CIĄGŁY NIELINIOWY UKŁAD BELKI PODWÓJNEJ JAKO WIARYGODNY MODEL DYNAMIKI DROGI KOLEJOWEJ[1]

Piotr Kozioł
PhD, DSc, Assoc. Prof., Cracow University of Technology, Faculty of Civil Engineering, Chair of Roads, Railways and Traffic Engineering, e-mail: pkoziol@pk.edu.pl

Rafał Pilecki
MSc, Eng., former student of Cracow University of Technology, e-mail: rafal.pilecki@gmail.com

Streszczenie. *Współczesny transport kolejowy napotyka na nowe wyzwania związane z rosnącymi wymogami bezpieczeństwa. Te stają się konieczne z powodu spodziewanego rozwoju skutkującego wysokimi prędkościami pociągów i częstotliwością połączeń, razem z trudnościami w planowaniu nowych linii umiejscowionych w rejonach o warunkach geotechnicznie wymagających. Utrzymanie istniejących trakcji staje się równie ważne ponieważ ich intensywne użytkowanie prowadzi do szybszej degradacji i zmniejszonej efektywności. Rozważając stan infrastruktury będącej w użyciu jeszcze kilka lat temu, można zauważyć, że poszukiwanie nowych metod analizy tworzonych układów jest obecnie nawet bardziej istotne. Poza problemami związanymi z wytrzymałością i zużyciem, należy zwracać uwagę na dynamiczne właściwości konstrukcji, głównie ze względu na ich rosnące znaczenie w odniesieniu do wpływu nieliniowych i stochastycznych charakterystyk. Nowe materiały używane w konstrukcjach drogi kolejowej mają bardzo często nieliniowe właściwości fizyczne, w sposób oczywisty potwierdzone i zbadane laboratoryjnie. Jednak ich wpływ na odpowiedź dynamiczną układów transportowych pozostaje dotychczas niewystarczająco zbadany. Obecny artykuł opisuje podejście do analitycznego modelowania dynamiki drogi kolejowej oparte na układach „wielobelkowych" posiadających nieliniowe właściwości. Wstępne wyniki obliczeniowe pokazujące dynamikę układu powiązaną z ruchomym obciążeniem złożonym z sił zmieniających się z różnymi częstotliwościami mogą być uważane za oryginalny wkład prezentowany w artykule.*

Słowa kluczowe: *dynamika nieliniowa, modelowanie drogi szynowej, rozwiązanie pół-analityczne, metody hybrydowe*

1. Introduction

Although the modelling of rail track is an extremely difficult process, some features of structure can be analysed by using relatively simple models, depending on assumptions made at the beginning of study [1]. This kind of approach is based on heuristic philosophy, which along with hybrid computational methods makes it a rarely used, although relatively known tool. However, new discoveries both in engineering and pure sciences give new possibilities for developments and practical applications in the analysis of considered models and description of their desired characteristics. The main goal recognized as a key problem in the analysis

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of the rail track dynamic response is directed towards modelling of vertical vibrations generated "at the source" [2]. This means that other sources, like e.g. a noise or environmental vibrations are omitted. Such an approach is a consequence of a necessity to reduce mechanical vibrations generated by interaction in the vehicle-structure system in order to protect environment and to increase a passengers' safety.

Moving load problems form a very wide subject of mechanics. One can find a huge library of published papers including important results giving essential contribution to the field. Therefore making a brief literature review seems impossible. Even though new results still appear giving opportunity to extend carried out studies and find more reliable characteristics or new phenomena associated with modern structures and increased velocities. One of the most interesting ideas is that one based on beam-foundation systems, including multi-beam structures, with special emphasis on double-beam models. These models should be extended by inclusion of nonlinear and stochastic properties associated with vertical imperfections in rails rolling surface or foundation stiffness [2-4].

There are several approaches to modelling the rail track vertical vibrations. These are based on numerical, semi-numerical, semi-analytical or analytical methods. This paper focuses on semi-analytical approach supported by tools based on developments type of hybrid (heuristically developed) [1,5,6]. In the case of nonlinear systems such an approach allows to obtain new results helping in an implementation process of innovative design solutions, e.g. fastening systems or under sleeper pads.

2. Modelling

Different simplified analytical models are considered for modelling vertical vibrations of rail track. The simplest one is described by an Euler-Bernoulli equation of motion for a beam resting on elastic or viscoelastic foundation [2]:

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + kw = Q(x,t) \tag{1}$$

where

EI [N/m^2] – bending stiffness of rail steel;

m [kg/m] – rail unit mass;

k [N/m^2] – linear foundation stiffness;

c [Ns/m^2] – foundation viscous damping;

[N/m] – force generated by moving train;

w [m] – beam deflection;

x [m] – space variable along a beam;

t [s] – time variable.

This equation can be enriched with an assumption of nonlinear foundation stiffness [N/m^4] [6]:

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + kw + k_N w^3 = Q(x,t). \tag{2}$$

The load generated by train can be modelled either as a series of moving loads or a series of moving masses. Models using masses are difficult to solve due to appearing inertial properties and therefore their solutions are published relatively rarely. In the case of loads, one should pay attention to their distribution density. One considers point loads, rectangular representations, or loads distributed on some interval with density described by using continuous functions, such as e.g. sine, Gauss or cosine square functions. It is proven that using these different representations of distribution does not lead to significant differences in shape and numerical results for vibration generation, at least in the case of linear track properties (Eq. 1) [2]. In the case of nonlinear foundation stiffness (Eq. 2), such an analysis is still unsolved issue, however usually a concentrated load is used for its relatively good applicability to known analytical approaches. However, the results are usually far from realistic values, mainly due to limited number of loads considered. Whereas in the linear model single separated loads can be analysed and then the entire solution can be obtained by the superposition method, in the nonlinear model the whole train should be analysed for a proper representation of the track dynamics. Therefore appropriate methods of solution for such complex models are still sought and sufficiently exact results are relatively rare. In order to avoid significant errors generated during computational procedures, one must omit numerical approximations in each possible way. Applying cosine square function (Eq. 3) representing each of loads generated by train axles allows avoiding unnecessary additional estimation, leading by that to a more precise solution with appropriate class of continuity [6]:

$$P_k(x,t) = \frac{P}{r}\cos^2\left(\frac{\pi(x-Vt)}{2r}\right)H(r^2-(x-Vt)^2) \tag{3a}$$

$$Q(x,t) = \sum_{k=1}^{K} P_k(x,t)\exp(i\Omega_k t) \tag{3b}$$

A series of loads represented by equations (3a-b), designed to model the whole train or even its part (i.e. a few axles) makes the system so complicated that another approximations must be applied for obtaining solution. One of such methods is a hybrid approach based on wavelet approximation combined with Adomian's decomposition allowing to deal with nonlinear factors [1,5-11]. Solution for a beam resting on a viscoelastic nonlinear foundation, based on a one-layer system, with parameters taken from real rail structures, is validated for a wide range of different factors, including the train speed [6].

This heuristically developed hybrid method was also validated for two-layer model, in which the first layer (E-B beam) describes rails and the second layer (E-B beam with zero bending) represents sleepers (u and w are indexes used for upper and lower beams, respectively) [3,4]:

$$EI_u \frac{\partial^4 u}{\partial x^4} + m_u \frac{\partial^2 u}{\partial t^2} + c_u \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) + k_u(u-w) - k_{Nw} w^3 = Q(x,t) \tag{4a}$$

$$m_w \frac{\partial^2 w}{\partial t^2} + c_w \frac{\partial w}{\partial t} + k_w w - c_u \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) - k_u(u-w) + k_{Nw} w^3 = 0 \tag{4b}$$

The results for this model are verified in linear case () and in the case of beam-foundation system resting on nonlinear foundation.

Adding two nonlinear factors (in foundation stiffness and in the layer between beams) makes the system difficult to solve due to several approximations combined within computational process. The analytical approximations used to obtain the estimated solution allow it to be called semi-analytical. A use of heuristically supported hybrid methods leading to semi-analytical solutions allows solving the system with several nonlinearities, under assumptions of appropriate errors control and increased order of approximations. The higher order of approximation needs a significantly longer time of computations. Another effort must be made then in order to improve the developed computational technique.

Before direct application of the proposed system to railway structures, one must carefully check all properties of theoretical mechanical model used as a base for building the two-layer nonlinear track representation. For this purpose, a double-beam model resting on viscoelastic foundation is solved and checked, especially in the case of a load consisting of several forces varying with constant but different frequencies [12,13]:

$$EI_u \frac{\partial^4 u}{\partial x^4} + m_u \frac{\partial^2 u}{\partial t^2} + c_u \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) + k_u(u-w) + k_{Nu} u^3 - k_{Nw} w^3 = \sum_{k=1}^{K} P_k(x,t) \exp(i\Omega_k t) \tag{5a}$$

$$EI_w \frac{\partial^4 w}{\partial x^4} + m_w \frac{\partial^2 w}{\partial t^2} + c_w \frac{\partial w}{\partial t} + k_w w - c_u \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) - k_u(u-w) + k_{Nw} w^3 - k_{Nu} u^3 = 0 \tag{5b}$$

The system was solved and discussed so far for a series of forces varying harmonically with similar frequencies and moving with constant velocities but only for a number of forces limited to 3. Some additional configurations were shown (a load with a middle force possessing lower frequency compared to two others) by way of example to present computational capabilities of the developed technique [12,13].

How the nonlinear properties of layers and the mentioned configuration of forces (including their number and the distance between them) influence the solution, remains an open problem. In this paper, new examples are given for different configurations of loads acting on upper beam. These studies makes closer an evaluation of the range of applicability of the developed model, in terms of its stability and the solution convergence. This analysis is necessary for appropriate solution possible to apply to railway problems, making the model a reliable representation of railway track dynamics under assumption of considering their nonlinear properties.

3. Examples and discussion

The system of parameters used for computational experiments in this paper is chosen for being a continuation of previously applied characteristics. The analysis of being consequently extended model is carried out with keeping relatively similar numerical values of coefficients important for its dynamic behaviour investigation. This is due to the fact that keeping all characteristics close to previously studied allows to control even small changes arising from applied modifications and additional assumptions made in each next stage of complexity enrichment process.

Thus, in examples given below a set of 3 forces is considered with an assumption of different frequencies of harmonical variations for each of them. Additionally, two different distances between separated forces are applied. The chosen configuration aims at recognition of nonlinear effects accumulation confirmed previously for simpler one-layer and double-beam models in the case of a set of identical forces moving along a linear system resting on nonlinear foundation. The following system of parameters is used in computational examples [12-14]:

$P = 5 \cdot 10^5$ N/m, $k_{Nu} = 10^{15}$N/m^4, $k_{Nw} = 4 \cdot 10^{13}$N/m^4, $EI_u = 10^7$Nm^2, $m_u = 100$ kg/m, $k_u = 4 \cdot 10^7$N/m^2, $c_u = 0.06 \cdot \sqrt{k_u \cdot m_u}$, $EI_w = 1.5 \cdot 10^9$Nm^2, $m_w = 3500$ kg/m, $k_w = 5 \cdot 10^7$N/m^2, $c_w = 0.06 \cdot \sqrt{k_w \cdot m_w}$, $V = 70$ m/s, $\Omega_k = 2\pi \cdot f_{\Omega_k}$, $K = 3$.

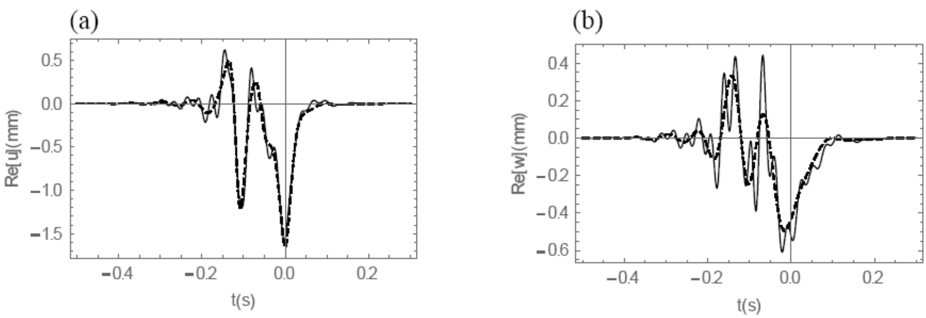

Fig.1. Vertical vibrations in the case of 3 forces moving at the distance of 4 m between them, with frequencies 2 Hz, 4 Hz and 10 Hz starting from the front of the load (linear – dashed, nonlinear – solid): (a) upper layer; (b) lower layer.

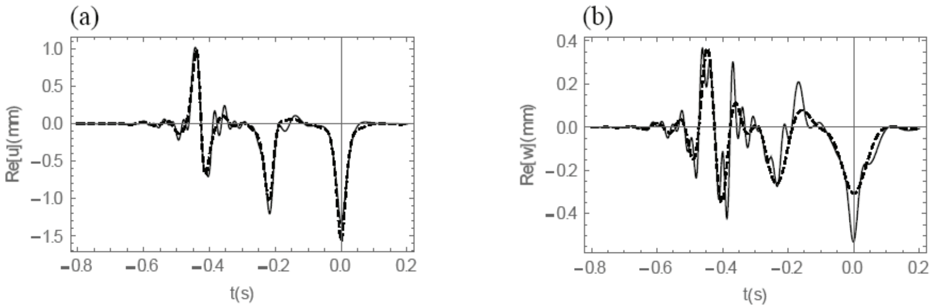

Fig.2. Vertical vibrations in the case of 3 forces moving at the distance of 15 m between them, with frequencies 2 Hz, 4 Hz and 10 Hz starting from the front of the load (linear – dashed, nonlinear – solid): (a) upper layer; (b) lower layer.

Figures 1 and 2 show vertical vibrations of the considered double-beam system in the case of 3 forces moving at the constant distance between them, with frequencies 2 Hz, 4 Hz and 10 Hz starting from the front of the load. One can see that the considered nonlinearity influences stronger the lower beam, although the vibration amplitude of this beam is lower, which corresponds to diminished maximal response of sleepers due to the energy absorption of the fastening system. Figure 1 shows that the direct effect of the middle force is practically invisible when the distance to other forces remains relatively small (comp. Fig. 2). It can be observed that overall character of the nonlinear response (in terms of nonlinear vibrations intensity) does not differ significantly when the distance between forces decreases, besides the maximal response of the lower beam becoming slightly stronger (comp. Figs. 1b and 2b). On the other hand, one can suppose that the frequency of separated forces has an important meaning for the double-beam reaction. This aspect, however, is left for future work. Further analysis should be carried out taking into account other systems of parameters, with special emphasis on frequency, load velocity and distance between moving forces.

Conclusions

Another step towards the nonlinear double-beam system analysis has been done by taking into account a load consisting of separated forces varying with different frequencies. The model is solved analytically and some computational examples are prepared based on previously validated hybrid semi-analytical approach. After a deep analysis of the system behaviour, the developed procedure can be used in a direct analysis of railway track.

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