Global and Local Trend Analysis and Change-Point Analysis of Selected Financial and Market Indices

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Abstract

From the macroeconomic point of view, the stock index is the best indicator of the behavior of the stock market. Stock indices fulfill different functions. One of their most important functions is to observe developments of the stock market situation. Therefore, it is crucial to describe the long-term development of indices and also to find moments of abrupt changes. Another interesting aspect is to find those indices that have evolved in a similar way over time. In this article, using trend analysis, we will uncover the global evolution of selected indices. After evaluating the global trend in the series we compare the results with local trend analysis. Other goal is to detect the moments in which this development suddenly changed using the change-point analysis. By means of cluster analysis, we find those indices that are most similar in long-term development. In each analysis, we select the most appropriate methods and compare their results.

Keywords: *Trend Analysis, Change-Point Analysis, Cluster Analysis*

1. Introduction

Stock market indices express the value of a section of the stock market. By observing the historical development of market indices, we can determine the trend of their long term development. It can be useful for construing predictions of the future development of the valuation process. Locating change-points is also essential factor in the analysis of the development of indices. Understanding the long-term development and abrupt changes in the prices of indices is a key factor for the investor in the decision making about where to invest. Therefore, our aim is to reveal the presence of the trend and identify its nature in the time series of 11 selected indices. We use non-parametric tests based on the fact that the data follows non-normal distribution. Comparing the results of the Cox-Stuart test, Mann-Kendall test, and Spearman's rho test we seek to find trend and its character. The power of the trend will be expressed by Sen's slope. This will be compared with the values of Kendal's tau and Spearman's rho.

Another important goal is to find change-points in the series. First we obtain single change-points for each time series using the Pettitt's test. Next we use multiple change-point analysis using divisive and agglomerative estimation. We compare the results of the three procedures and we will look for common change-points.

In the last part indices, which long-term development are similar, and which development are the most different from the others will be found. For this purpose agglomerative techniques of cluster analysis will be used.

2. Statistical Methods

2.1. Trend Analysis

The trend analysis in the time series of stock market indices has been evaluated using the following nonparametric tests: Cox-Stuart test, Mann-Kendall test and Spearman's rho test. We will denote $X_1, X_2, \ldots, X_n$ as a sample of n independent variables. The above tests are testing the null hypothesis that there is no trend in the data, against the alternative hypothesis that there is a statistically significant increasing/decreasing trend. Positive/negative values of the statistics implies increasing/decreasing trend.

Cox-Stuart test. The Cox-Stuart test is based on the signs of the differences

$$\begin{aligned} y_1 &= x_{1+d} - x_1 \\ y_2 &= x_{2+d} - x_2 \\ &\vdots \\ y_d &= x_n - x_{n-d} \end{aligned} \tag{1}$$

where $d = n/2$, if the size n is odd, otherwise $d = (n + 1)/2$. Assign $y_1, y_2, \ldots, y_m$ the sample of positive differences. The test statistic of Cox-Stuart test is

$$T = \sum_{i=1}^{m} sign(y_i) \tag{2}$$

On the significance level α we reject the null hypothesis if $|T| > t_\alpha$, where t_α is the quantile of binomial distribution. For $m > 20$, we can approximate t_α with the α-quantile w_α of the standard normal distribution

$$t_\alpha = \frac{1}{2}\left[m + w(\alpha)\right]\sqrt{m}. \text{ [1]} \tag{3}$$

Mann-Kendall test. The Mann-Kendall test statistic is defined as

$$S = \sum_{i=1}^{n} \sum_{j=1}^{i-1} sign\left(x_i - x_j\right) \quad (4)$$

For $n > 8$, S can be approximated by normal distribution, thus the standardized test statistic is given:

$$Z = \begin{cases} \frac{S-1}{\sqrt{D(S)}} & S > 0 \\ 0, & S = 0 \\ \frac{S+1}{\sqrt{D(S)}} & S < 0 \end{cases} \quad (5)$$

We reject the null hypothesis, if $Z > Z_{1-\alpha/2}$, and that means there is increasing trend in the series, or if $Z < -Z_{\alpha/2}$ what means decreasing trend. $Z_{1-\alpha/2}$ and $Z_{\alpha/2}$ are the critical values of the standard normal distribution. [2], [3].

Spearman's rho test. The test statistic of Spearman's rho test is given

$$D = 1 - \frac{6\sum_{i=1}^{n}\left(R_i - i\right)^2}{n\left(n^2 - 1\right)} \quad (6)$$

where R_i is the rank of the i-th observation in time series. The standardized statistic is given

$$Z_{SR} = D\sqrt{\frac{n-2}{1-D^2}} \quad (7)$$

If $|Z_{SR}| > t_{(n-2,1-\alpha/2)}$, the trend exists. $t_{(n-2,1-\alpha/2)}$ is the critical value of Student's t distribution [4].

2.2. Change-Point Analysis

First we checked the homogeneity of our time series using Wald-Wolfowitz test and then we used Pettitt's test for single change-point detection, then we detected multiple change-points.

Wald-Wolfowitz test. It is a nonparametric test for verifying homogeneity in time series. The null hypothesis says that a time series is homogenous between two given times. The test statistic is given

$$R = \sum_{i=1}^{n-1} x_i x_{i+1} + x_1 x_n \quad (8)$$

where $x_1, x_2, ..., x_n$ are the sampled data. For $n > 10$ we can make an approximation

$$Z = \frac{R - E(R)}{\sqrt{D(R)}}. \text{ [5]} \quad (9)$$

Pettitt's test. The null hypothesis of this test is that there is no change in the series against the alternative hypothesis there is change. The test statistic of Pettitt's test is

$$\hat{U} = max|U_k| \quad (10)$$

$$U_k = 2\sum_{i=1}^{k} r_i - k\left(n+1\right) \quad (11)$$

where $k = 1, 2, ..., n$ and r_i are the ranks of X_i. The most probably change-point is located where $\hat{U}$ reaches maximum value [6].

Hierarchical divisive estimation E-divisive. This method applies single change-point detection iteratively. Details on the estimation of these change-point's locations can be found in [7].

Hierarchical agglomerative estimation E-agglo. This method assumes an initial segmentation of the data. If there are no initial segmentations defined, then each observation can be considered as a separated segment. In this method bordering segments are sequentially merged to maximize the goodness-of-fit statistic. The estimated locations of change points are assigned by the iteration which maximized the penalized goodness-of-fit statistic. More details about this method can be found in [7].

2.3. Cluster Analysis

Cluster analysis belongs to multidimensional statistical methods used to seek out similar objects and grouping them into clusters. Clusters contain objects with the highest degree of similarity, while high dissimilarity among each cluster is desirable. Results of cluster analysis can be the best shown by *dendrogram* which represents each object and the linkage distance of these objects. It is a figure which arranges the analyzed objects so that individual joining of objects to clusters can be observed.

Since there are several aggregation methods, each of which generally yields different results, it is necessary to determine the most appropriate method of aggregation. Such measure is the *cophonetic correlation coefficient CC*. The cophonetic correlation coefficient is defined as the Pearson coefficient of correlation between actual and predicted distance. For the most suitable agglomeration method, we choose the one for which the cophonetic correlation coefficient is the highest [8].

In hierarchical clustering, we can choose the appropriate number of clusters from the dendrogram by cutting through its branches at the selected distance level on the corresponding axis. For this several indices has been developed as a criteria. Detailed criteria used to select the number of clusters can be found in [9].

3. Analysis of the Development of Selected Stock Indices

In this paper we analyzed 11 stock market indices: SPX (measure performance of the broad US economy), CCMP (a broad-based capitalization-weighted index of stocks in all three NASDAQ tiers: Global Select, Global Market and Capital Market), INDU (a price-weighted average of 30 blue-chip stocks that are generally the leaders in their industry), DAX (The German Stock Index), UKX (index of the 100 highest capitalized companies of the London Stock Exchange), CAC (the most widely-used indicator of the Paris market), NKY (a price-weighted average of 225 top-rated Japanese companies of the Tokyo Stock Exchange), HSI (weighted index of a selection of companies from the Stock Exchange from Hong Kong), LEGATRUU (a measure of global investment grade debt from twenty-four local currency markets), SPGSCITR (the leading measure of general commodity price movements in the world economy), CCI (an equal-weighted geometric average of commodity price levels relative to the base year average price). We analyzed monthly time series from January 1995 to January 2018.

To understand the basic relationship between the indices we calculated Kendall's correlation coefficient. The correlations are illustrated in *Figure 1*. We can see that most of the correlation coefficients are positive. That indicates similar nature of development (increasing/decreasing) of the series through time. The highest correlation with the value of *0.87* is observed between indices *SPX* and *CCMP* and also between *SPX* and *INDU* indices. Other high level of correlation is between *CCMP* and *INDU*, *DAX* and *SPX*, *CCMP* and *INDU*. From the indices of the European market there is strong positive correlation between the indices *UKX* and *DAX*. These indices also strongly correlate with the indices *SPX, CCMP* and *INDU*. Weak correlation is between *SPGSCITR, NKY* and the rest of the indices. Weak negative correlation is only between NKY and the indices *LEGATRUU, SPGSCITR* and *CCI*.

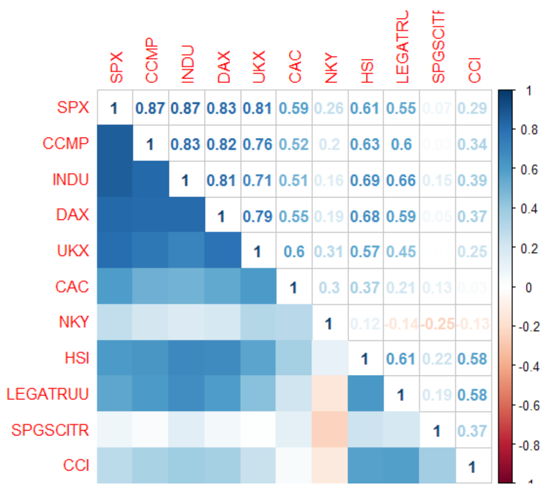

Fig. 1. Kendall's correlation coefficient

After observing the correlation during the whole time period, we were interested in the development of the local correlations. For this purpose we chose seven intervals with length of 72 months overlapping by 36 months with the neighbouring intervals. The results of this analysis can be found in *Figure 2*. Correlations greater than 0.7 are highlighted. In the legend of the figure we can see the couples of indices with significant positive correlations. These indices have very similar development through the intervals and also during the whole observed time period. *SPX* is strongly correlated with indices *CCMP, INDU, DAX, UKX* and *CAC* during all intervals. The German stock market index *DAX* also gain positive correlation with more of the indices, i.e. *CCMP, INDU, UKX* and *CAC*. *UKX* is beside above mentioned indices also highly correlated with *INDU* and *CAC* indices. Significant positive correlation is also between indices *CCMP* and *INDU*.

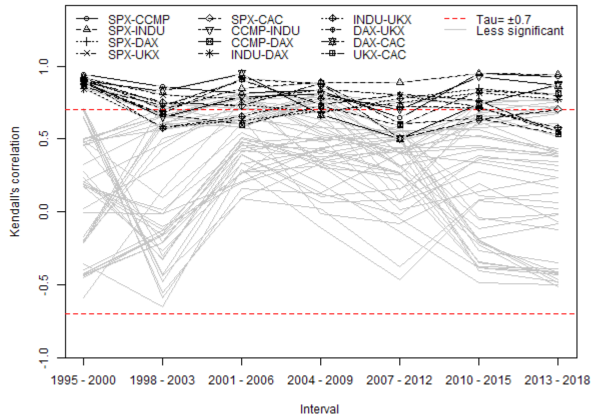

Fig. 2. Local Kendall's correlation coefficients

3.1. Global Trend Analysis

Trend analysis plays vital role in various fields of study since researcher are often interested in the long term development of processes. Describing the long term character of the stock indices can reflect the progress of market efficiency. For this purpose we analyzed the presence and the character of the trend of 11 stock market indices: *SPX, CCMP, INDU, DAX, UKX, CAC, NKY, HSI, LEGATRUU, SPGSCITR* and *CCI* index. We analyzed monthly time series from January 1995 to January 2018.

Since the multidimensional normality was rejected by testing, for this purpose Mann-Kendall test, Cox-Stuart test and Spearman's rho test was used. These tests were evaluated in software *R*, using the packages: *trend* [10] for the Mann Kendall test and Cox-Stuart test and *pastecs* [11] for Spearman's rho test. The results of the three trend tests are listed in *Table 1*. From this table it is found that the Mann-Kendall test indicates the presence of a statistically significant monotonic trend for all the observed indices. Cox and Stuart test and Spearman's rho test rejected the presence of trend at the level 0.05 only for the *NKY index*.

The character (increasing/decreasing) of the trend was obtained by Sen's slope using the R package *TTAinterfaceTrendAnalysis* [12]. All of the indices except *JPY* have a long-term increasing tendency. *JPY index* indicated decreasing trend. According to the magnitude of Sen's slope *61.1* the *HSI index* has the highest rising tendency among all the analyzed indices. Other high level of increase was observed in the *INDU Index* with the value of *45.05* and *DAX Index* with the value of slope *29.4*. The only decreasing trend in *NKY Index* reached the value of *-6.73*. P-values of Sen's slope magnitudes indicate that all of the magnitudes

Tab. 1. Results of Trend Analysis

Index	Cox-Stuart test		Mann-Kendall test			Spearman's test		Sen's slope	
	T	p-value	Z	p-value	Tau	Rho	p-value	Magnitude	p-value
SPX	9.75	$< 10^{-6}$	15.30	$< 10^{-6}$	0.62	0.75	$< 10^{-6}$	4.98	$< 10^{-6}$
CCMP	9.75	$< 10^{-6}$	16.58	$< 10^{-6}$	0.67	0.78	$< 10^{-6}$	12.25	$< 10^{-6}$
INDU	9.75	$< 10^{-6}$	17.97	$< 10^{-6}$	0.72	0.86	$< 10^{-6}$	45.05	$< 10^{-6}$
DAX	9.75	$< 10^{-6}$	16.44	$< 10^{-6}$	0.66	0.81	$< 10^{-6}$	29.40	$< 10^{-6}$
UKX	8.29	$< 10^{-6}$	12.02	$< 10^{-6}$	0.48	0.62	$< 10^{-6}$	9.91	$< 10^{-6}$
CAC	4.75	$< 10^{-6}$	6.95	$< 10^{-6}$	0.28	0.35	$< 10^{-6}$	6.51	$< 10^{-6}$
NKY	0.38	0.70	-2.06	0.04	-0.08	-0.11	0.06	-6.74	0.04
HSI	9.75	$< 10^{-6}$	16.01	$< 10^{-6}$	0.64	0.84	$< 10^{-6}$	61.10	$< 10^{-6}$
LEGATRUU	9.75	$< 10^{-6}$	22.37	$< 10^{-6}$	0.90	0.98	$< 10^{-6}$	1.30	$< 10^{-6}$
SPGSCITR	3.30	$< 10^{-4}$	4.17	$< 10^{-4}$	0.17	0.20	$< 10^{-3}$	6.43	$< 10^{-4}$
CCI	9.75	$< 10^{-6}$	13.67	$< 10^{-4}$	0.55	0.79	$< 10^{-6}$	1.26	$< 10^{-6}$

are statistically significant at the 0.05 level. We obtained the same results considering the signs of the statistics of Mann-Kendall test. Other aspect can be the value of Mann-Kendall's tau and Spearman's rho, which all indicate decreasing trend for *NKY* and increasing trend for all the other indices. Remarkably, according to these last three criteria, *LEGATRUU index* shows the highest level of increasing trend and on the other hand *SPGSCITR* and *CAC* the lowest increasing level.

3.2. Local Trend Analysis

Our next goal was to illustrate the partial development of the trend in the series. For this purpose we tested the presence of the trend over chosen intervals. We chose the same intervals as we chose for the local correlation. We evaluated the above mentioned trend tests for each index for all seven interval. Our aim was to compare the results of the global analysis with the local trend analysis results.

Similarly to the previous section, the decision making criterion whether to reject the null hypothesis or not, was the p-value. If we rejected the null hypothesis; that means there is statistically significant trend in the selected interval; then using Sen's slope we evaluated the character of the trend. The results of these tests for the *SPX* index are organized in *Table 2*.

Tab. 2. Local trend test results for SPX index

SPX				
Interval	CS test	MK test	SP test	Trend
1995–2000	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
1998–2003	0.0002	0.0002	$< 10^{-6}$	↘
2001–2006	0.004	0.003	$< 10^{-6}$	↗
2004–2009	0.22	0.11	0.46	–
2007–2012	0.41	0.32	0.39	–
2010–2015	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
2013–2018	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗

Tab. 3. Local trend test results for LEGATRUU index

LEGATRUU				
Interval	CS test	MK test	SP test	Trend
1995–2000	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
1998–2003	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
2001–2006	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
2004–2009	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
2007–2012	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	↗
2010–2015	0.004	$< 10^{-6}$	$< 10^{-6}$	↗
2013–2018	$< 10^{-6}$	0.002	0.001	↗

The null hypothesis is rejected for intervals except the one from 2004 to 2012. It means that there is statistically significant monotonic trend for all of the other intervals. In the interval from 2004 to 2012 the development of the *SPX* index price was constant. This time period overlaps with the duration of the Great Recession. Where it appears a significant trend the prices were increasing, except the period from 1998 to 2003, when the prices were decreasing. The overall trend test for this index led to a significant increasing trend in the series.

We evaluated the local trends for the other indices as well. The following patterns were observed. For most of the indices there is a period with a constant trend in the series. This constant trend was from 2004 to 2012. Here belong all of the indices except the *LEGATRUU* index. The development of this index is increasing for all of the intervals. The result for this index is organized in *Table 3*. Other common period occurred for the indices *SPX, CCMP, DAX, UKX* and *CAC* from 1998 to 2003. The trend for these indices was decreasing during this interval. They were decreasing for no other interval. *INDU* and *HSI* indices are significant because except the period with constant trend they were increasing for all of the intervals. Constant trend was observed for indices *DAX* and *CAC* also during the period from 2001 to 2006. *INDU* index revealed constant trend from 1998 to 2003. Two intervals with decreasing trend and one with constant

Tab. 4. Results of Change-point detection

Index	Pettitt's test		Divisive estimation		Agglomerative estimation
	Position	p-value	Position	p-value	Position
SPX	191	$< 10^{-6}$	34,80,133,163,223	0.01	31,226
CCMP	188	$< 10^{-6}$	37,74,127,159,190,223	0.01	47,74,119,225
INDU	130	$< 10^{-6}$	37,75,106,136,166,218,248	0.01	230
DAX	140	$< 10^{-6}$	37,90,129,162,218,248	0.01	60,80,131,227
UKX	191	$< 10^{-6}$	33,90,126,162,218,248	0.01	25,91,117,165,175,217
CAC	48	$< 10^{-6}$	38,90,126,165,225	0.01	39,91,125,165,230
NKY	69	$< 10^{-6}$	34,72,129,165,197,227	0.01	79,129,157,217
HSI	138	$< 10^{-6}$	51,81,140,175,216	0.01	144
LEGATRUU	139	$< 10^{-6}$	40,97,155,188,229	0.01	91,192
SPGSCITR	96	$< 10^{-6}$	63,111,166,196,240	0.01	110,239
CCI	135	$< 10^{-6}$	40,102,132,189,238	0.01	136

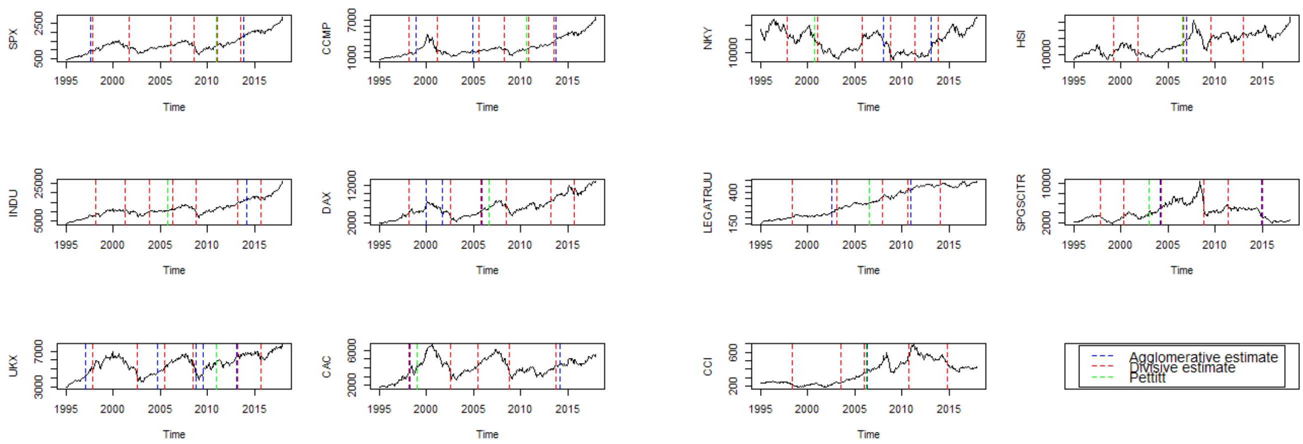

Fig. 3. Results of the change-point detection

trend was obtained for the *NKY* index, which from the global point of view was the only index with decreasing trend. Even if the overall trend test indicated increasing trend in the series, analyzing the local trend we also received intervals with constant and decreasing trend for all of the series except *LEGATRUU* index.

3.3. Change-Point Analysis

Presence of a change-point in time series is a vital question in the development of various processes. Our aim was to find abrupt changes in the time series of each index. First we used Pettitt's test from package *trend* for single change-point detection.

After finding the single change-points we carried out multiple change-point analysis using divisive estimation and agglomerative estimation of change-points from the *ecp* [13] package. Results of this analysis are in *Table 4* and *Figure 3*. According to Pettitt's test three indices- *SPX*, *CCMP* and *UKX* have a significant change-point in 2010. Other common significant change was detected in *INDU, DAX, HSI, LEGATRUU* and *CCI* indices from October 2005 to August 2006. *CAC index* has a significant change in December 1998. *NKY index* in September 2000, *SPGSCITR* in December 2012.

Next, the results of multiple change-point analysis were compared. As we can see in general we obtained more results using the divisive estimation. Some of the results are similar to the agglomerative estimation, although there are differences. All the detected change-points obtained by divisive estimation are statistically significant. We can see some pattern in the positions of the change-points. Most of the indices have the first abrupt change from September 1997 to April 1998. Other significant period can be considered from December 2000 to September 2001. *DAX, UKX* and *CAC* changed abruptly in June 2002. Further common changes were observed in the period from June 2005 to August 2006. Another significant changes in most of the indices was from March 2008 to July 2009. *LEGATRUU, SPGSCITR, CCI* and *CCMP* has significant changes from August 2010 to April 2011. Other changes was found from the end of 2012 to the beginning of 2014. For most of the indices it was the last change-point. For *INDU, DAX* and *UKX*, the last change-points were detected in August of 2015. For *CCI* and *SPGSCITR* indices it was in October and December 2014. As we can see in *Figure 3*, for most of the indices, divisive estimation allocated one of the multiple change-points near to the ones found by Pettitt's test. Also in most case the results of agglomerative estimation are close to the ones gained by divisive estimation.

3.4. Cluster Analysis

In practice it is common to seek for similar objects and explain the relationship between these objects. Thus our other goal was to identify the indices which development are similar in time and can be separated into clusters. We used hierarchical methods from the *stats* package [14], because of the low number of the clustering objects. Since indices are measured in various currencies, first we standardized the values of the series. To determine the best cluster analysis method we calculated the cophonetic correlation coefficient from package *stats*, for each method using Euclidean distance. We chose the method which contains the highest cophonetic correlation coefficient. According to this coefficient the best clustering methods to use are the average linkage method and Ward´s method. In average linkage method the average distance between objects of each cluster is used as distance between clusters. Ward´s method is based on minimization of the within-cluster variance. The number of clusters was determined using the *NbClust* package based on 30 indices as criteria. Most of the indices proposed the three cluster solution. The results of this analysis was creating using *dendextended* package [15] and can be found on the dedndrogram in *Figure 4*.

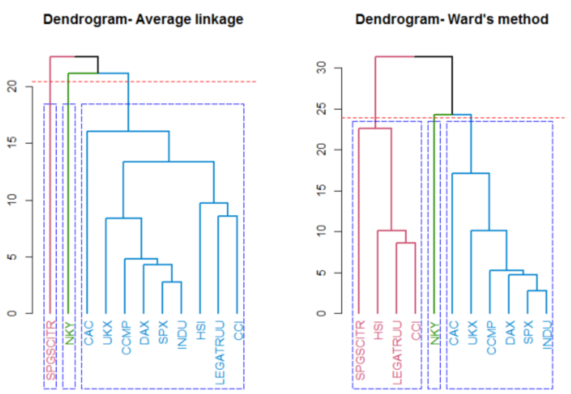

Fig. 4. Results of cluster analysis- average linkage method and Ward's method

As we can see the results have something in common. On the lowest linkage distance *SPX* and *INDU indices* are joined into a cluster. *DAX* and *CCMP* are very similar to them. The most different from other indices are *NKY* index and *SPGSCITR* index. *NKY* index was the only index showing decreasing trend. It also has a unique change-point found with Pettitt's test. In *SPGSCITR* some different change-point can be found from the change-points of other indices. We can also see that correlation analysis gave similar results. We can also observe the basic behavior of Ward's clustering method, that objects are joined into existing clusters and new cluster is created just in case of high dissimilarity.

4. Conclusion

The aim of this paper was to analyze the development of 11 market indices. Indices are very important indicator of market development. Our first goal was to reveal the trend in time series of indices. We compared the results of three nonparametric methods to determine the trend: Cox-Stuart test, Mann-Kendall test and Spearman's rho test. The Cox-Stuart test showed a significant increasing trend for all of the indices except *NKY index*. Mann-Kendall test and Spearman's test showed a statistically significant trend for all of the indices. Except *NKY index* it was a statistically significant increasing trend. The magnitude of the trend was calculated by Sen's slope. According to this statistics *HSI index* has the highest increasing tendency. All magnitudes are statistically significant on the level of 0.05.

Next we analyzed the trend locally to see its development. We chose seven intervals with length of 72 months overlapping by 36 months with the neighbouring intervals. Testing each interval using the previous tests we obtained the local trends. For most of the indices there was a period with a constant trend in the series. This constant trend was from 2004 to 2012. *LEGATRUU* index had all of the local trends increasing. In the development of other indices there was also at least a constant trend.

Other important point of view on the development of indices is finding change-points. Single change-point detection was carried out by Pettitt's test. Single change-points was found in 1998, 2000, from 2005 to 2006, in 2010 and 2012. Multiple change-point analysis was performed by using divisive and agglomerative estimation. The results of these methods are similar although a little biased. Also the agglomerative estimation proposes less change-points then divisive methods. The results of divisive estimation are statistically significant on the level of 0.05. Also the change-points found by Pettitt's test are located near to the ones obtained by divisive estimation. These change-points can be caused by changes in the components of the indices or by economic depression.

In the third part of this paper we found indices which development is similar in time. Results of average linkage method and Ward's method basicly give very similar clusters. The most similar development has *INDU* and *SPX indices*. Very similar to them are *DAX, CCMP* and *UKX*. The lowest level of similarity is between *NKY* and the other indices. We determined the three cluster solution as the most appropriate.

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