TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 124, 2012, 17–39

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On the internal efficiency of a turbine stage: classical and computational fluid dynamics definitions

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Abstract

Almost entire fleet of steam turbines in Poland was designed between 1950–1980 with the use of the so-called zero-dimensional (0D) calculation tools. For several years, design and modernization of the turbines occur in assistance with the state-of-the-art methods that describe working fluid flow field based on three-dimensional (3D) models and computational fluid dynamics (CFD) codes. This cooperation between 0D and 3D codes requires exchange of overall, integral information such as: power, efficiency, heat and mass fluxes. In consequence the question arises regarding the cohesion of definitions, and particularly regarding the correctness of the definition for internal efficiency of the turbine's stage and the turbine as a whole. In the present paper we formulate basic definitions reason of efficiency that are naturally adapted to the numerical 0D and 3D models. We show that the main reason of differences between the definitions in 0D and 3D is the definition of the theoretical work of the stage l_t. In the classical 0D models, mostly employed is the isentropic approach, and hence the isentropic efficiency occurs. Meanwhile, in the increasingly common 3D approach (most likely by CFD), we use more physically correct pathway by subtracting energy loss from the available energy, that leads to the polytropic definition of efficiency. We show an example of computing the efficiency and the 3D losses, denoted with additional subscript CFD, we also discuss benefits of this definition in comparison with the isentropic classical definition in 0D.

Keywords: Turbine stage; Stage power; Stage efficiency; Losses

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Nomenclature

0D	–	algebraic model of flow based on integral balances of mass, momentum and energy
3D	–	three-dimensional model based on differential equations, that requires complete geometry of a flow channel
CFD	–	uniform method of simultaneously solving fluid flow governing equations based on finite element or finite volume method discretization
A	–	area of section, mm^2
A_u	–	blade moving area on which medium exerts action, mm^2
$\mathbf{c}$	–	average velocity vector in 0D derived from momentum conservation equation, m/s
c	–	average velocity vector length (speed), m/s
c_n	–	normal to surface average velocity component magnitude in 0D, m/s
c_t	–	theoretical average velocity vector length, m/s
c_o, c_1, c_2	–	absolute average velocity vector length at points 0, 1, 2, respectively, m/s
$\mathbf{D}$	–	diffusive stresses tensor, Pa
$\mathbf{e}_r$	–	radial direction
$\mathbf{e}_u$	–	circumferential direction
$\mathbf{e}_z$	–	axial direction
i	–	medium enthalpy, kJ/kg
$\mathbf{I}$	–	unit diadic; $\mathbf{I} = \mathbf{e}_r \otimes \mathbf{e}_r + \mathbf{e}_u \otimes \mathbf{e}_u + \mathbf{e}_z \otimes \mathbf{e}_z$
$\mathbf{J}_{UV}$	–	Umov-Volter mechanical energy flux, N/ms = Pa m/s
J_r	–	radial clearance, mm
J_z	–	axial clearance, mm
l_t	–	theoretical unit work of a stage, kJ/kg
l_s	–	isentropic unit work of a stage, kJ/kg
l_p	–	polytropic unit work of a stage, kJ/kg
l_u	–	circumferential unit work of a stage, kJ/kg
Δl_i	–	internal work losses of a stage, kJ/kg
Δl_u	–	circumferential work losses of stage, kJ/kg
$\dot{m}$	–	mass flow rate, mass flux, kg/s
$\dot{m}_t$	–	theoretical mass flow rate, kg/s
$\mathbf{n}$	–	unit normal vector
N	–	mechanical power, MW
N_u	–	circumferential power of a stage, MW
N_t	–	theoretical power of a stage, MW
p	–	pressure, MPa
$\dot{p}$	–	rate of pressuree change, Pa/s
Pu	–	Puzyrewski number
$\mathbf{P}_u$	–	blade drift force, N
r	–	pitch radius, mm
$\mathbf{r} = r\mathbf{e}_r$	–	radial vector, mm
$\mathbf{R}$	–	turbulent stresses tensor, Pa
s	–	medium entropy, kJ/kg °C
$\bar{T}$	–	average temperature, K
T	–	medium temperature, K

$\mathbf{t}$	–	Cauchy stress tensor, Pa
$v_n = \mathbf{v} \cdot \mathbf{n}$	–	fluid velocity normal component, m/s
$\mathbf{u} = \boldsymbol{\omega} \times \mathbf{r} = u(r)\mathbf{e}_u$	–	blade drift velocity vector, m/s
$\mathbf{v}$	–	fluid absolute velocity vector, m/s; $\mathbf{v} = \mathbf{u} + \mathbf{w}$
$\mathbf{w}$	–	fluid relative velocity vector, m/s

Greek symbols

ζ	–	loss coefficient
η	–	efficiency $\eta = 1 - \zeta$
μ	–	mass flow rate coefficient
ρ	–	volumetric density of a medium, kg/m^3
$\tilde{\rho}$	–	average volume density of a medium, kg/m^3
$\boldsymbol{\tau}$	–	viscous stresses tensor, Pa
τ	–	time, s
φ, ψ	–	velocity reduction coefficients nozzle and rotor, respectively
$\boldsymbol{\omega} = \omega \mathbf{e}_z$	–	angular velocity vector, rpm
ξ	–	losses from leak, kJ/kg

Subscripts

0,1,2	–	real points of process
1s,2s	–	points of isentropic process
1p,2p	–	points of politropic process
0c,2c	–	real points of process with kinetic energy
in	–	inlet
out	–	outlet
p	–	politropic
s	–	isentropic
t	–	theoretical
exp	–	experimental
u	–	unit, circumferential
n	–	normal
r	–	rotor
s	–	stator
leak	–	leak losses

1 Stage efficiency — classical and CFD definition

Definition for the cycle efficiency either of a heat engine or of a turbine stage, has its origins in the work of Sadi Carnot [4,1], since which the distinction between the heat machine power and its efficiency begins. Starting from the works by Pambour, Zeuner, Schichau [14,26,27,12] efficiency is related to one kilogram of working fluid, which 'behaves' in a better or worse way during conversion of heat energy into mechanical energy. Carnot discoveries are still important, since they set up a paradigm for constructing and thinking of a heat engine – by introducing the ideal conversion of heat into work in which, in Carnot's words, there are no 'blank passages of the caloric', that is, the energy conversion in the working fluid into work occurs with conserved entropy. Then, also has a paradigm been set of thinking about determining the maximum allowable work to be achieved from thermal energy of the ideal working medium, while the information of the real working medium being nonideal, suffering from losses in its accumulated energy, has been embraced in the Carnot coefficient, known today as efficiency (denoted commonly with Greek letter η). The difference between an ideal medium and a real medium that defines inevitable losses, was mathematically described with the medium loss coefficient μ.

The concepts of losses and efficiencies are essential in the process of designing of any thermal engine also in these days. Similarly to the practice of Carnot and other pioneers of turbine technology, the very approach in designing is still the same — first an object for an ideal medium is designed, medium possessing no viscosity, no thermal conductivity, no turbulence or diffusivity, etc. and then the real gas corrections are implemented through the methods more or less correctly chosen [5,18–20,22]. Owing to repeatedly corrected knowledge, magnitudes of the losses attributed to all spectra of devices are anticipated today with increased accuracy. Difficulties emerge only in the case of new constructions or in cases of solutions required by modernization.

Carnot approach may be summarized in the following verbal rule [4]:

$$\begin{pmatrix}\text{device}\\ \text{power}\end{pmatrix} = \begin{pmatrix}\text{amount of kilograms}\\ \text{of medium per second}\end{pmatrix} \cdot \begin{pmatrix}\text{efficiency}\\ \text{of a real}\\ \text{conversion}\end{pmatrix} \cdot \begin{pmatrix}\text{theoretical work}\\ \text{of one}\\ \text{kilogram}\end{pmatrix} \qquad (1)$$

or mathematically (employing notations by Szewalski [20])

$$N = \dot{m}\eta l_t \,, \qquad (2)$$

where N is the mechanical power, $\dot{m}$ is the real mass flow rate, η is the efficiency of the stage and l_t is the theoretical unit work of a stage.

Theoretical work of the turbine stage or the entire turbine achieved from a unit mass of the medium was denoted with subscript t, that stands for 'theoretical' and indicates an ideal conversion of available energy of steam [5,18,21]. Equation (2) reflects the character of the traditional approach to designing first the thermodynamic cycle which is being designed for 1 kg of an ideal working fluid, next the resulting work is decreased by the factor of the efficiency of the real working fluid and, to acquire the desired power, it is then multiplied by the resultant amount of working fluid kilograms to perform the work. Efficiency, η, is an empirical parameter, known a priori for similar devices. However, deriving the magnitude of cross-sections to possess the required mass flow rate, $\dot{m}$, is performed based on the theoretical mass flow rate, $\dot{m}_t$, to be $\dot{m} = \mu\dot{m}_t$, where mass flow coefficient, μ, is to restrict the value (coefficient) of the theoretical mass flow rate accordingly to the real velocity profile.

In accordance with the current computational tools possibilities based on the three-dimensional (3D) modeling, natural question occurs on the agreement between the computational fluid dynamics (CFD) and the classical line of reasoning in traditional design workflow. The question is if concepts such as efficiency or losses may be strictly defined in the 3D approach, in other words — are these concepts general, independent on the tool being used. In the following sections we propose consistent definitions for efficiency and losses, to allow for transforming computed integral quantities received from 3D models into zero-dimensional (0D) codes.

2 Geometry of the stage and its parameters common with 0D model

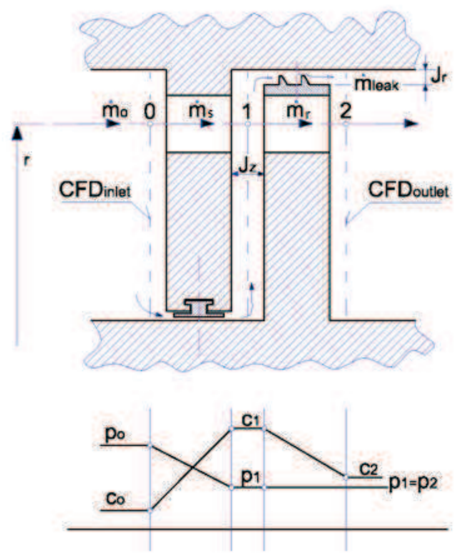

Figure 1. Geometry of an axial impulse stage of a turbine with working medium parameters written for pitch radius, where: $\dot{m}_0$ – the mass flow rate of the steam flown in point 0, $\dot{m}_s$ – the mass flow rate of the steam flown through stator diaphragm channel, $\dot{m}_r$ – the mass flow rate of the steam flown through the rotor stage channel, $\dot{m}_{leak}$ – mass flow rate of the steam flown through the leak, p – pressure, c – absolute averaged velocity vector length, J_r – radial clearance, J_z – axial clearance, r – pitch radius.

Figure 1 presents the classical scheme of the impulse stage in an axial turbine. It comprises stator diaphragm (subsonic nozzles) and the rotor wheel. It has very easy to define geometrical parameters (3D), such as pitch radius, r, axial clearance, J_z, radial clearance J_r. In 0D modelling it is assumed, that the steam parameters (p, c) are averaged in sections, are given at pitch radius, and are denoted – accordingly to tradition initiated by the pioneers and the masters of turbine technology – by 0,1,2, respectively. In a 0D model no information on the geometry or fluid parameters on the segment 0-1 and 1-2 is provided; nonetheless it is tacitly assumed, that those change linearly.

3 Mass flow rate

The question on the mass flow rate of the stage, or the entire turbine, is fundamental for determination of its performance and working parameters. It is important to remember, that the turbine is a device that serves for energy conversion of the working medium internal energy into mechanical work, which is then received at the moving surfaces. For example, that internal energy of the working (fluidic) medium first needs to be converted into kinetic energy in the expansion device called the nozzle, and then is forwarded and absorbed at the moving surfaces of the rotor blades. Thus, differently from reciprocating engines, where working medium is at some temporal instance at relative rest, in the axial and radial turbines it is constantly flowing through the device. Mathematically it is the mass flow rate $\dot{m}$. The overdot, according to Rankine's proposal, means the material time derivative of mass — it is consistent with the material velocity (in the Lagrange sense) of the volumetric mass density, $\dot{\rho}$, found in 3D models [2]. In other words, energy conversion occurs in the medium flow through an expansive device (nozzle, guide vanes) and a mechanical device (rotor).

To enforce the constant mass flow rate, $\dot{m} = const$, of the medium through the stage passage and its clearances a constant pressure difference between inlet and outlet stages needs to be sustained, i.e., between p_{in} and p_{out}. Parameters denoted by CFD $\dot{m}$, p, $\mathbf{v}$, T, subscripts *in* and *out* may be also determined at points 0 and 2 (Fig. 1), respectively. For the entire turbine we usually know both pressures, as the pressure of the life steam, and the pressure in the condenser or extraction point, and is identified with the measured static pressure. Usually it is assumed, that these pressures are constant in the measurement cross sectional area, which allows to identify the pressures measured at the wall (or inside an insulation) with the 0D model pressure. The assumption on equal pressure across the characteristic cross sectional area is not made in CFD modelling, we however agree, that energetically weighted pressure of the medium in the cross section in question should be identical to the 0D pressure. This assumption relates to the averaged CFD pressure be equal to the measured pressure. The three-dimensional CFD model allows for 'reading' the value of the pressure precisely at the measurement point. It is possible now to explain the difference between the 0D and measured pressures – those differences are significant and may reach even up to 0.2 MPa [3].

Other basic differences between 0D and 3D models need reminding. Firstly, the given pressure drop, $\Delta p = p_{in} - p_{out}$, propelling the flow, is used in an ultimately different manner in the classical 0D approach compared with the numerical 3D approach. In 0D, the given pressure drop results in enthalpy drop of the medium, which induces some kinetic energy, for the sake of energy conversion principle to be met. From the kinetic energy, velocity vector length is established, denoted with c_{out}. Let it be recalled that in an ordinary converging channel, where heating and working diminish the energy equation per one kilogram of a working medium has a 0D form [11,13,25]

$$i_{in}(p_{in}, T_{in}) + \frac{c_{in}^2}{2} = i_{out}(p_{out}, T_{out}) + \frac{c_{out}^2}{2} \ . \tag{3}$$

This equation allows the outlet velocity vector length to be determined [11,13]

$$c_{out} = \sqrt{2(i_{in} - i_{out}) + c_{in}^2} \ . \tag{4}$$

Equation (4) is readily used by measurement engineers, since it only requires measured pressures and temperatures at both channel inlet and outlet, so that tabularized equations of state could provide enthalpies i_{in} and i_{out}. The energy equation (3) is correct during compression as well as during expansion [8]. Average velocity vector length c_{out} energetically averaged, is slightly higher from the vector length derived from momentum conservation equation and averaged with respect to momentum, and hence $c_{out} \geq |\mathbf{c}_{out}|$ [2].

In 0D computation practice determination of enthalpy i_{out} for the real flow is impossible to be made directly, since the temperature T_{out} is unknown. Hence calculations are based on a hypothetical quantity, governing entire further design process — it is the ideal velocity, in the literature known as 'theoretical' velocity [5,18], which is computed from a different enthalpy drop from the real one occurring in Eq. (4). Rejecting quantities unable to establish readily, such as heat escape from steam through channel walls, turbulent heat transfer along the channel between its inlet and outlet, turbulent momentum transport, frictional resistance against channel's walls, friction between fluid streams, etc., finally a picture emerges of a 'theoretical' enthalpy value $i_{out,t}$ of such an ideally elastic (reversible) conversion.

Certain approximation of ideal enthalpy value is an enthalpy mostly employed in 0D models, defined to be the enthalpy of an isentropic conversion (expansion, compression); it is calculated with the use of equations of state

$$i_{out,s} = i_{out,t}(p_{out}, s_{in}) \ . \tag{5}$$

Here, ideal conversion is enforced by the assumption, that it processes in the entire channel with a constant entropy value of the medium $s_{in} = s_{out} = const.$

It is distinguished from others by adding the subscript s. It is hence excessively stiffening the condition for conversion — entropy of the medium flowing through the channel changes anyway, both through elastic conversion described with equations of state and through internal sources 'producing entropy', originating from the above described processes.

Another theoretical enthalpy is the enthalpy, which value evolves only as a result of any arbitrary change in equations of state, while the changes from irreversible processes are rejected due to the fluid being assumed ideal. The definition is then

$$i_{out,p} = i_{out,t}(p_{out}, s_{out,p}) \ . \tag{6}$$

Throughout the paper it will be called enthalpy of ideal polytropic process (it is distinguished from others by adding the subscript p), entropy for which is computed with neglecting any irreversible phenomena in the flow.

The real velocity c_{out} of the steam leaving the nozzle is lower than the theoretical velocity $c_{out,t}$, regardless enthalpy definition employed, either isentropic Eq. (5) or polytropic Eq. (6). In both cases, same losses decrease the velocity, that is the friction between the molecules and the vessel walls. The only measure for practical estimation of those 'discrepancies from the thought (referential) ideality' (these are not losses *per se* in any case) is the assumed in the 0D model, and calculated in a 3D model, velocity reduction coefficient, denoted in literature by φ (nozzles, guide vanes) or ψ (rotors). In this case the real length of the velocity vector (direction still remains unknown) acquired from the energy conservation equation satisfaction condition is:

$$c_{out} = \varphi c_{out,t} \ . \tag{7}$$

First empirical research on determining the φ coefficient were conducted by Christlein, Fluggel, Stodola, Brown and Boveri [5,18]. Its values depend strongly on the ideal velocity and arrange themselves in the so called nozzle characteristic $\varphi = 0.95$, 0.968 at 400 and 800 m/s, respectively. Deviation from the ideality (hypothetical losses) of kinetic energy are determined by the loss coefficient $\zeta = 1 - \varphi^2$ [14].

Owing to the fact, that in 0D model most accurately is computed the theoretical velocity, it is easier to formulate the theoretical mass flow rate than the real mass flow rate. Hence, the real mass flow rate, which according to the velocity profile is always lower, may be described with the empirical mass flow rate coefficient μ:

$$\dot{m} = \mu \dot{m}_t \ . \tag{8}$$

In the 0D modelling practice, mass flow rate coefficient determination bases on computing the theoretical mass flow rate, $\dot{m}_t$, and measuring the real mass flow rate, $\dot{m} = \dot{m}_{exp}$,

$$\mu_{0D} = \frac{\dot{m}_{exp}}{\dot{m}_{t,0D}} \ . \tag{9}$$

An opposite situation is the case in 3D modeling CFD – the real mass flow rate is accurately computed, and interpretational difficulties arise with the theoretical one. When both mass flow rates are computed with the use of the same numerical tables of state, for the polytropic change of state, the mass flow rate coefficient may be determined for each turbine stage or the entire turbine as

$$\mu_{CFD} = \frac{\dot{m}_{CFD}}{\dot{m}_{t,CFD}} \ . \tag{10}$$

The μ_{CFD} coefficient is slightly smaller than μ_{0D}, nonetheless, for design purposes it may be assumed that $\mu_{0D} = \mu_{CFD}$.

It requires noting, that the mass flux computed in CFD is the basic integral parameter of the 3D flow, which is controlled during the numerical process — it is possible to be read in any section, A, (not necessarily planar) of a turbine flow domain, oriented with a normal vector, $\mathbf{n}$, as [2]

$$\dot{m}_{CFD} = \iint_A \rho v_n dA = \iint_A \rho \mathbf{v} \cdot \mathbf{n} dA \ , \tag{11}$$

where the normal to surface velocity component magnitude, v_n, occurs, not the entire magnitude of the vector. Transition to the 0D model is as follows [2,16]:

$$\dot{m}_{0D} = \iint_A dA \frac{\iint_A \rho dA}{\iint_A dA} \frac{\iint_A \rho v_n dA}{\iint_A \rho dA} = A\tilde{\rho} c_n = A\tilde{\rho}\mu_{0D} c_t = \mu_{0D}\dot{m}_{t,0D} \ , \tag{12}$$

where $\tilde{\rho}, \rho$ are the averrage volume and volume density of a medium respectively, and c_t is the theoretical average velocity vector length, c_n is the normal to surface velocity component magnitude in 0D.

The flow coefficient depends on both the fact, that the average velocity, $c = \varphi c_t$, determines the constant velocity profile within the section, and that its value and the direction of the averaged vector $\mathbf{c}$ is unknown, that enforces assuming $c_n = \mathbf{c} \cdot \mathbf{n} \cong c$ [2][1].

4 Power of the stage

Recall, that the fundamental task ahead of a working medium is its contribution in the process of energy conversion from thermal to mechanical — either in the form of fluid storing energy at one place and then transferring this energy into

[1] Full discussion of this problem one can find in monograph [2]

another place, or in the form of (ideal) converter of energy forms. Mathematically, mechanical energy flux, expressed by the working medium parameters, is defined by the Umov-Volterra energy flux [2] $\mathbf{J}_{UV} = \mathbf{tv}$, that is the product of momentum flux tensor, $\mathbf{t}$ (known as the Cauchy stress tensor) and the material velocity vector of the fluid $\mathbf{v}$. It is a sufficiently general definition embracing turbulent, diffusive and thermal stresses. Mechanical power received through the Umov-Volterra energy flux on a moving surface of a blade A_u, oriented with a normal vector, $\mathbf{n}$, is (Badur [1,2])

$$N_{u,CFD} = \iint_{A_u} \mathbf{J}_{UV} \cdot \mathbf{n} dA = \iint_{A_u} \mathbf{tv} \cdot \mathbf{n} dA \; . \tag{13}$$

Here, $\mathbf{t} = -p\mathbf{I} + \boldsymbol{\tau} + \mathbf{R} + \mathbf{D} + \ldots$ is a symmetric tensor of entire momentum flux, comprising, respectively, elastic pressure tensor, traceless viscous stress tensor, Reynolds turbulent stress tensor, diffusive stress tensor, rarely employed with respect to working fluids (e.g., radiation stress tensor in gas turbines) and other. In CFD models of fluids without velocity slip on a wall, fluid velocity, $\mathbf{v}$, used in Eq. (13) is equal to velocity, $\mathbf{u}$, of the moving surfaces (Badur [1,2]).

Even though the $N_{u,CFD}$ is included in the energy equation, in CFD computation is determined entirely and solely from the momentum balance equation. Thus, in case of the fluid tensor model, limited only to the elastic Euler fluid, i.e., $\mathbf{t} = -p\mathbf{I}$, where pressure is computed from a polytropic process. It be expected, that the circumferential power for the Euler fluid, say $N_{u,tCFD}$, would be far greater than the power acquired for a viscous, turbulent and nonadiabatic fluid, Eq. (13)

$$N_{u,tCFD} > N_{u,CFD} \; . \tag{14}$$

Note, that computation of the circumferential power in the 0D model, based on the 0D momentum balance, has a similar character as in Eq. (13); it is the product of the average force in the circumferential direction, $\mathbf{P}_u$, and the $\mathbf{u}$ velocity.

Summarizing, the definition of the power of stage is defined differently, depending on the stage of the design process (Szewalski [20], Perycz [15], Badur [1,2], Puzyrewski [16], Kosowski [9,10])

$$N_u = \begin{cases} \dot{m}_r(i_{in} - i_{out}), & \text{0D design without any geometry,} \\ \mathbf{P}_u \cdot \mathbf{u}, & \text{0D verification with designed geometry,} \\ \iint_{A_u} (-p\mathbf{I} + \boldsymbol{\tau} + \mathbf{R})\mathbf{n} \cdot \mathbf{u} dA, & \text{3D verification with exact geometry.} \end{cases} \tag{15}$$

Recall, that $\dot{m}_r$ stands for the mass flow rate of the steam flown through rotor stage channel (hence the r subscript), i_{in}, i_{out} are the enthalpies of the steam at the inlet, and outlet of the stage, respectively (averaged in the 0D sense), $\mathbf{P}_u = \dot{m}_r(c_{in} - c_{out})\mathbf{e}_u$ is the circumferential force at the rotor. Surface integral in

(15), including all blades of the rotor stage, is computed as an average work of the pressure p and the surface tensile forces $\boldsymbol{\tau}$ on the variable over the blade height velocity $\mathbf{u} = u(r)\mathbf{e}_u$ (Fig. 2). Unit vector $\mathbf{n}$ is the normal vector of the blade surface, and it generally changes significantly over the blade's circumference, but since 3D blades are employed, vector $\mathbf{n}$ also changes over the blade's height. The above definitions of the stage power are equivalent. Equivalence of the Eqs. $(15)_1$ and $(15)_2$ is shown in [11,16]. Transition from Eq. $(15)_3$ to $(15)_2$ is also often proved qualitatively and quantitatively [2,16].

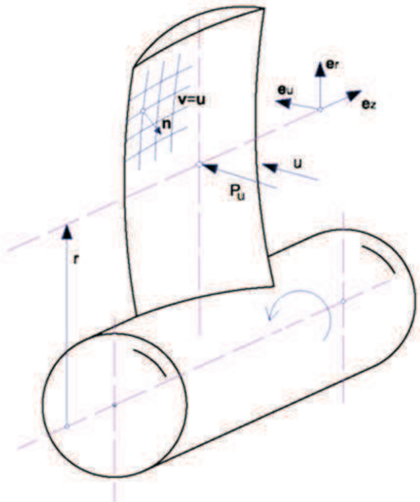

Figure 2. Illustration of the computation of the rotor stage power with use of momentum conservation equation, where: $\mathbf{r} = r\mathbf{e}_r$ – radial vector, $\mathbf{P}_u$ – blade drift force, $\mathbf{u} = \boldsymbol{\omega} \times \mathbf{r} = u(r)\mathbf{e}_u$ – blade drift velocity, $\mathbf{n}$ – unit normal vector.

In the case of a turbine being designed, for which neither the geometry nor the cross-sections of the blade channel are known, the power of a stage is computed based on energy balance equation $(15)_1$ and a given theoretical pressure drop (theoretical power) or a given, measured, enthalpy drop (measured real power). The real power in the 0D model is acquired by employing the 0D momentum balance equation, which is based on over-stiffened idealization, that is isentropic expansion [5,18,22], and its accuracy depends on determination of free parameters of the Puzyrewski model [16]:

- $\alpha_{u,0}, \alpha_{r,0}$ – angles orienting velocity vector $\mathbf{c}_0$ – considering length of the vector $\mathbf{c}_0$ should be consistent with the mass flow rate, $\dot{m}$; only two angles orienting this vector may be independent unknowns of a 0D model;
- velocity coefficient for stator and rotor, φ and ψ, respectively;
- mass flow rate coefficient, μ.

In practice comparison of the real powers computed in 0D and 3D model is a very delicate matter. In the 0D model there are four free parameters, that are determined through empirical closures, verified inside the construction offices of the modern turbine factories. In the 3D approach, the number of free parameters is several times greater – the designer may improperly pick one of those with an error, in consequence, far greater than in the standard calibrated 0D model.

5 Circumferential work of the stage

The basic quantity for comparison of thermodynamic quality of the projected stage is the unit work on the stages circumference, that is, the work received from unit of mass of a working medium [11,13,15]

$$l_u = \frac{N_u}{\dot{m}_r} \,. \tag{16}$$

Only the mass flow rate flowing through the blading passage is taken into account, omitting the flow through clearances; $\dot{m}_r = \dot{m} - \dot{m}_{leak,r}$. Circumferential power N_u may be computed or measured, hence the definitions (16) have fundamental character.

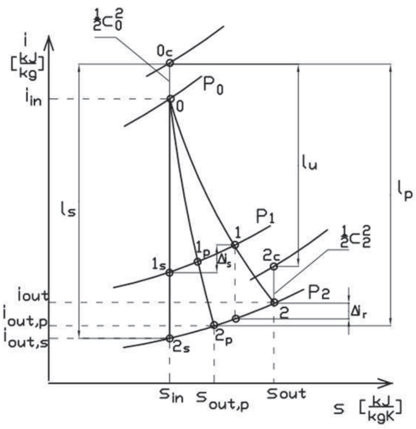

Figure 3. Geometrical depiction of the energy balances in the turbine stage presented in the i-s diagram, where: 0,1,2 – real points of process; 1s,2s – points of isentropic process; 1p,2p – points of politropic process; 0c,2c – points of process with kinetic energy; l_u – circumferential unit work of a stage; l_s – isentropic unit work of a stage; l_p – polytropic unit work of a stage; Δi_s – losses of the stator diaphragm channel; Δi_r – losses of the rotor channel; $\Delta i_{out} = \frac{1}{2}c_2^2$ – outlet loss connected with the unused kinetic energy of the fluid; $\frac{1}{2}c_0^2$ – kinetic energy of the fluid in point 0.

In Fig. 3 visualization of both theoretical process (isentropic and politropic) compared to real process with geometrical description of the energy balances in the turbine stage. In this figure enthalpies: isentropic $i_{2,s} = i_{out,s}$, and polytropic $i_{2,p} = i_{out,p}$ calculated for a single turbine stage is to be find.

6 Theoretical circumferential work

Theoretical unit work on the circumference, say l_t, is computed similarly to the real circumferential work l_u, as a ratio of theoretical power to theoretical mass flow rate

$$l_t = \frac{N_{u,t}}{\dot{m}_{r,t}} \ . \tag{17}$$

Depending on the employed model, ideal work becomes either isentropic (0D) or polytropic (3D, CFD). In the 0D model, theoretical circumferential unit work is computed from the Eq. $(15)_1$, with the difference of considering isentropic expansion for a change, where $i_{out} = i_{out,s}$ (Fig. 3). Thus, subscripts $t \equiv s$ are identified and there is:

$$l_{s,0D} = \frac{N_{u,s}}{\dot{m}_{r,s}} \ . \tag{18}$$

In the 3D model, computation of $N_{u,t}$ and $\dot{m}_{r,t}$ bases on CFD numerical analysis of the Eulerian elastic fluid and polytropic numerical quantities of tables of state, so $i_{out} = i_{out,p}$ (Fig. 3). Therefore $t = p$

$$l_{p,CFD} = \frac{N_{u,p}}{\dot{m}_{r,p}} \ . \tag{19}$$

Many works accomplished at the Energy Conversion Department IFFM PAS[2] show, that the results always satisfy the condition $l_{p,CFD} > l_{u,CFD}$ which means, that the unit work of the ideal fluid is greater than the unit work of the real fluid. This comes from the fact, that the ideal fluid (Eulerian) does not possess turbulence, viscosity and shear stresses.

7 Circumferential stage efficiency

In agreement with the Carnot paradigm, the ratio of the circumferential unit work to the theoretical circumferential work, l_t, is called the stage circumferential efficiency [11,15]:

$$\eta_u = \frac{l_u}{l_t} = 1 - \frac{\Delta l_u}{l_t} = 1 - \zeta_u < 1 \ , \tag{20}$$

where $\Delta l_u = l_t - l_u$ [kJ/kg] and ζ_u are the circumferential losses comes from resistance and include blade losses extended by the outlet loss (Fig. 3) and circumferential losses coefficient, respectively.

[2]References for these works one can find in 'Annual Reports of IFFM PAS' in years 1995–2012

It may be defined correctly both in the sense of 0D and 3D approximations. The real work can be related to some idealized situation process, where hypothetical losses Δl_u occur. For example in 0D codes such idealized process is the isentropic expansion process, on the other hand in 3D (CFD) codes this idealized process is the polytropic expansion. This paradigm of thinking originates from traditions in European teaching, that relates themselves to Platon's world of perfection, and where disciples are directed to strive for perfection. Contemporarily it is understood that the losses ζ are merely a mathematical manipulation employing easy to compute referential point. In other words, the ideal work is only a useful fiction that cannot be measured by any means.

Quantity Δl_u occurs in Eq. (20), falsely called in literature [3,16] circumferential losses, comes from the flow resistance resulting from molecular (material) and operational (turbulent) viscosity and includes blade losses. In case of the presence of the following stages, losses need to be extended by the outlet loss connected with the unused kinetic energy of the fluid. It is proposed, for clarity, to use the notion $\eta_{u,CFD}$ in case of losses or efficiencies computed in the CFD methodology.

8 Internal efficiency of the stage

Extremely important is the definition of internal efficiency of the stage — let it be denoted with $\eta_{i,CFD}$. It includes all non-blade losses, in this particular case in form of shroud leaks. Hence, instead of the equation employed in 0D modelling: $l_i = l_t - \Delta l_i$, where Δl_i denotes losses of the stage extended by the outlet loss along with non-blade losses from leaks, a more primal definition of internal unit stage work may be used:

$$l_i = \frac{N_u}{\dot{m}} \,, \tag{21}$$

where power ($N_u = N_i$) is related to the entire mass flow rate flowing through the stage, not only to the mass flux over the blading passages: $\dot{m} = \dot{m}_s + \dot{m}_{leak,s} = \dot{m}_r + \dot{m}_{leak,r}$, where $\dot{m}_s$ – mass flow rate of the steam flown through the stator diaphragm channel, $\dot{m}_r$ – mass flow rate of the steam flown through rotor channel and $\dot{m}_{leak,s}$ – mass flow rate of steam flown through the leak of the stator diaphragm (guide diaphragm) channel, $\dot{m}_{leak,r}$ – mass flow rate of steam flown through the leak of the rotor channel, respectively. Since leaks are always greater than zero, $l_{i,CFD} < l_{u,CFD}$. Similarly, theoretical unit work may be computed:

$$l_t = \frac{N_{u,t}}{\dot{m}_t} \,. \tag{22}$$

Difference acquired in the CFD between the real power, N_u, and the theoretical power, $N_{u,t}$, may serve for determination of work losses Δl_i, which, on the other hand, are empirically determined in the classical 0D model [21,22].

Internal (casing) efficiency of the stage is defined as a ratio of internal stage work to the ideal stage work (Perycz [15], p. 33, Eq. 1.28)

$$\eta_i = \frac{l_i}{l_t} = \frac{l_t - \Delta l_i}{l_t} = 1 - \frac{\Delta l_i}{l_t} \ , \tag{23}$$

where $\Delta l_i = l_t - l_i$.

Depending on the model employed, two situations may result — 0D model, if the over constrained isentropic expansion is assumed, $t \equiv s$ or 3D model, if the polytropic expansion is assumed, $t \equiv p$. Let it be recalled, that in the 0D model, internal stage losses comprise enthalpy losses of the stage along with the non-blade losses from leaks [17,19]

$$\Delta l_{i,0D} = \Delta i_s + \Delta i_r + \Delta i_{out} + \xi_z + \xi_r \ , \tag{24}$$

where Δi_s, Δi_r and Δi_{out} are the losses of the stator diaphragm channel, losses of the rotor channel and $\Delta i_{out} = \frac{1}{2}c_2^2$ outlet loss connected with the unused kinetic energy of the fluid, respectively, and here ξ_z are the losses from axial leaks between the hub and the guide diaphragm, ξ_r are the radial losses between the blade tip and the casing. Numerous equations available in literature for particular losses may be easily found [11,15,22].

On the contrary, 3D model losses $\Delta l_{i,CFD}$ may not be separated and attached to particular processes, and are usually given aggregately:

$$\Delta l_{i,CFD} = \frac{1}{\dot{m}_t \dot{m}} (\dot{m} N_{u,t} - \dot{m}_t N_u) \ . \tag{25}$$

9 Example — CFD efficiency of the control stage

Figures 4–8 contain an illustration to the $N_{u,CFD}$, $N_{u,tCFD}$, $l_{i,CFD}$, $l_{t,CFD}$, $\eta_{i,CFD}$ definitions in the CFD given by equation 13, 14, 21, 22, 23. $N_{u,CFD}$, $N_{u,tCFD}$, $l_{i,CFD}$, $l_{t,CFD}$, $\eta_{i,CFD}$, $\zeta_{i,CFD}$ are computed for the control (impulse) stage of a 100 MW steam turbine [3]. The CFD calculation assumes that: the stage is fed with steam through four nozzle boxes, of which one is opened in 80% of its mass flow rate. In this case of calculation, the guide diaphragm leak is absent $\dot{m}_{leak,s} = 0$, while there appear losses from uneven feeds. The 3D model does not require any particularly special approach.

All calculations were performed for the same pressure drop equal to $p_{in} - p_{out} = 2.402$ MPa. The inlet pressure was preserved constant at the level of $p_{in} = 9.445$ MPa, and the fresh steam fed into the turbine had the temperature $T_{in} = 811.15$ K. Numerical analysis included axial clearances $J_z = 2, 3, 9, 13$ mm, and radial clearances $J_r = 2, 9, 14$ mm[3].

[3]These higher values of clearances follows from untypical strength condition for high rotating blade foot, see [3].

Figures 4 and 5 present stage real power $N_{u,CFD}$ and power aquired for Euler fluid $N_{u,tCFD}$, respectively.

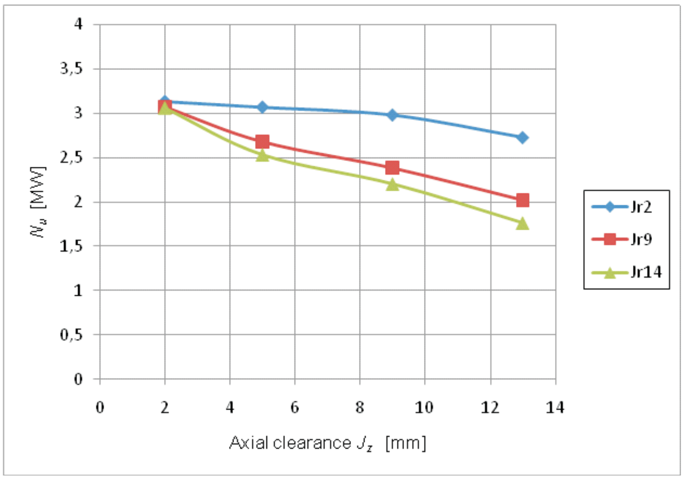

Figure 4. Power, $N_{u,CFD}$, of the control stage as a function of clearances.

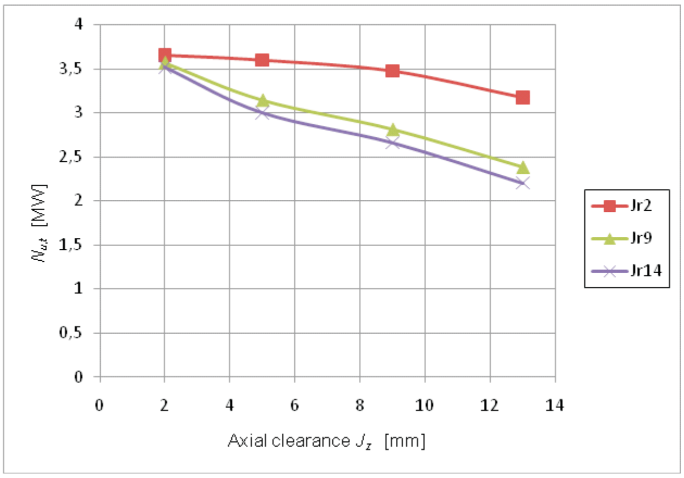

Figure 5. Circumferential power for the Euler fluid $N_{u,tCFD}$, of the ideal control stage as a function of clearances.

Figures 6 and 7 show internal unit work $l_{i,CFD}$ and the theoretical unit work $l_{t,CFD}$, respectively. The casing (internal) efficiency of the stage $\eta_{i,CFD}$ and the losses $\zeta_{i,CFD}$ are shown in Figs. 8 and 9, respectively. It concludes, that casing efficiency in CFD varies between 0.88 and 0.81, which is correct according to the practical knowledge of this particular turbine's element. From the results given in Tab. 1 it concludes that the coefficient μ_{CFD} for the impulse control stage changes according to the axial clearance J_z and radial clearance J_r, and, for instance, is $\mu_{CFD} = 0.92$ for $J_z = 2$ mm and $J_r = 3$ mm.

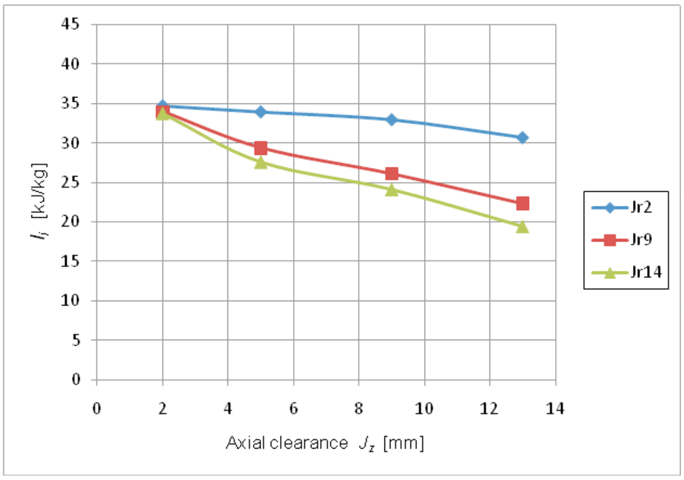

Figure 6. Internal unit work, $l_{i,CFD}$, in the control stage as a function of clearances.

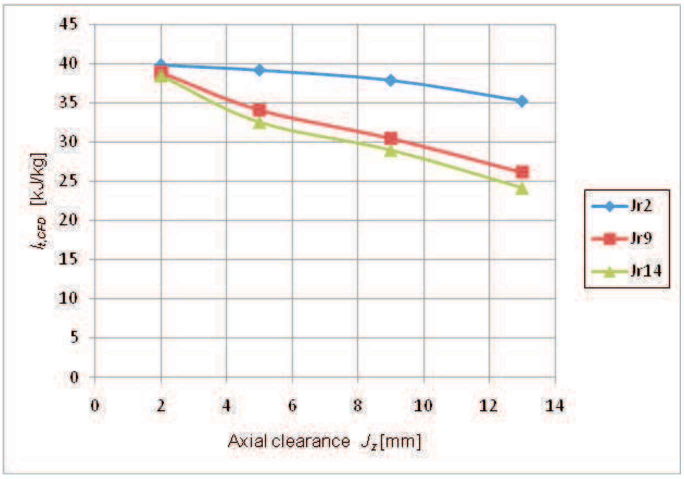

Figure 7. Theoretical unit work, $l_{t,CFD}$, in the control stage as a function of clearances.

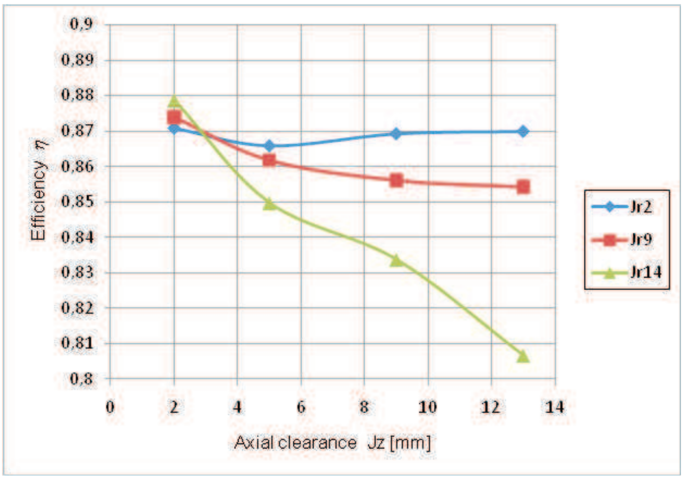

Figure 8. Control stage efficiency $\eta_{i,CFD}$ as a function of clearances.

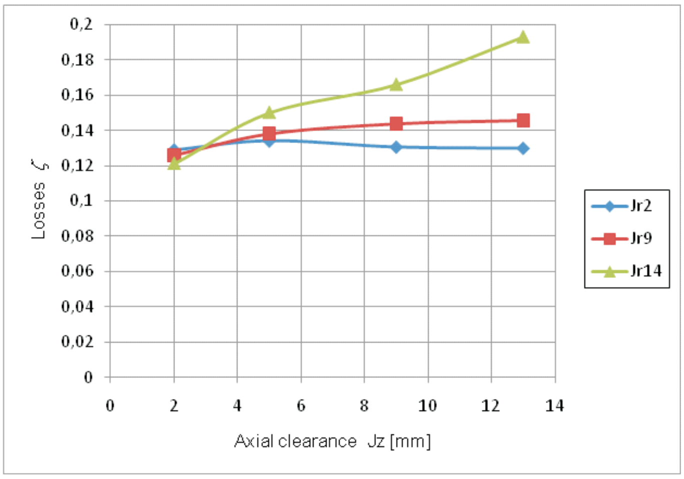

Figure 9. Control stage losses $\zeta_{i,CFD}$ as a function of clearances.

10 Isentropic and polytropic efficiency

To explain differences between isentropic and polytropic efficiencies, it is comfortable to consider this aspect apart from the turbine stage, and look at it in a wider perspective. For this purpose let's consider efficiency in flow, where no heated and cooled or moving surfaces occur. It should be pointed out, that the flow efficiency is connected with the general concept of the ratio of utilization of the energy stored, carried by the unit of mass of a working medium; it describes the relation of real enthalpy, i, change of the medium to some reference (theoretical)

Table 1. Mass flow rate of the axial control stage for a given pressure drop $p_{in} - p_{out} = 2.402$ MPa, $p_{in} = 9.445$ MPa, $T_{in} = 811.15$ K, revolution speed 10400 rpm.

Mass flow rate [kg/s]		Radial clearance J_r [mm]											
		2				9				14			
		Axial clearance J_z [mm]											
		2	4	9	13	2	4	9	13	2	4	9	13
Real	$\dot{m}$	90.2	90.3	90.3	88.7	90.4	91.1	91.2	90.4	90.4	91.5	91.1	90.4
	$\dot{m}_{leak,r}$	4.9	6.4	7.7	8.3	6.5	14.4	19.6	24.0	6.5	15.8	22.6	29.3
	$\dot{m}_r$	85.3	83.9	82.6	80.4	83.9	76.7	71.7	66.4	83.9	75.6	68.5	61.1
Ideal	$\dot{m}_t$	91.6	91.7	91.6	90.0	91.8	92.2	92.3	91.2	91.5	92.3	91.8	91.1
	$\dot{m}_{t,leak,r}$	5.0	6.2	7.6	8.1	6.7	13.5	18.3	22.3	6.8	14.4	20.4	26.2
	$\dot{m}_t, r$	86.6	85.5	84.0	81.9	85.1	78.7	74.0	68.9	84.7	77.9	71.5	64.9

change of energy, e, of the medium, which mathematically is

$$\eta_e = \frac{\int_{\tau,in}^{\tau,out} i d\tau}{\int_{\tau,in}^{\tau,out} e(s,v) d\tau} = \frac{i_{in} - i_{out}}{\int_{\tau,in}^{\tau,out} e(s,v) d\tau} , \tag{26}$$

where $e(s,v)$ may be an internal energy, free energy, free enthalpy or any other energy expressing elastic properties of the fluid depends from entropy destruction s, and volume change v in the time from start of the process τ_{in} to the end of process τ_{out}, and τ is the time of process.

Classical isentropic efficiency, η_s, of the flow, computed in 0D modelling, employs the isentropic drop of enthalpy [5,7,14] ($s_{in} = s_0 = s_{2s} = s_{out,s}$) (Fig. 3)

$$\eta_s = \frac{i_{in} - i_{out}}{i_{in} - i_{out,s}} . \tag{27}$$

This definition is practical for the 0D model, since it is related to the ideal state described by enthalpy $i_{out,s}$ – it is easily interpreted in the Molier i-s chart (Fig. 3) and does not require the temperature, T_{out}, to be known. However, the definition for the isentropic efficiency does not have a solid physical fundamentals, since the process $i_{in} - i_{out,s}$ may not be acquired in the real fluid flow [23]. On the other hand, the polytropic definition also does not require the temperature T_{out} to be known initially; energy dissipation intensity is calculated in the 3D model solely for the real process, $i_{in} - i_{out}$, or in the case of Eulerian fluid — for available energy drops to zero – the process becomes polytropic, $i_{in} - i_{out,p}$. Hence, in the

3D modelling based in CFD methodology, the polytropic efficiency definition is

$$\eta_{p,CFD} = \frac{N_{u,CFD}}{N_{u,tCFD}} = \frac{\iint_{A_u}(-p\mathbf{I}+\boldsymbol{\tau}+\mathbf{R})\mathbf{n}\cdot\mathbf{u}dA}{\iint_{A_u}(-p\mathbf{I}\mathbf{n}\cdot\mathbf{u}dA} \cong \frac{(i_{in}-i_{out})\dot{m}}{(i_{in}-i_{out,p})\dot{m}_t} . \tag{28}$$

However as proposed by R. Puzyrewski [16, p.51] for 0D approach polytropic efficiency $\eta_{p,0D}$ seems more natural than η_s and is definited

$$\eta_{p,0D} = \frac{i_{in}-i_{out}}{\int_{\tau,in}^{\tau,out}\frac{1}{\rho}\dot{p}d\tau} , \tag{29}$$

where $\dot{p}$ is the rate of pressure change, that is valid for any channel geometry, both convergent and divergent.

From Eq. (29) it concludes, that the difference between isentropic efficiency, η_s, and polytropic efficiency, $\eta_{p,0D}$, lies in a different referential state theoretical for the energy drop [16,6,7]. In Puzyrewski monograph [16,p.53] evaluation of the difference is found

$$\eta_s = \eta_{p,0D}\left(1+\zeta_s\frac{\bar{T}_{in-out}-\bar{T}_{out,s-out}}{\bar{T}_{out,s-out}}\right) , \tag{30}$$

where the over bar denotes averaged temperature of the thermodynamic process (Fig. 3) and $\zeta_s = 1-\eta_s$ is isentropic loss. From Eq. (30) it reads that both efficiencies are close to one another by their values, if [16]

$$\eta_s = (1+\mathrm{Pu})\eta_{p,0D} , \tag{31}$$

where Pu is the Puzyrewski number.

On the other hand for CFD we can write

$$\eta_s = (1-\mathrm{Pu})\eta_{p,CFD} . \tag{32}$$

For both cases Pu can be written as

$$\mathrm{Pu} = \left(\frac{\bar{T}_{in-out}}{\bar{T}_{out,s-out}}-1\right)\zeta_s \approx 0 . \tag{33}$$

For turbine stages Pu varies from 0.002 to 0.0001, what in practice means, that the computed polytropic efficiency $\eta_{p,CFD}$ is by 0.2 to 0.01% higher from the isentropic efficiency η_s. It is closely enough to justify the assumption $\eta_{s,0D} \cong \eta_{p,CFD}$.

11 Conclusion

Internal (casing) efficiency of the stage and the entire turbine is the basic criterion in estimating machine perfection, which the manufacturer guarantees before the very first start-up. Since the efforts of the designer to achieve the highest possible efficiency a natural question arises for the integrity of efficiencies derived from the two methods, namely 0D and 3D. Intuitively it seems, that 3D models are able to better (more accurately) determine the flow parameters and hence its efficiency. In the paper the task of comparing the definitions for efficiency in 0D and 3D was undertaken. It turns out that both definitions, aside from real power estimation accuracy, differ substantially in case of establishing the referential theoretical powers, to which the real power is then related. Therefore it is appropriate to speak of polytropic CFD efficiency and isentropic 0D efficiency as quantities different in the very definitions. Both discussed definitions of efficiencies give similar results differing by the term specified by the Puzyrewski equation (33). If the number Pu $\to 0$, then identity ow both definitions occurs and we may write $\eta_{s,0D} \cong \eta_{p,CFD}$.

In the future, if the procedures for checking the new turbine designs and modernisations of old turbines will require CFD analyses as a standard, classical efficiencies $\eta_{s,0D}$, burdened with simplifications of the 0D approach, should be replaced with the CFD efficiency, $\eta_{p,CFD}$, and thus it would become the definition of efficiency to use when defining turbine technical parameters. It is also attractive from the commercial point of view, since the CFD efficiency is, by definition being polytropic, higher than the 0D isentropic efficiency.

Acknowledgements The authors would like to thank to Prof. Krzysztof Kosowski from Gdańsk University of Technology for discussion during the article's preparation.

Received in November 2012

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