r} + l_1^2 e^{-y} + \frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + 2K_r Se^{-y} + K_r^2 e^{-y}\right)\frac{t^3}{3!} +$$
$$+\left(\frac{e^{-y}}{P_r^3} + \frac{3l_1 e^{-y}}{P_r^2} - \frac{S^2 e^{-y}}{P_r}\frac{3l_1^2 e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2l_1 Se^{-y}}{P_r} - l_1^2 Se^{-y} - l_1 S^2 e^{-y} + l_1^3 e^{-y} + \right. \tag{3.29}$$
$$+\frac{e^{-y}}{S_c^3} - \frac{3Se^{-y}}{S_c^2} - \frac{2K_r e^{-y}}{S_c^2} + \frac{6K_r Se^{-y}}{S_c} + \frac{3S^2 e^{-y}}{S_c} + \frac{3K_r^2 e^{-y}}{S_c} - \frac{K_r e^{-y}}{S_c} +$$
$$\left. -K_r S^2 e^{-y} - 3K_r^2 Se^{-y} - K_r^3 e^{-y}\right)\frac{t^4}{4!},$$

$$P^1; u_1(y,t) = L^{-1}\left\{\frac{1}{s}\, L\left\{S\frac{\partial u_0}{\partial y} + \frac{\partial^2 u_0}{\partial y^2} + \alpha\frac{\partial^3 u_0}{\partial y^2 \partial t} + \beta_a \frac{\partial^4 u_0}{\partial y^2 \partial t^2} + \beta_b \left(u_0'\right)^2\left(u_0''\right) + \right.\right.$$
$$\left.\left. +\gamma_a \frac{\partial^5 u_0}{\partial y^2 \partial t^3} + \gamma_b\left[2u_0''' u'_{0t} + \left(u_0'\right)^2\left(u_0'' u_{0t}'\right)\right] - l_2 u_0\right\}\right\}, \tag{3.30}$$

or

$$u_1(y,t) = \left(e^{-y} - Se^{-y} + \alpha G_c e^{-y} + \alpha G_r e^{-y} + \frac{\beta_a e^{-y}}{S_c} - \beta_a K_r e^{-y} + \frac{\beta_a e^{-y}}{P_r} - \beta_b e^{-y} + \right.$$
$$+\frac{\gamma_a e^{-y}}{S_c^3} - \frac{2\gamma_a Se^{-y}}{S_c} - \frac{2\gamma_a K_r e^{-y}}{S_c} + 2\gamma_a K_r Se^{-y} + \gamma_a K_r^2 e^{-y} + \frac{\gamma_a e^{-y}}{P_r^2} + \frac{2\gamma_a l_1 e^{-y}}{P_r} +$$
$$\left. +\gamma_a l_1^2 e^{-y} - 2\gamma_a G_c e^{-2y} + 2\gamma_a G_r e^{-2y} + \gamma_b G_c e^{-3y} + \gamma_b G_r e^{-3y} - l_2 e^{-y} - \lambda l_2\right)t +$$
$$+\left(G_c e^{-y} + G_r e^{-y} - G_c Se^{-y} - G_r Se^{-y} + \frac{\alpha e^{-y}}{S_c} - 2\alpha Se^{-y} - \frac{\alpha e^{-y}}{P_r} + \alpha l_1 e^{-y} + \frac{\beta_a e^{-y}}{S_c^2} + \right.$$
$$-\frac{2\beta_a Se^{-y}}{S_c} - \frac{2\beta_a K_r e^{-y}}{S_c} + 2\beta_a K_r Se^{-y} + \beta_a K_r^2 e^{-y} + \frac{\beta_a e^{-y}}{P_r^2} - \frac{2\beta_a l_1 e^{-y}}{P_r} + \beta_a l_1^2 e^{-y} +$$
$$+3\beta_b G_c e^{-3y} + 3\beta_b G_r e^{-3y} + \frac{\gamma_a e^{-y}}{S_c^3} - \frac{3\gamma_a e^{-y}}{S_c^2} - \frac{6\gamma_a K_r Se^{-y}}{S_c} + \frac{6\gamma_a S^2 e^{-y}}{S_c} + \tag{3.31}$$
$$-6\gamma_a K_r S^2 e^{-y} - 3\gamma_a K_r^2 Se^{-y} - \gamma_a K_r^3 e^{-y} + \frac{\gamma_a e^{-y}}{P_r^3} + \frac{3\gamma_a e^{-y}}{P_r^2} - \frac{\gamma_a S^2 e^{-y}}{P_r} +$$
$$+\frac{3\gamma_a l_1^2 e^{-y}}{P_r} - \frac{\gamma_a Se^{-y}}{P_r} - \gamma_a l_1^2 Se^{-y} - \gamma_a l_1 S^2 e^{-y} + \gamma_a l_1^3 e^{-y} - 2\gamma_b G_c^2 e^{-3y} +$$
$$\left. -3\gamma_b G_r^2 e^{-3y} - 4\gamma_b G_C G_r e^{-3y} - l_2 G_c e^{-y} - l_2 G_r e^{-y}\right)\frac{t^2}{2!} +$$
$$+\left(\frac{e^{-y}}{S_c} - K_r e^{-y} + \frac{e^{-y}}{P_r} + l_1 e^{-y} - \frac{Se^{-y}}{S_c} + K_r Se^{-y} - \frac{Se^{-y}}{P_r} + Sl_1 e^{-y} + \frac{\alpha e^{-y}}{S_c^2} + \right.$$

$$
\begin{aligned}
&-\frac{2\alpha Se^{-y}}{S_c}-\frac{2\alpha K_r e^{-y}}{S_c}+2\alpha K_r Se^{-y}+\alpha K_r^2 e^{-y}+\frac{\alpha e^{-y}}{P_r^2}+\frac{2\alpha l_1 e^{-y}}{P_r}+\alpha l_1^2 e^{-y}+\\
&+\frac{\beta_a e^{-y}}{S_c^3}-\frac{3\beta_a Se^{-y}}{S_c^2}-\frac{2\beta_a K_r e^{-y}}{S_c^2}+\frac{6\beta_a K_r Se^{-y}}{S_c}+\frac{3\beta_a S^2 e^{-y}}{S_c}+\frac{3\beta_a K_r^2 e^{-y}}{S_c}+\\
&-\frac{\beta_a K_r e^{-y}}{S_c}-3\beta_a K_r S^2 e^{-y}-3\beta_a K_r^2 Se^{-y}-\beta_a K_r^3 e^{-y}+\frac{\beta_a e^{-y}}{P_r^3}+\frac{3\beta_a l_1 e^{-y}}{P_r^2}+\\
&-\frac{\beta_a S^2 e^{-y}}{P_r}+\frac{3\beta_a l_1^2 e^{-y}}{P_r}-\frac{\beta_a Se^{-y}}{P_r}-\frac{2\beta_a l_1 Se^{-y}}{P_r}-\beta_a l_1^2 Se^{-y}-\beta_a l_1 S^2 e^{-y}+\\
&+\beta_a l_1^3 e^{-y}+\beta_a S^3 e^{-y}+12\beta_b G_c G_r e^{-3y}-6\beta_b G_c^2 e^{-3y}-6\beta_b G_r^2 e^{-3y}+\\
&-\frac{l_1 e^{-y}}{S_c}+l_2 K_r e^{-y}-\frac{l_2 e^{-y}}{P_r}+6\gamma_b G_c^2 G_r e^{-3y}+6\gamma_b G_c G_r^2 e^{-3y}+2\gamma_b G_c^3 e^{-3y}+\\
&+2\gamma_b G_r^3 e^{-3y}-l_1 l_2 e^{-y}-\frac{l_2 e^{-y}}{P_r}+6\gamma_b G_c^2 G_r e^{-3y}+6\gamma_b G_c G_r^2 e^{-3y}+2\gamma_b G_c^3 e^{-3y}+\\
&+2\gamma_b G_r^3 e^{-3y}-l_1 l_2 e^{-y}\Big)\frac{t^3}{3!}+\Big(\frac{Se^{-y}}{S_c^2}+\frac{2S^2 e^{-y}}{S_c}+\frac{2K_r Se^{-y}}{S_c}-2K_r S^2 e^{-y}-K_r^2 Se^{-y}\\
&-\frac{Se^{-y}}{P_r^2}-\frac{2l_1 Se^{-y}}{P_r}-l_1^2 Se^{-y}+\frac{e^{-y}}{S_c^2}-\frac{2Se^{-y}}{S_c}-\frac{2K_r e^{-y}}{S_c}+2K_r Se^{-y}+K_r^2 e^{-y}+\\
&+\frac{e^{-y}}{P_r^2}+\frac{2l_1 e^{-y}}{P_r}+l_1^2 e^{-y}+\frac{\alpha e^{-y}}{S_c^3}-\frac{3\alpha Se^{-y}}{S_c^2}-\frac{2\alpha K_r e^{-y}}{S_c^2}+\frac{6\alpha K_r Se^{-y}}{S_c}+\frac{3\alpha S^2 e^{-y}}{S_c}+\\
&+\frac{3\alpha K_r^2 e^{-y}}{S_c}-\frac{\alpha K_r e^{-y}}{S_c}-3\alpha K_r S^2 e^{-y}-3\alpha K_r^2 Se^{-y}-\alpha K_r^3 e^{-y}+\frac{\alpha e^{-y}}{P_r^3}+\frac{3\alpha l_1 e^{-y}}{P_r^2}+\\
&-\frac{\alpha S^2 e^{-y}}{P_r}+\frac{3\alpha l_1^2 e^{-y}}{P_r}-\frac{\alpha Se^{-y}}{P_r}-\frac{\alpha l_1 Se^{-y}}{P_r}-\alpha l_1^2 Se^{-y}-\alpha l_1 S^2 e^{-y}+\alpha l_1^3 e^{-y}+\\
&+6\beta_b G_c^3 e^{-3y}+18\beta_b G_c G_r^2 e^{-3y}+6\beta_b G_r^3 e^{-3y}-\frac{l_2 e^{-y}}{S_c^2}+\frac{l_2 Se^{-y}}{S_c}+\frac{2l_2 K_r e^{-y}}{S_c}+\\
&-2l_2 K_r Se^{-y}-l_2 K_r^2 e^{-y}-\frac{l_2 e^{-y}}{P_r}-\frac{2l_1 l_2 e^{-y}}{P_r}-l_1^2 l_2 e^{-y}\Big)\frac{t^4}{4!}+\Big(\frac{e^{-y}}{S_c^3}-\frac{3Se^{-y}}{S_c^2}+\\
&-\frac{2K_r e^{-y}}{S_c^2}+\frac{6K_r Se^{-y}}{S_c}+\frac{3S^3 e^{-y}}{S_c}+\frac{3K_r^3 e^{-y}}{S_c}-\frac{K_r e^{-y}}{S_c}-3K_r S^2 e^{-y}-3K_r^2 Se^{-y}\\
&-K_r^3 e^{-y}+\frac{e^{-y}}{P_r^3}+\frac{3l_1 e^{-y}}{P_r^2}-\frac{S^2 e^{-y}}{P_r}+\frac{3l_1^2 e^{-y}}{P_r}-\frac{Se^{-y}}{P_r}-\frac{2Sl_1 e^{-y}}{P_r}-l_1^2 Se^{-y}-l_1 S^2 e^{-y}+\\
&+l_1^3 e^{-y}-\frac{Se^{-y}}{S_c^3}+\frac{3s^2 e^{-y}}{S_c^2}+\frac{2K_r Se^{-y}}{S_c^2}-\frac{6K_r S^2 e^{-y}}{S_c}-\frac{3s^3 e^{-y}}{S_c}-\frac{3K_r^2 Se^{-y}}{S_c}+\\
&+\frac{K_r Se^{-y}}{S_c}+3K_r S^3 e^{-y}+3K_r^2 S^2 e^{-y}+K_r^3 Se^{-y}-\frac{Se^{-y}}{P_r^3}-\frac{3l_1 Se^{-y}}{P_r^2}+\frac{s^3 e^{-y}}{P_r}+
\end{aligned}
\tag{cont.3.31}
$$

$$-\frac{3l_1^2Se^{-y}}{P_r}+\frac{S^2e^{-y}}{P_r}+\frac{2l_1S^2e^{-y}}{P_r}+l_1^2S^2e^{-y}-l_1S^3e^{-y}-l_1^3Se^{-y}-\frac{l_2e^{-y}}{S_c^3}+\frac{3l_2Se^{-y}}{S_c^2}+$$
$$+\frac{2l_2K_re^{-y}}{S_c^2}-\frac{6l_2K_rSe^{-y}}{S_c}-\frac{3l_2S^2e^{-y}}{S_c}-\frac{3l_2K_r^2e^{-y}}{S_c}+\frac{l_2K_re^{-y}}{S_c}+3l_2K_r^2Se^{-y}+ \quad \text{(cont.3.31)}$$
$$+3l_2K_rS^2e^{-y}+l_2K_r^3e^{-y}-\frac{l_2e^{-y}}{P_r^3}-\frac{3l_1l_2e^{-y}}{P_r^2}+\frac{l_2S^2e^{-y}}{P_r}-\frac{l_2Se^{-y}}{P_r}+\frac{2l_1l_2Se^{-y}}{P_r}+$$
$$+l_1^2l_2Se^{-y}+l_1l_2S^2e^{-y}-l_1^3l_2e^{-y}\Big)\frac{t^5}{5!}.$$

Therefore, the solution to Eq.(2.16) is;

$$u(y,t)=u_0(y,t)+u_1(y,t)+\cdots \tag{3.32}$$

where, $u_0(y,t)$ and $u_1(y,t)$ are defined in Eq.(3.29) and (3.31) respectively.

The physical momentum, heat and mass properties such as the skin friction, Nusselt and Sherwood number, which are elucidated in Arifuzzaman *et al.* [8] are:

$$\begin{cases} C_f=-\dfrac{1}{2\sqrt{2}}G_r^{-3/4}\left(\dfrac{\partial u}{\partial y}\right)_{y=0}, \\ N_u=\dfrac{1}{2\sqrt{2}}G_r^{-3/4}\left(\dfrac{\partial \theta}{\partial y}\right)_{y=0}, \\ S_h=-\dfrac{1}{2\sqrt{2}}G_r^{-3/4}\left(\dfrac{\partial C}{\partial y}\right)_{y=0}. \end{cases} \tag{3.33}$$

4. Results and discussion

An unsteady MHD flow of a fourth-grade fluid in a horizontal parallel plates channel with thermal radiation, chemical reaction and suction effects has been analyzed. The impact of thermal radiation, chemical reaction, suction, third and fourth-grade parameters along with other pertinent flow parameters are plotted graphically on different flow fields. The default values for the pertinent flow parameters are taken as (Arifuzzaman [8])

$$\lambda=0.30,\quad \alpha=0.20,\quad \beta_a=0.05,\quad \beta_b=0.05,\quad \gamma_a=0.05,\quad \gamma_b=0.0.05,\quad S_c=0.50,$$

$$G_r=5,\quad G_c=5,\quad P_r=0.71,\quad Ha=0.30,\quad \delta=.05,\quad Da=1.00,\quad K_r=0.50.$$

To validate the present work, a numerical comparison is provided in Tab.1. It can be seen there is no significant difference. The impact of flow parameters on the skin friction C_f, Nusselt number N_u, Sherwood number S_h is also investigated and presented in Tab.2.

Table 1. Validation of present study against Arifuzzaman [8] for the steady state Nusselt number, where $P_r = 0.71$.

δ	Nusselt number N_u (Present work)	Nusselt number N_u (Arifuzzaman *et al.* [8])	Difference
0.05	*0.54190*	*0.49308*	*0.05*
0.10	*0..63411*	*0.58207*	*0.05*
0.15	*0.40762*	*0.39764*	*0.01*
0.20	*0.40981*	*0.37984*	*0.03*

Table 2. Computational values of the skin friction C_f, Nusselt number N_u and Sherwood number S_h.

s	δ	Ha	P_r	α	$\beta_a = \beta_b$	$\gamma_a = \gamma_b$	K_r	S_c	C_f	N_u	S_h
0.20	*0.05*	*0.30*	*0.71*	*0.05*	*0.05*	*0.05*	*0.50*	*0.50*	*2.0045*	*0.5519*	*0.3494*
0.30									*1.9544*	*0.5383*	*0.3401*
0.40	*0.10*								*1.9092*	*0.5358*	*0.3312*
0.50	*0.15*								*1.8639*	*0.5328*	*0.3227*
	0.20	*0.60*							*1.8959*	*0.5885*	*0.3494*
		0.80							*1.7986*	*0.5519*	*0.3494*
		1.00	*1.00*						*1.6727*	*0.5510*	*0.3494*
			1.50						*1.9283*	*0.6102*	*0.3494*
			2.00	*0.10*	*0.10*				*2.2777*	*0.7171*	*0.3494*
0.80				*0.20*	*0.20*				*2.8109*	*0.4615*	*0.2996*
				0.30	*0.30*	*0.10*			*3.3347*	*0.5519*	*0.3494*
						0.20			*3.8775*	*0.5519*	*0.3494*
						0.30	*1.00*		*4.2177*	*0.5519*	*0.3064*
							1.50	*0.70*	*1.9216*	*0.5519*	*0.2034*
							2.00	*0.80*	*1.9021*	*0.5519*	*0.1493*
								1.20	*1.9107*	*0.5519*	*0.0911*

Table 2 presented the effect of flow parameters on the skin friction C_f, Nusselt number N_u, Sherwood number S_h. It is seen that the skin friction develops due to the increase in $\delta, P_r, \alpha, \beta_a, \beta_b, \gamma_a$ and γ_b but diminishes due to an increase in S, Ha, K_r and S_c. The Nusselt number increases with an increase of P_r and decreases with an increase in S and δ. The Sherwood number diminishes due an increase in S, K_r and S_c. Figures 2 and 3 depict the velocity and temperature fields for the increment in the thermal radiation parameter δ $\left(0.05 \le \delta \ge 0.40\right)$. Thermal radiation is known as electromagnetic radiation or the conversion of thermal energy which generates the thermal motion of particles in matter. Thermal radiation could be attributed to thermal excitation. Both velocity and temperature fields are affected significantly by on increase in the thermal radiation parameter (δ).

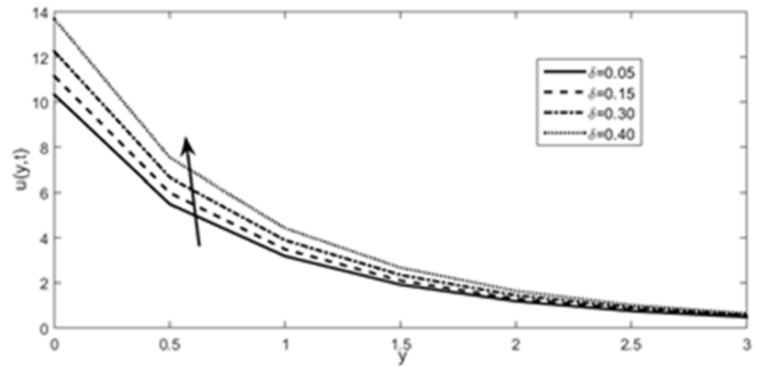

Fig.2. Effect of δ on Velocity profile u.

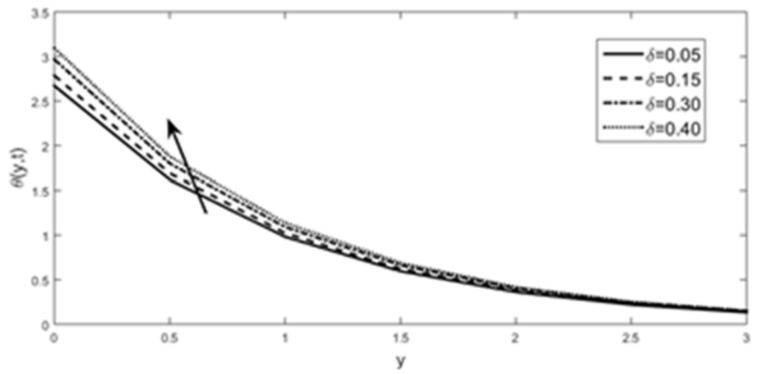

Fig.3. Effect of δ on temperature distribution θ.

The effect of the chemical reaction parameter (K_r) on velocity and concentration profiles is depicted in Figs 4 and 5 respectively. Due to the rise of the chemical reaction (K_r) from $0.50 \le K_r \ge 2.00$, the velocity field decreases, and the concentration field also decreases. Physically, chemical reaction occurs with more disturbance which develops the molecular motion and upsurges the heat transport phenomena and as a result retards the velocity of the flow.

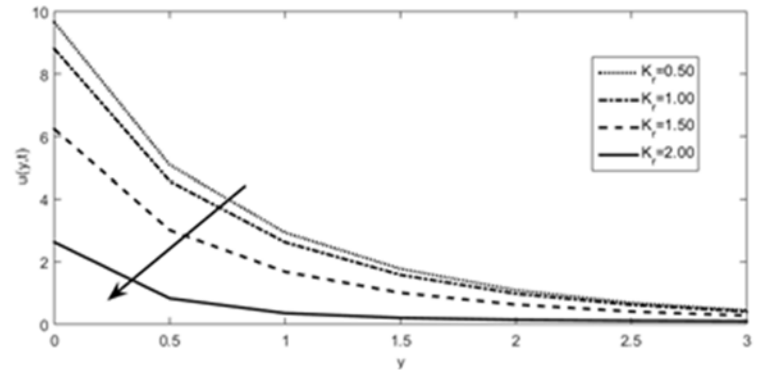

Fig.4. Effect of K_r on velocity profile u.

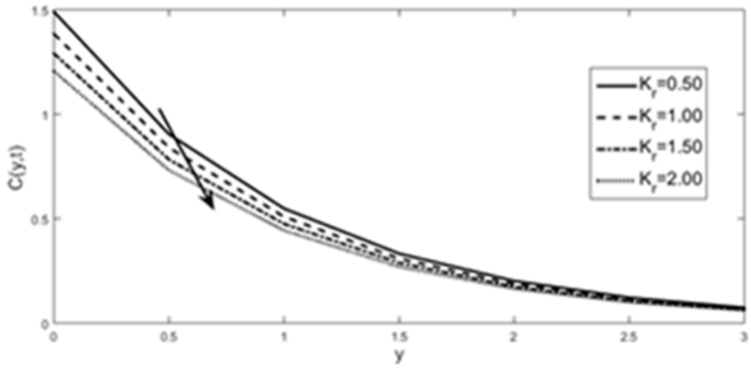

Fig.5. Effect of K_r on concentration distribution C.

Figure 6 illustrates the drag force effect on the fluid flow. The velocity profile decreases with an increment in the Hartmann number $(0.10 \le Ha \ge 0.90)$. The role of the Hartmann number, which is the magnetic parameter, is to suppress turbulence. Physically, when a magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming a viscous elastic solid. It is of great interest that yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of an electromagnet which give rise to many possible control-based applications, including MHD power generation, electromagnetic casting of metals, MHD propulsion, etc.

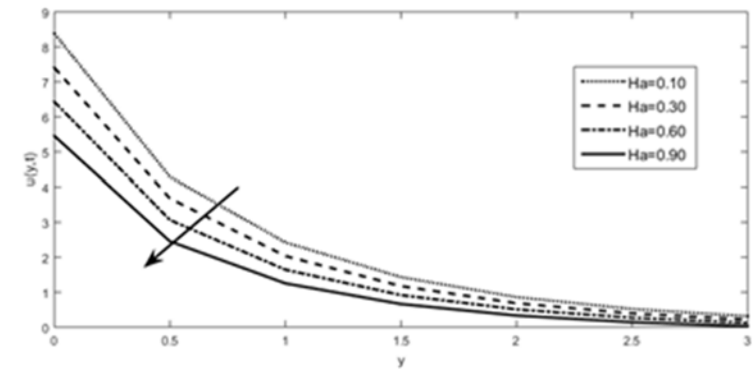

Fig.6. Effect of Ha on velocity profile u.

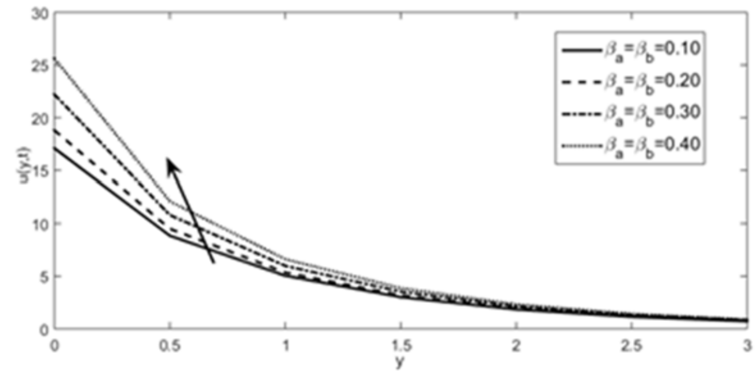

Fig.7. Effect of β_a and β_b on velocity profile u.

The effect of third-grade and fourth-grade parameters on velocity profiles are respectively illustrated in Figs 7 and 8. It is observed that the velocity profile develops with an increase in both third-grade $\left(0.10 \leq \left(\beta_a = \beta_b\right) \geq 0.40\right)$ and fourth-grade ($0.10 \leq \left(\gamma_a = \gamma_b\right) \geq 0.40$) parameters.

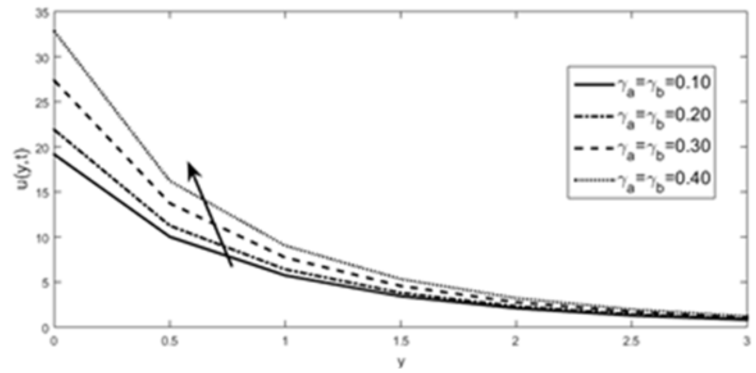

Fig.8. Effect of γ_a and γ_b on velocity profile u.

The impact of the suction parameter S on velocity, temperature and concentration profiles is illustrated in Figs 9, 10 and 11 respectively. It is clearly seen that velocity, temperature and concentration profiles diminish with an increase of S $\left(0.10 \leq S \geq 0.40\right)$. This is due to the porosity of plates.

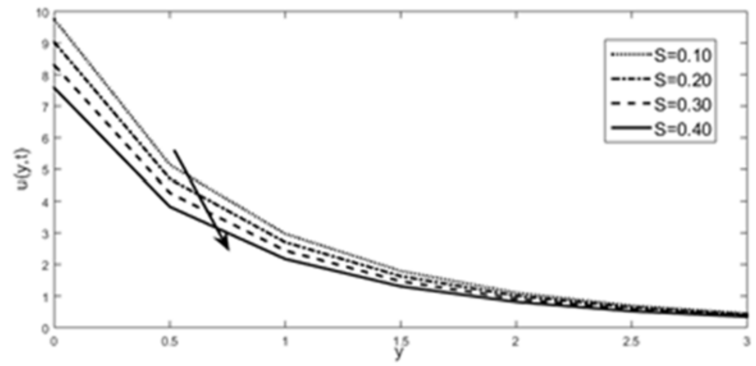

Fig.9. Effect of S on velocity profile u.

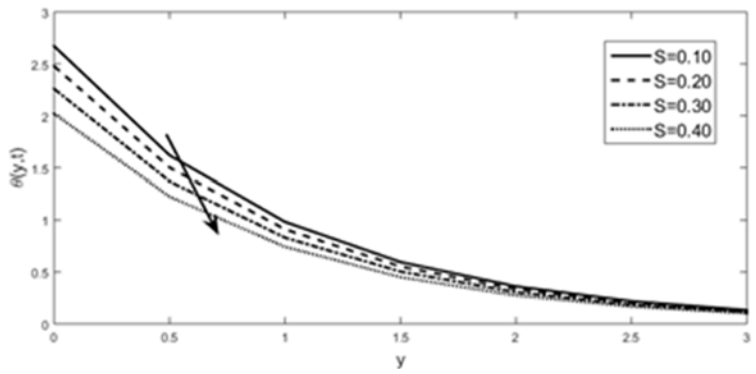

Fig.10. Effect of S on temperature distribution θ.

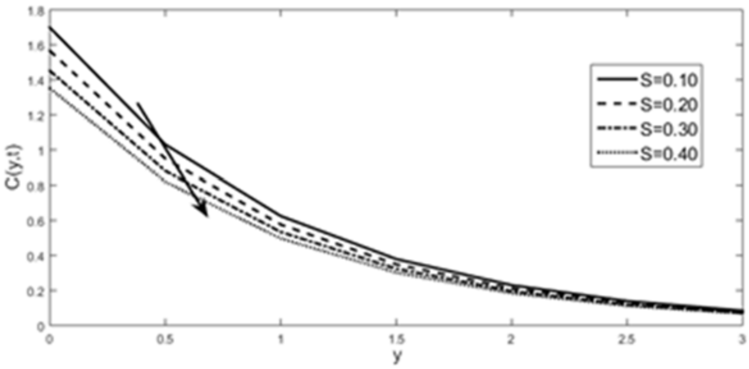

Fig.11. Effect of S on concentration distribution θ.

5. Conclusion

An unsteady MHD flow of fourth-grade in a horizontal parallel plates channel with thermal radiation, chemical reaction and suction effects has been investigated. The solutions for the nonlinear partial differential equations are obtained by the He-Laplace scheme. The effects of flow parameters on velocity, temperature and concentration profiles are depicted in figures and discussed. From the results obtained, the findings are:

- Velocity and temperature fields rise due to the increment the thermal radiation parameter.
- For upsurging data of chemical reaction, velocity and concentration fields diminish.
- Velocity, temperature and concentration fields diminish due to the increment in the suction parameter.
- Velocity profile goes up when third and fourth-grade parameters get to rise.
- Velocity and skin friction fields decline due to the increment in the magnetic parameter.
- Nusselt number distribution rises due to the enhancement in the thermal radiation parameter and drops due to an increase in the suction parameter.
- Large values of thermal radiation parameter, Prandtl number, second, third and fourth-grade parameters increase the skin friction while higher values of the Schmidt number, chemical reaction and suction parameters diminish the skin friction.
- The rate of mass transfer diminishes due to the rise in the Schmidt number, chemical reaction parameter and suction parameter.

The results of this work are applicable in many industrial production processes such as in the drilling of oil and gas wells, polymer extrusion from dye, glass fiber, paper production and draining of plastics films, etc.

This work can be extended to a plane Poisueille flow of a fourth grade fluid by taking into account the viscous dissipation and buoyancy effects.

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Nomenclature

B_0 – external magnetic field
C – species concentration
C_p – specific heat capacity
C_f – skin friction

C_w – concentration at the surface
C_∞ – concentration as $y \to \infty$
G_c – Grashof number due to mass transfer
G_r – Grashof number due to heat transfer
Ha – Hartmann number
K_r – chemical reaction parameter
N_u – Nusselt number
P_r – Prandtl number
q_r – radiative heat flux
S – suction parameter
S_c – Schmidt number
S_h – Sherwood number
T – temperature of the fluid
T_w – temperature at the surface
T_∞ – ambient temperature as $y \to \infty$
u – fluid velocity
x, y – cartesian coordinates
α – second grade parameter
β – thermal expansion coefficient
β_a, β_b – third grade parameters
β_c – concentration expansion coefficient
γ_a, γ_b – fourth grade parameters
δ – thermal radiation parameter
μ – coefficient of shear viscosity
ν – kinematic viscosity
ρ – density of the fluid
σ – Stefan Boltzmann constant

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