

Bartosz POWALKA<sup>1</sup>  
Marcin CHODZKO<sup>1</sup>  
Krzysztof JEMIELNIAK<sup>2</sup>

## **STABILITY ANALYSIS IN MILLING BASED ON OPERATIONAL MODAL DATA**

Prediction of process stability in milling is usually based on experimental frequency response functions of machine tool and cutting force model. Alternatively, operational modal analysis may be applied for the prediction of a stability diagram. It does not require modal impact test but uses vibration signal acquired during actual cutting to identify modal parameters. Such an approach considers boundary conditions that may be different during cutting. On the other hand synthesis of frequency response function is not feasible due to the lack of scaled modal residues. Thus, in this paper modal mass obtained by means of impact test is used to calculate frequency response functions. Estimation of modal damping and natural frequency is carried out using only acceleration signals measured during flexible workpiece machining.

### **1. INTRODUCTION**

Milling of flexible parts is a challenging task due to the occurrence of regenerative chatter. The mechanism of regenerative chatter was explained by Tlustý and Poláček [1], and by Tobias and Fishwick [2]. A typical approach to avoid chatter vibration is the selection of spindle rotation speed and depth of cut based on a stability lobes diagram [2]. Since its introduction by Tobias, numerous methods of stability lobes construction calculation have been developed. A comprehensive review of the methods has been given by Altintas and Weck [3]. Generally, in order to generate a stability lobes diagram, it is necessary to have information regarding the cutting process and machine tool dynamics. The cutting process is frequently described by the mechanistic force model, which assumes the proportionality of the cutting force to the chip cross-sectional area. Usually, for practical applications, the dynamics of a machine tool are represented in terms of a frequency response function (FRF) matrix. Typically, an FRF matrix is identified through the impact test. This test not only requires trained personnel but is also a time-consuming task. Therefore, it may not always be feasible in an industrial environment. Also, modal parameters, i.e. modal damping and natural frequencies obtained by means of impact test, might differ from those obtained during cutting due to the speed-dependent dynamics of the

---

<sup>1</sup> West Pomerania University of Technology, Szczecin, Poland

<sup>2</sup> Warsaw University of Technology, Poland

spindle and different boundary conditions. Several researchers have performed impact tests with a rotating spindle to take into account its speed-dependent dynamics [4],[5],[6]. Such an approach is, however, not recommended due to injury risk and the excessive machine tool immobilization that is required to identify dynamics at various spindle speeds. This limitation was overcome by Gagnol et al. [7], who constructed an accurate speed-dependent model of the spindle and used it for calculating a stability diagram. Abele and Fiedler [8] presented the method of calculating dynamic behaviour during milling that also considered the change in boundary conditions. Their method requires a dynamometer to measure cutting forces and thus has limited industrial applicability. Zaghbani and Songmene [9] proposed a different approach that requires no measurement of cutting forces. Instead, they employed the operational modal analysis (OMA) to extract modal damping and natural frequencies from the acceleration signal measured during actual machining. They applied OMA to find dynamic parameters and then used them to predict tool-spindle based chatter instability. In order to eliminate the so-called ‘virtual modes’ that correspond to the harmonic excitation, they proposed a complicated algorithm. Their algorithm was limited to the estimation of modal damping and natural frequency, whereas modal residues were not identified, which made it impossible to synthesize an FRF matrix using only output data. In this paper, a modified OMA which takes into account harmonic excitations is applied. This significantly simplifies the selection of real structural modes.

## 2. METHOD OF GENERATION OF STABILITY LOBES DIAGRAM BASED ON OPERATIONAL DATA

The proposed method consists of the following steps:

Measurement of acceleration signals in the close vicinity of the tool-workpiece interface  
 Extraction of modal parameters from acceleration signal, i.e., modal damping, natural frequency  
 Generation of stability lobes diagram using synthesized FRF matrix. Modal residues are replaced by modal masses obtained from the impact test.

### 2.1. ESTIMATION OF MODAL PARAMETERS

In this paper, a modified Eigensystem Realization Algorithm (ERA) [10] method in the presence of harmonic excitation was applied in order to extract modal parameters from the output-only data. The ERA method is based on the Natural Excitation Technique [11]: if the system is excited by stationary white noise, the auto- and cross-correlation functions between the response signals are similar to the impulse response signal and can be expressed in a discrete time domain as:

$$\mathbf{R}(k\Delta t) = \sum_{r=1}^{2N} \mathbf{C}_r \exp(s_r k\Delta t) \quad (1)$$

$$s_r = -\omega_r \xi_r + i\omega_r \sqrt{1 - \xi_r^2}$$

or in frequency domain

$$\mathbf{S}_{xx} = \sum_{r=1}^{2N} \frac{\mathbf{C}_r}{i\omega - s_r} \quad (2)$$

where  $\mathbf{R}$  represents matrix built of auto and cross-correlation functions at  $k$ th discrete time,  $N$  is the number of modes in the system,  $\mathbf{C}_r$  is a matrix of constants associated with the  $r$ th mode,  $\omega_r$  and  $\zeta_r$  are the  $r$ th non-damped natural frequency and damping ratio,  $\mathbf{S}_{xx}$  is the power spectral density matrix of the response signal. The OMA version of ERA consists of building Hankel matrices from the matrices given by equation (1) for different discrete time intervals. This is used to compute a system matrix. Eigenvalues of this matrix are used to extract damping ratio and natural frequencies. In the presence of harmonic excitation, in addition to the random loads, the correlation functions will include non-decaying components. If the system is excited by  $m$  harmonic components of frequencies  $\omega_j$  ( $j=1,2,\dots,m$ ) then application of classical OMA algorithms will yield  $m$  additional non-damped modes with poles  $s_j = \pm i\omega_j$ . Theoretically, in order to identify virtual modes, a zero-damping criterion may be used, but in practice, damping ratio is different than zero. To cope with this difficulty, Mohanty and Rixen [12] proposed a modified version of the ERA which forces the modal solution to have non-damped poles with natural frequencies equal to excitation frequencies. The frequency of harmonic excitation is assumed to be known a priori as it is in the case of milling. In the present study, harmonic components were assumed to be multiples of the spindle rotation frequency. Consequently, the stabilization diagram does not include virtual modes but only potential structural modes (numerical modes might still be present). Stable poles are then selected from the stabilization diagram.

The FRF matrix which is required for stability analysis may be expressed in terms of modal parameters as:

$$\mathbf{G}(i\omega) = \sum_{r=1}^{2N} \frac{\mathbf{A}_r}{i\omega - s_r} \quad (3)$$

Modal residues  $\mathbf{A}_r$  can be derived from the relationship between power spectral density, FRF matrix and power spectral density of the excitation forces:

$$\mathbf{S}_{xx} = \mathbf{G}(i\omega) \mathbf{S}_{FF} \mathbf{G}(i\omega)^H \quad (4)$$

When the system is excited by stationary white noise with variance  $\sigma^2$ , the power spectral density matrix  $\mathbf{S}_{xx}$ , which is a Fourier transform of the  $\mathbf{R}$  matrix takes form:

$$\mathbf{S}_{xx} = 2\sigma^2 \Delta t \mathbf{G}(i\omega) \mathbf{G}(i\omega)^H \quad (5)$$

Equations (5), (3) and (2) relate modal residue matrices  $\mathbf{A}_r$  to matrices of constants  $\mathbf{C}_r$  as

$$\mathbf{C}_r = 2\sigma^2 \Delta t \frac{\mathbf{A}_r \mathbf{A}_r^H}{2\omega_r \zeta_r} \quad (6)$$

which is valid when the damping is light.

Since variance of the excitation forces is unknown, modal residues are not known exactly and, therefore, an FRF matrix cannot be accurately synthesized. The scaling factor may be found through the analytical model of the tested structure or from the mass change effect. These approaches require either an accurate model of the structure or carrying out an additional test after introducing mass changes at chosen measurement point. Both requirements are not feasible in an industrial environment. In this study a modal mass identified in the impulse test was used to synthesize OMA-based FRF. Circle fit method was adopted for estimation of modal masses independently in two orthogonal directions X and Y. FRF matrix was then constructed as

$$\mathbf{G}(i\omega) = \begin{bmatrix} G_x(i\omega) & 0 \\ 0 & G_y(i\omega) \end{bmatrix} \quad (7)$$

where

$$G_x(i\omega) = \frac{1/m_x}{-\omega^2 + i\omega\zeta_r\omega_r + \omega_r^2}, \quad G_y(i\omega) = \frac{1/m_y}{-\omega^2 + i\omega\zeta_r\omega_r + \omega_r^2} \quad (8)$$

## 2.2. GENERATION OF STABILITY LOBES USING OPERATIONAL MODAL DATA

Analytical prediction of stability in milling proposed by Altintas and Budak [13] was applied to generate stability lobes. The characteristic equation for finding limit depth of cut  $a_p$  is:

$$\det(I - a_p \mathbf{G}(i\omega_c) \bar{\mathbf{G}}_{cp} (1 - \exp(-i\omega_c T))) = 0 \quad (9)$$

where  $\bar{\mathbf{G}}_{cp}$  is the immersion dependent matrix, which is a function of cutting coefficients, and T is tooth period. In order to build this matrix, specific cutting coefficients for machined material – cutting tool are required. FRF matrix  $\mathbf{G}(i\omega_c)$  obtained from an impact test expresses a response of the structure to a unit harmonic excitation.

This equation is used to find limit stability conditions, i.e., depth of cut, spindle speed and chatter frequency.

## 3. EXPERIMENTAL VERIFICATION

Figure 1 shows an experimental stand used for the verification of OMA-based stability prediction. The stand imitates a flexible workpiece that is responsible for the occurrence of chatter vibration. The FRF matrices in the X- and Y-axis direction required for conventional stability analysis were first identified through impact modal test. The most flexible mode of the workpiece is at 1162 Hz (see Fig. 2). Modal damping of this mode is 0.491%. FRF measured at tool tip was also measured, and it turned out that the tool is significantly stiffer than the workpiece. Cutting force coefficients were estimated from the calibration tests performed for various feed rates. Cutting forces were measured using

rigidly mounted Kistler dynamometer type 9752. The cutting material was aluminum PA6, and the cutter was 20 mm 2-fluted Iscar HM90E90AD20-2-W20\_XL with inserts HM 90APCT100302R-PDR made of IC28. The estimated specific cutting force coefficients in radial and tangential direction are  $K_r=401.9 \text{ N/mm}^2$  and  $K_t=1028.5 \text{ N/mm}^2$ , respectively.

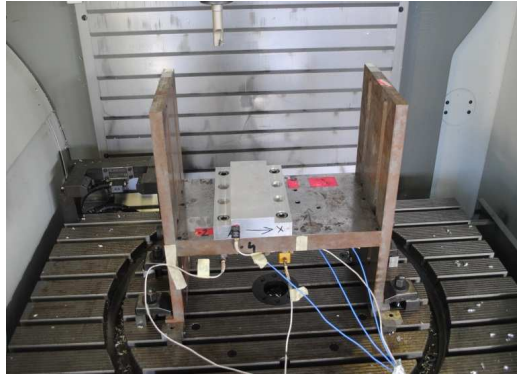


Fig. 1. View of the experimental stand

Experimentally identified FRF matrix and cutting force model coefficients were used to generate stability lobe diagram for full immersion cutting. Such generated stability lobes diagram is compared with diagrams constructed using in-operation acceleration signals. In order to predict stability limit based on operational data two cutting tests were performed. Feed direction during both cutting tests corresponded to the X-axis which is marked on the machined workpiece (Fig. 1).

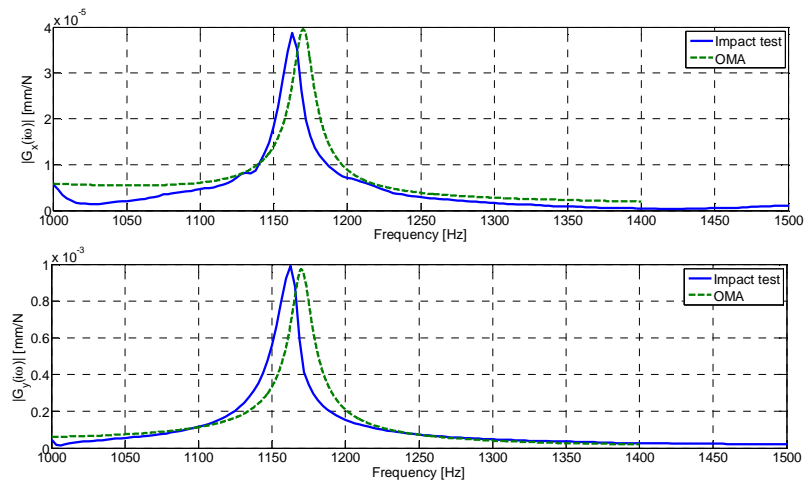


Fig. 2. Comparison of FRFs obtained through impact test and synthesized from signals acquired during full-immersion aluminum cutting at spindle speed 8100 RPM, feed 0.1 mm/tooth and depth of cut  $a_p=2 \text{ mm}$

Depth of cut  $a_p$  was set to 2 mm, and the spindle speed was set to 8100 RPM to provide stable cutting. During cutting tests, the acceleration signals were measured by the same set of accelerometers that was used in impact testing. Estimation of modal parameters

was carried out using X- and Y-axes acceleration signals measured by a sensor fixed directly to the workpiece. It must be noted that the transient vibration signal, as the tool entered the workpiece, was not included in the analysis. Stabilization diagrams constructed on the basis of operational data are shown in Fig. 3. In each analysis, 20 first multiples of spindle rotation frequency (135Hz) included in the modal solution were removed from the stabilization diagram. The stabilization diagrams contain only so-called ‘stable poles’, i.e., poles that do not change frequency more than 1% and damping more than 20% through consecutive model orders. OMA-based FRFs were synthesized according to the formula (8) and compared with the impact test results in Fig. 2. It turned out that only the most flexible mode had an impact on the synthesized FRF. The other identified poles contributed insignificantly to the FRF. Modal damping and natural frequency obtained from the operational data were 0.45% and 1172 Hz respectively.

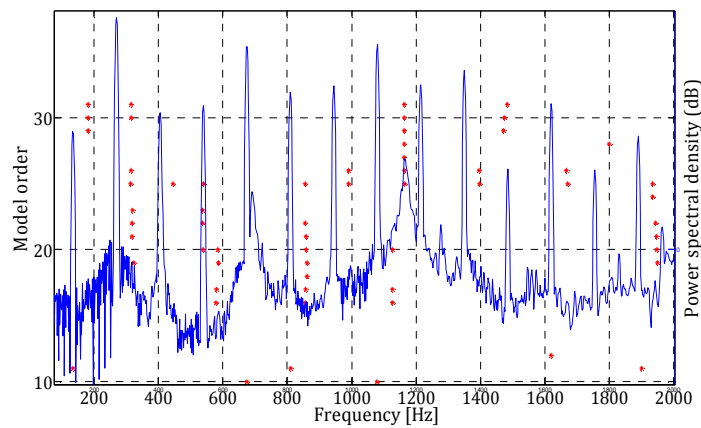


Fig. 3. Stabilization diagrams obtained from the signals acquired during full-immersion aluminum cutting at spindle speed 8100 RPM, feed 0.1mm/tooth and depth of cut  $a_p=2$ mm

Figure 4 visualizes the difference between stability diagrams from impact test and from actual cutting at depth 2 mm, respectively. It can be observed that lobes from the impact test and the depth of cut 2 mm are very similar.

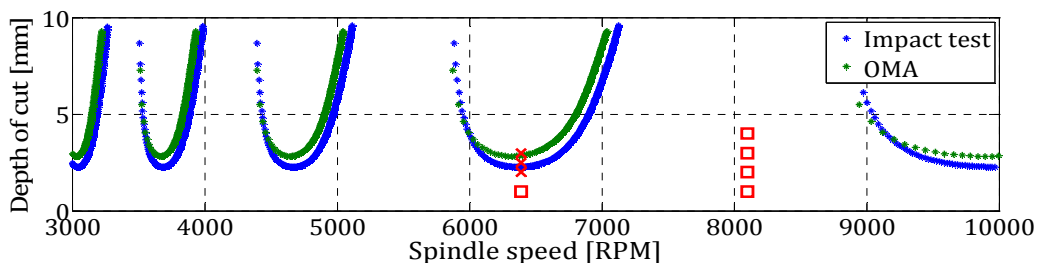


Fig. 4. Analytical stability lobe diagrams from impact test and from signals acquired during full-immersion aluminum cutting at spindle speed 8100 RPM, feed 0.1mm/tooth and depth of cut  $a_p=2$ mm. Square ( ) and cross (x) indicate stable and unstable cutting conditions during experimental test, respectively

In order to verify the stability diagram, additional cutting tests were carried out. In this study, acceleration and the quality of the machined surface were inspected visually to determine chatter occurrence. We observed acceleration and inspect visually the quality of machined surface. Fig. 5 shows acceleration signal in the feed direction for various depths of cut at 6390 RPM. This speed corresponds to the bottom of the stability lobes. Visual analysis (Fig. 6) of the surface quality classified cutting tests with 2, 2.5 and 3 mm at 6390 RPM as chatter, while the test with 1mm resulted in a chatter-free surface. All cutting tests performed at 8100 RPM ( $a_p=3, 4$  mm) were stable.

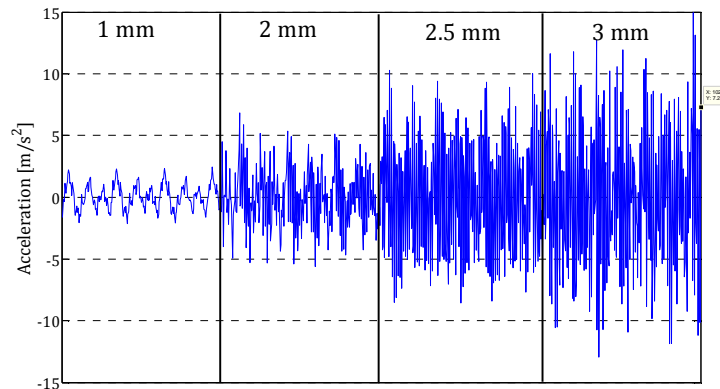


Fig. 5. Acceleration signal in the feed direction for various depths of cut at 6390 RPM

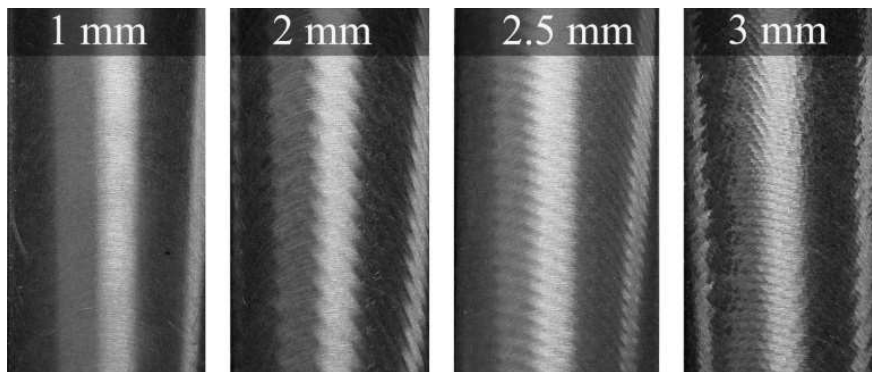


Fig. 6. Surface photos for various depths of cut at 6390 RPM

#### 4. CONCLUSION

Stability analysis that utilizes modal parameters obtained from operational data is presented. Results of the analyses agree with conventional chatter prediction. A discrepancy in absolute limit depth of cut (bottom of lobes) is observed. This discrepancy is caused by different values of modal damping obtained by means of impact test and operational data that may be due to different boundary conditions. Accurate finding of limit depth of cut exclusively (in this study modal mass from the impact test was assumed) from the OMA based stability analysis will be a subject of further research. It requires the development of a method for finding scaled values of modal residues. In our future work, we will also

focus on applying operational data for chatter prediction when it is impossible to place sensors in close vicinity of the tool-workpiece interface.

#### ACKNOWLEDGMENTS

*Financial support of Structural Funds in the Operational Programme - Innovative Economy (IE OP) financed from the European Regional Development Fund - Project "Modern material technologies in aerospace industry", Nr POIG.01.01.02-00-015/08-00 is gratefully acknowledged*

#### REFERENCES

- [1] TLUSTY J., POLACEK M., 1963, The Stability of Machine Tools Against Self-excited Vibrations in Machining, Proceedings of the ASME International Research in Production Engineering, 465-474.
- [2] TOBIAS S. A., FISHWICK W., 1958, Theory of Regenerative Machine Tool Chatter, The Engineer, London, 205/199-203.
- [3] ALTINTAS Y., WECK M., 2004, Chatter Stability in Metal Cutting and Grinding, Annals of the CIRP, Key Note Paper of STC-M, 53/2/619-642.
- [4] FAASSEN, R. P. H., VAN DE WOUW, N., OOSTERLING, J.A.J., NIJMEIJER, H., 2003, Prediction of Regenerative Chatter by Modelling and Analysis of High-speed Milling, International Journal of Machine Tools and Manufacture, 43/1437-1446.
- [5] SCHMITZ, T., ZIEGERT, J., AND STANISLAUS, C., 2004, A Method for Predicting Chatter Stability for Systems with Speed-Dependent Spindle Dynamics, SME Technical Paper TP04PUB182, Transactions of NAMRI/SME, 32/17-24.
- [6] CHEN C., WANG K., SHIN Y., 1994, An Integrated Approach Toward the Dynamic Analysis of High-Speed Spindles, Part 1: System Model, ASME J. of Vibrations and Acoustics, 116/506-513.
- [7] GAGNOL V., BOUZGARROU B. C., RAY P., BARRA B., 2007, Model-based Chatter Stability Prediction for High-speed Spindles, International Journal of Machine Tools and Manufacture, 47/1176-1186.
- [8] ABELE E., FIEDLER U., 2004, Creating Stability Lobe Diagrams during Milling, Annals of the CIRP, 53/1/309-312.
- [9] ZAGHBANI I., SONGMENE V., 2009, Estimation of Machine-Tool Dynamic Parameters during Machining Operation through Operational Modal Analysis, International Journal of Machine Tools and Manufacture, 49/12-13/947-957.
- [10] JUANG J. N., PAPPAS R. S., 1985, An Eigensystem Realization Algorithm for Model Parameter Identification and Model Reduction, Journal of Guidance Control and Dynamics, 8/5/620-627.
- [11] JAMES G. H., CARNE T. G., LAUFFER, J. P., 1993, The Natural Excitation Technique for Modal Parameters Extraction from Operating Wind Turbines, SAND92-1666, UC-261.
- [12] MOHANTY, P. RIXEN, D.J., 2006, Modified ERA method for Operational Modal Analysis in the Presence of Harmonic Excitations, Mechanical Systems and Signal Processing, 20/1/114-130
- [13] ALTINTAS, Y., BUDAK, E., 1995, Analytical Prediction of Stability Lobes in Milling, Annals of the CIRP, 44/1/357-362.