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EXAMPLES OF REASONS OF USING ICT IN CLASSROOM FOR CREATIVE PUPILS ACTIVITY

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Abstract. We can create many activities when we place geometrical problems to solve in class. For example: analyzing questions, making observation, discovering property of geometrical objects, searching the dependences among them, making hypotheses, making argumentation, proving, investigation of special cases and many others. Using traditional didactic resources causes that the abovementioned activities may be difficult for pupils at every level of education. It is also hard to explore the considered problems. That is why it is legitimate to search how to use ICT to support pupils in this process. Using GeoGebra based materials is an interesting idea for this purpose. Well made interactive materials can be used during normal lessons as well as during additional activities. Such a solution may increase understanding of the problem, and also allows a teacher to discuss with pupils extracurricular mathematics contents.

1. Introduction

The process of teaching mathematics is very difficult one. However, when we look at it globally, we can distinguish, with a considerable simplification, four components [1]:

- forming mathematical terms,
- mathematical reasoning,
- task solving,
- establishing mathematical language.

The main aim of this paper is not to show examples of using ICT at lessons of mathematics, but to show reasons of such utilization in almost each component. Our research is based on the course of geometrical properties of triangles. It consist of interactive GeoGebra based didactic materials. The investigation took place in the 1st and 2nd classes of high school (64 pupils being 16-18 years old). The pupils who approached investigation are from university classes with mathematical - physical - computer science profile. During everyday work they apply ICT in the wide meaning of this word. Utilization of supplies of e-learning platform as well as Internet or graphic calculators and computer programmes for these pupils are as natural as in traditional classes handiwork of auxiliary figure on sheet of paper. So as the figure is not the proof for pupils in the typical class, so for these classes simulations and inspecting tens of examples for given problem is not sufficient. However, they use GeoGebra for realization of illustration, for visualization their hypotheses willingly. Let us see what kind of didactic material they work on preferably.

2. Interactive definition

The interactive material made in GeoGebra programme is able to show an object of the definition in any but fixed instance and also to illustrate individual properties of this term. Such a help leads pupil across the term in dynamic way. Suitably prepared didactic material can turn pupils' attention to all conditions of the definition in the visual way. The traditional definition causes that very often pupils have large problems in retrieval of the essence of the definition. As a result, it is hard to them to show an example, instance or contrary instance for defined object. They have also problems to decide whether the given object fulfils the definition or not. Our interactive definition turns pupils special attention to necessity of fulfilment of all conditions described in definiens (so that definiendum sets) (Figure 1).

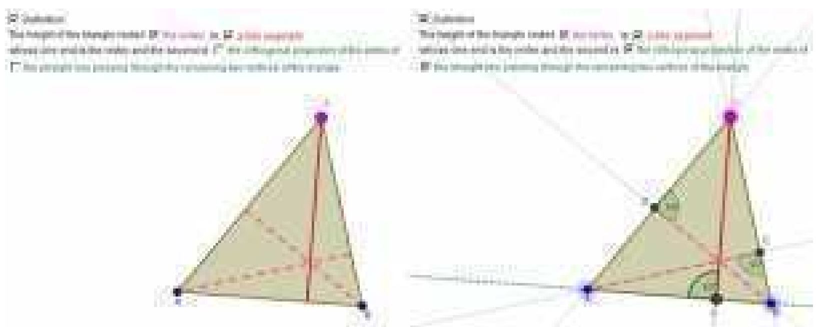

Figure 1. Dynamic definition of the height of a triangular.

With such interactive definition, pupils can easily observe what is essential in the definition. Not only colors (which can be used in the traditional definition) takes important part in this material. The possibility of testing any combination of conditions, which the defined object has to fulfil, is essential for this didactic help. The "transparent clear figure" describing the object, on which elements (properties) chosen by a pupil appear, allows for more solid understanding of the definition. Such a form of the performance definiendum facilitates deciding whether the given mathematical object is an instance, contrary instance or extreme case of the defined object. The possibility of discovering a property of terms directly appearing from the definition is another important aspect of this material. The example of observation of such a dependence is the fact that the midperpendicular marks the midpoint of the segment as the central symmetry figure.

3. Discovering properties of terms and formulating hypotheses

The observation is the most frequent aspect of ICT used by teachers at mathematics lessons. Simulations provoke formulating hypotheses and can help with their verification. Pupils during the work with presented materials also formulate hypotheses which they prove during the learning of new material. Pupils encounter the possibility of observing property of mathematical object and then formulate hypothesis (for example, when studying midperpendicular). The aim of pupils' investigation is to explore that points from midperpendicular are evenly distant from both ends of the segment (Figure 2).

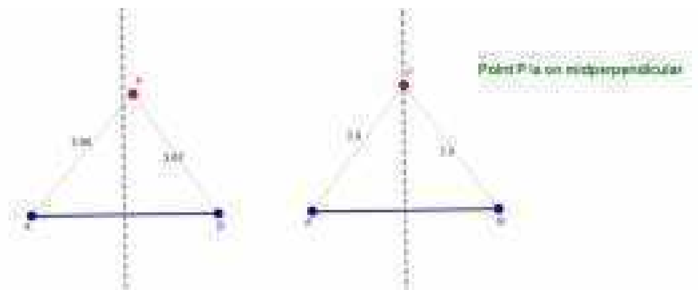

Figure 2. Simulation provokes formulating hypotheses.

During investigation of properties of mathematical objects, the main aim is not to concentrate on the possibility of observation of very wide spectrum of examples, which lead to deep visualization and assurance of the noticed hypothesis, but to prove it in the formal way. GeoGebra can help to create such a tool which will lead pupils across entire formal process of prove.

4. Proof (proving of discovered properties)

The traditional proof represents entire progress of reasoning at ones (as you can see in Figure 3).

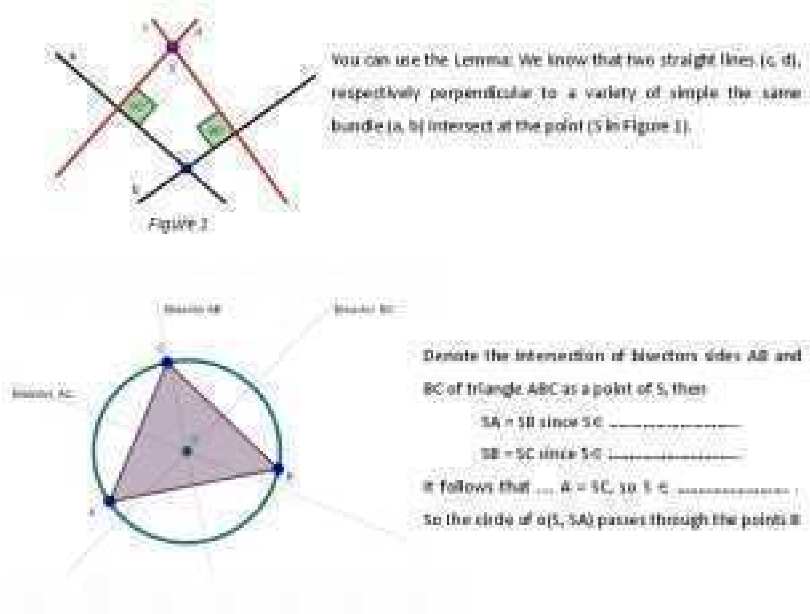

Figure 3. Example of "traditional proof" card to supply.

In such tasks pupils focus on supplement of lacking symbols, often making this "thoughtlessly" because, e.g. one of the steps is inexplicable (very often). We can also present traditional proofs using the interactive help made with GeoGebra (Figure 4).

The active card presenting the proof leads pupils by certain deductive progress of reasoning step by step, illustrating the geometrical interpretation on dynamic figure simultaneously. Pupils cannot see the entire proof at once, so it does not distract their attention. They can focus on the essential elements of the proof at the moment. Pupils have to pass all the steps (earlier stages) to see the last one. Such a dynamic proof can block next steps if there is, for example, inappropriate position of one of mathematical objects (Figure 5).

The next difficulty is the fact that the majority of proofs in mathematics begins from words, e.g.: Let ABC be any fixed triangle ... This formulation "any fixed" very often causes (even for students) feeling of internal contradiction. We establish any case for which we guide the proof (in normal proofs). Establishing any mathematical object, we consider the concrete instance but without concrete properties, except for those from the foundations. Fixed,

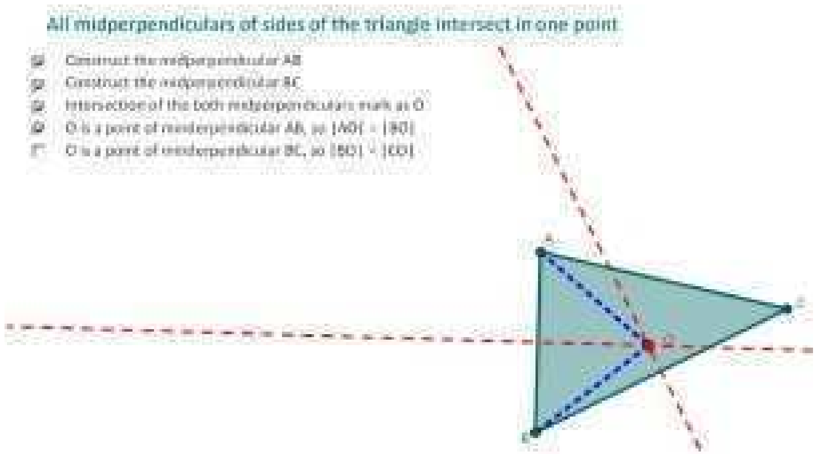

Figure 4. Example of GeoGebra's "interactive proof".

because we have to guarantee invariability of all the parameters during the whole proof (nothing can be changed during the proof). For example, we cannot accept the position of the point A at one moment and then alter its position in the midst of the proof. This will cause so many changesin objects depending on the point A that you really should begin the proof from the beginning. This is a difficult element of the proof even for students. It is inexplicable why the proof is correct for every (any) object but the proof is guided only for the fixed one. Pupils often feel the need of verifying the theorem for other cases (special cases) even if they are convinced of the truth of the theorem. Using the dynamic card made in GeoGebra, it makes possible the proof appears step by step and is equipped by suitable comments. It allows to turn attention on essentials of the given dynamic figure. In GeoGebra a figure is always dynamic, and it can be changed at any time without losing the idea of the proof. Pupils can test (during the proof) how the proof will behave for the special cases (e.g. for rectangular triangle or equilateral one). Such an approach allows schoolchildren to avoid fear that the proof is guided for one fixed (concrete) triangle, but not for all ones with the given properties. GeoGebra can help teaching argumentations and proving in such a way.

5. Tasks and mathematics problem solving

Pupils during their mathematical education meet various types of tasks. Teachers spend few time for constructional problems because of long time for their realization and often because of manual pupils skill (mainly during the construction itself). A pupil creates the range of mathematics through the prism of solved problems [2, p. 3], so it is important to dedicate them a little more

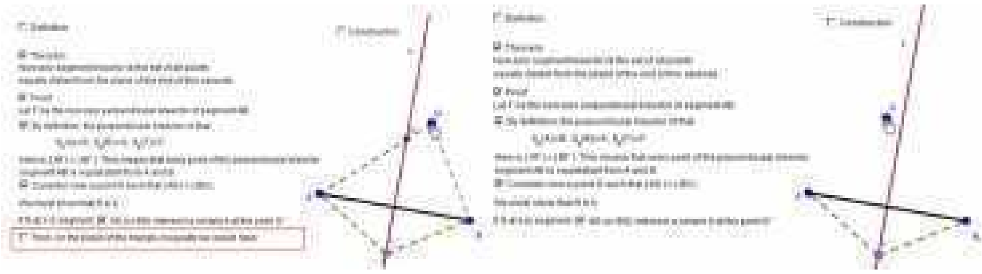

Figure 5. Example of GeoGebra's "interactive proof".

time. Pupils get to know what is mathematics? and what can it be? when they solve actively and suitably the well-chosen mathematical problems [2]. GeoGebra can be a generator of some tasks for practice, for testing, verifying and evaluating the assimilated material. We will show such an example of the constructional problem.

The solution of a constructional problem consists in the following stages: analysis of construction, description of construction, proof of correctness, discussion of existence of solution and the numbers of solutions, possibility of realization, construction [3, p. 78]. While reminding pupils the terms necessary to expand their knowledge of geometry of triangle using application representing the definition of mathematical objects, they replenish knowledge with construction of introduced definiendum. The construction can be described with dynamic commentary which will turn pupils' attention at stages of construction or any possible kind of simulations.

Struggling with the constructional problem, pupils first have to pass its analysis. In "traditional conditions" we find the existence of mathematical object described in content of the problem and approach to realization of auxiliary figure. Such a help is fixed for one concrete case, while the computer programme allows us to change quickly the initial foundations and to observe the consequences (changes in objects related with them). GeoGebra can show dynamic figures and allows to verify observations related with the positions of objects depending from each other. It is more easily to choose the proper necessary statements for proving hypothesis in this way. The active figure allows to verify quickly whether the chosen statement, which we want to use in a proof, is correct or not for actual situation. During change of initial points (or others properties of a figure) the program changes objects related with it automatically, but it never changes mathematical rules (based on the objects dependence and mathematics statements). Let us look at an example – the problem of the heights in a triangle.

Pupils usually identify the height of a triangle as the segment which is perpendicular to the base (at the "bottom" of the triangle) having the end at the opposite apex. When we make such a construction in an acute triangle, we usually do not perceivethe problem. The remaining heights give rise to the problem when a triangle is rectangular or obtuse.

When we make wrong construction of the perpendicular to a segment (but not to a line enclosed the base), the program displays that in this situation an obtuse triangle does not have three heights, but only one (in Figure 6 the segments marked as dotted just disappear). This fact should induce pupils to reflection and search of the essence of the mistake.

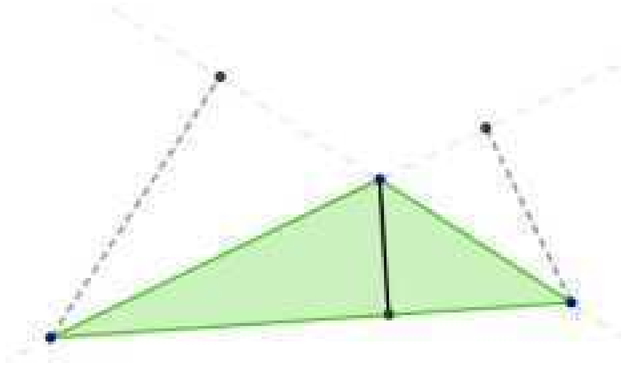

Figure 6. Heights in an obtuse triangle.

The description of construction is the next step of solving a constructional task. Usually pupils make many mistakes at this stage. GeoGebra based didactic material can also help with it. Let us look at another example – the circumcircle and the incircle of a triangle.

Teachers teaching circumscribed or inscribed polygons begin from the subject of the circumcircle and the incircle of a triangle. Very often pupils simplify both problems to a question of finding the position of the centers of the sought circles. But such an approach to the task can cause next mistakes.

At first construction pupils find the point of intersection of midperpendiculars of sides of a triangle and then plot the circumscribed circle. The construction is correct, because it suffices to guarantee that the circle crosses through vertices of a triangle (it is not necessary to know the radius of the circle). It is harder in the second situation, but pupils do the same. They find the point of intersection of interior angle bisectors of a triangle and try to plot the incircle immediately. The new problem appears because there is no point which the sought circle must enclose. Usually they forget to construct the tangency point. Our investigation shows that pupils often construct the incircle by:

- intersection the interior angle bisector and opposite side of a triangle;
- a point not belonging to a triangle, just to make the circle inside the triangle;
- a point belonging to the side of a triangle (Figure 7).

When we have an equilateral acute triangle, it seems to be the correct construction. However, when we change the position of vertices of a triangle, we will see our mistake. If we would proceed this wrong procedure working with a sheet of paper, we get the "correct looking" incircle in the incorrect way. We would not have consciousness that the solution of the constructional task had serious lacks.

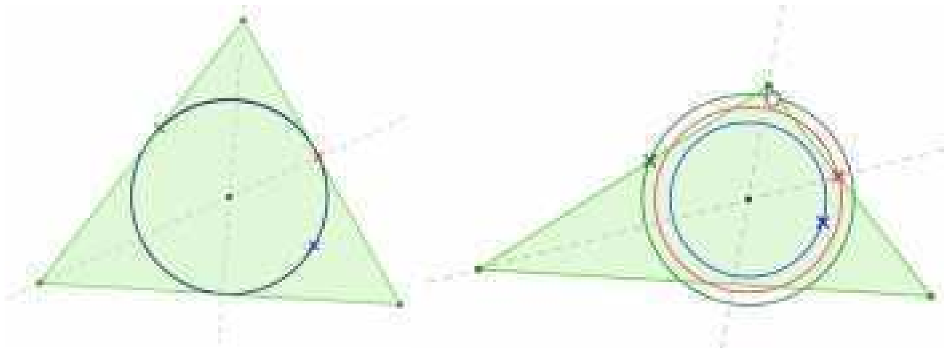

Figure 7. "Incircles" in a triangle.

GeoGebra (like other DGS programs) is ruthless and unfeeling in such situations. It shows all the shortcomings in the construction, informs about all the inaccuracies. Such an experience is particularly valuable for future teachers who will teach such mathematics which they know themselves. Next essential aspect of using DGS programs is the possibility to analyze the protocol of the construction (Figure 8).

Every object from description of the construction has exact connection with the object from protocol of the construction. Every mathematical object is registered in the protocol in such a way that it shows all the connections with other objects. Next steps of solution consist in the search of answers to questions relating quantities of solutions and feasibility of the construction. The properties of GeoGebra described above help in detecting conditions for both of them. A teacher should prepare suitably the didactic material to make it easy.

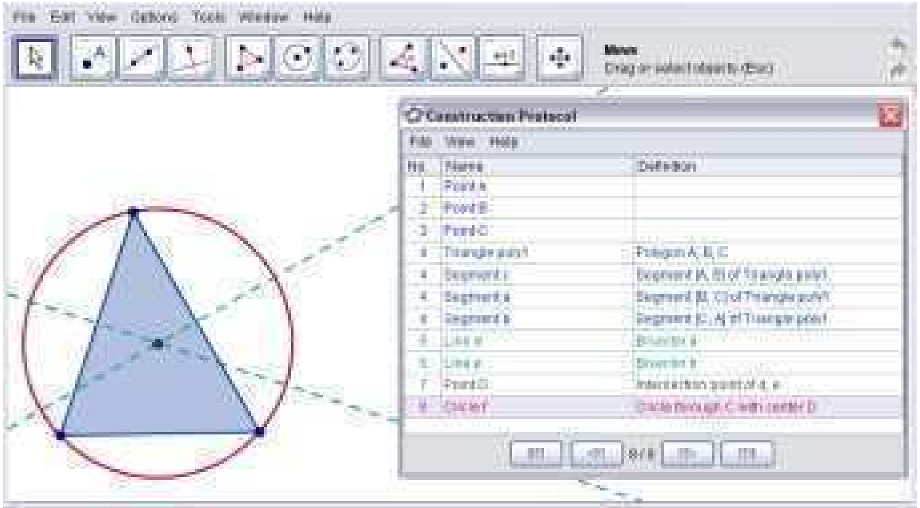

Figure 8. Protocol of the construction.

GeoGebra makes possible the limitation of access available tools (removing temporarily needless icons from interface). The program also allows us to create new tools more adapted to the concrete situation. Simulations with such a program also make easier proving correctness of solution that is the last element of constructional task. We teach perception of properties of mathematical objects and dependence between them with help of ICT in such a way.

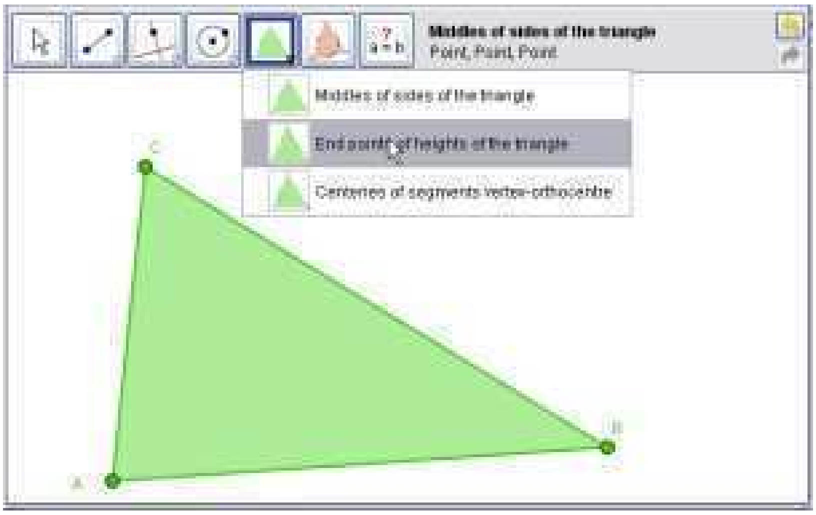

Figure 9. New interface of GeoGebra.

6. Conclusions

Our investigation showed that interactive didactic materials can be successful use for:

- illustration of new mathematical terms considering the necessary and sufficient conditions (for better understanding of definition);
- observation of properties of mathematical objects (to expand and intensify mathematics knowledge and to make reasoning);
- formulation of the noticed hypotheses and their verification (to teach proving);
- solution of tasks and mathematical problems (to make mathematics reasoning).

Didactic material based on Interactive GeoGebra can shape mathematical terms while formulating definitions and during the simulations. It can lead pupils via mathematical reasoning or provoke them to solve mathematics problems during simulation, proofs and constructions. Introduced materials interlace all the components of the process of teaching mathematics. Preparation of such applications is very time-consuming, but well prepared materials represent valid mathematical questions in the alternative, creative way for pupils. This kind of didactic material (as ours investigations showed) obtained the recognition of students because it enlarges their chances of understanding mathematics.

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