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LIMITATION OF CAUCHY FUNCTION METHOD IN ANALYSIS FOR DOUBLE ESTIMATORS OF FREE TRANSVERSAL VIBRATION OF CANTILEVER TAPERED SHAFTS

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Abstract

In this paper the Bernstein-Kieropian simplest Dunkerly estimators of natural frequencies of cantilever shafts with power variable flexural rigidity and attached concentrated mass were analyzed in a theoretical approach. The approximate solution of boundary value problem of transversal vibrations by means of Cauchy function and characteristic series method has gave getting functional dependence between natural frequency and variable parameters of shafts. Particular attention has been given to a singularity arising from the uncertainty of estimates of Bernstein-Kieropian. Limitation of Cauchy function method in analysis double estimators of natural frequencies of transversal vibration of cantilever tapered shafts exude to exact theoretical selection using by Bessel's function and experimental result received by Panuszka R., Uhl T.

Introduction

In process of designing many structures and structural components such as chimneys towers, head frames, masts, airplane wings and turbine blades it is important to emphasize that they can be modeled by means of a tapered cantilever beam of varying cross section – which works in extremely dynamic loads – we have to know dynamics characteristic like as values and forms of natural vibrations which are necessary to analysis of resonance effect until

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starting and braking. These elements can be modeled by cantilever beams with variable parameters in which they are more importantly distributed: rigidity, mass, module, resilience, cross section, rigidity of bearing, axial loads. Sometimes it is necessary to considered discrete inclusions in distributed masses and form base rigidity. In general terms the solution to those problems is difficult but possible to solve as listed in a few cases below.

In papers (LAN, SUPPINGER, TALEB 1945) have solved boundary value problem of cantilever tapered beam by Bessel's function. Exiting equation of transversal vibrations power variable of cross – these section might have variable Euler's form and then general integral is formulating by elementary function (LAN, PANUSZKA, UHL 1983).

More often studies beams like these are applying analytical and first of all numerical approximate methods. Applying these in theory it is difficult estimate accuracy calculated characteristics without knowing exact value for chosen special case.

Between numerical methods widely applying have finite elements methods and finite difference method (JAROSZEWICZ, ZORYJ 2000) and transfer matrix method which J. Jaroszewicz has applied earlier in his paper. The example of analytical approach is variation method – although it does not warrant high accuracy that include particular form of vibration – however, this gives very simple form of solution (GOEL, 1976, TIMOSHENKO, GERE 1963). It's crucial to remember the sequenced approximation. On this base of variable cross – section, which has more complicated form (MIKEŁADZE 1951). Application of Green's function method in frequency analysis of axisymmetric vibration of annular platter with elastic ring supports (KUKLA, SZEWCZYK 2005). The method of partial discretization in free vibration problems of circual plates with variable distribution of parameters (JAROSZEWICZ et a.l. 2006, JAROSZEWICZ, ZORYJ 2006). Furthermore free vibrations of a system of non-uniform beams coupled by elastic layers has been considered (KUKLA, ZAMOJSKA 2006). Free vibrations of axially loaded stepped beams has been analysed by using a Green's function method (KUKLA, ZAMOJSKA 2007).

Disregarding papers that considerate simultaneously few variable parameters, we move on to articles that deserves our attention, like work of L.M. Zoryj and J. Jaroszewicz that is applying influence function method and characteristic series to salving those problem. This proposition was developed in common works (JAROSZEWICZ, ZORYJ 1987, 2000). Spectral function method which were proposed by Bernstein was applied to analysis constant parameters system, without consideration friction (BERNSTEIN, KIEROPIAN 1960). L.M. Zoryj and J. Jaroszewicz have improved his method to boundary value problems of transversal and torsion vibrations of beam and shafts in there papers published in JAROSZEWICZ, ZORYJ (1994).

They received general formulas other scribed the coefficient of characteristic series for variable form of cantilever beam.

In paper (JAROSZEWICZ, ZORYJ 1990) have obtained formulas which can by applied to engineering practice for calculation fundamental natural frequency transversal vibrations non-homogeneous beams and shafts, and have formulated conditions of those applying.

In this paper the boundary value problem of cantilever with variable distributed flexural rigidity and mass along the cantilever axis, (M – concentrated mass, l – length of beam, x, y – coordinates) was presented. The result is general form characteristic equation allowing design estimates of the vibration and estimates of critical power (AUCIELLO 2001, JAROSZEWICZ, ZORYJ 1999).

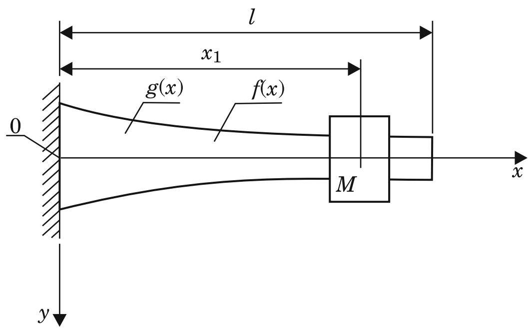


Fig. 1. Substituted model of cantilever of variable cross – section
Source: JAROSZEWICZ, ZORYJ (1997).

Presented substituted model is corresponding assembly shafts with tapered shaft neck which were applied in machines like grinding machines, radial fans and pumps (JAROSZEWICZ, DRAGUN 1994).

In classic exist to formulate boundary value problem of transversal vibration presented in figure 1 is necessary to considerate except conditions on ends ($x = 0$, $x = l$) fourth conditions in place of attached mass $\mu(x = x_1)$. In this method mass inclusion is considerate in exist equation by Dirac's function (GOEL, 1976, JAROSZEWICZ, ZORYJ 1997).

The proposed method allows more to solve problem.

Definition of the problem

Transversal free vibrations of the model of a cantilever of figure 1 are governed by the equation:

$$L[y] - \alpha_1 f(x_1)\delta(x - x_1)y(x_1) = 0 \tag{1}$$

where $L[y]$ denotes the differential operator

$$L[y] = y^{IV} + \frac{2f'(x)}{f(x)}y''' + \frac{f''(x)}{f(x)}y'' - \omega^2 v(x)y \tag{2}$$

and

$$f(x) = EI(x),\ \alpha_1 = f(x),\ \alpha_1 = \omega^2 M,\ (x) = \frac{g(x)}{f(x)}\ \mu = \frac{M}{m_0 l} \tag{3}$$

δ is the Dirac function and prime denotes derivative with respect to x, ω – parameter of frequency. The boundary conditions are

$$y(0) = y'(0) = 0,\ y''(l) = y'''(l) = 0 \tag{4}$$

The characteristic equation

The general solution to the equation (2) has the following form GOEL (1976), JAROSZEWICZ, ZORYJ (1997):

$$y(x, \alpha) = C_0 Q(x, \alpha) + C_1 \dot{Q}(x, \alpha) + C_2 \ddot{Q}(x, \alpha) + C_2 \dddot{Q}(x, \alpha) \tag{5}$$

where

$$Q(x, \alpha) = K(x, \alpha) + \alpha f(x_1)\ K(x_1, \alpha)\ K(x, x_1) \tag{6}$$

and C_0, C_1, C_2, C_3 are arbitrary constants, $K(x, \alpha)$ is the Cauchy function of the equation $L[y] = 0$, $\Phi\ (x, \alpha) = K(x, \alpha)\ \Theta\ (x - \alpha)$ is the influence function, $\Theta\ (x)$ denotes the Heaviside function α is a parameter, and dot denotes derivative with respect to the parameter α.

The result substituted of expressions (5) boundary conditions (4) received characteristic equation:

$$\begin{aligned}\Delta \equiv\ & [K''(x, \alpha)K'''(x, \alpha)\ K'''(x, \alpha)\ K(x, \alpha)] \\ & + \alpha_1 f(x_1)[K'''(x, \alpha)K''(x, x_1)K(x_1, \alpha) + K''(x, \alpha)K'''(x, x_1)K(x_1, \alpha)] = 0 \Bigg|_{\substack{x=l \\ \alpha=0}}\end{aligned} \tag{7}$$

where accepted designation:

$K(x_1, \alpha) = K_1\alpha\, K(x, x_1) = K_{x1}$ and $\alpha = 0$, what is possible, when rigidity $f(x)$ has positive value. Most often in practice concentrated mass meet attachment case of the free end of support $x = l$, from which equation has the form:

$$\Delta \equiv F_{01} - r_1 F'_4 = 0 \left| \begin{array}{l} x = x_1\, l \\ \alpha = 0 \end{array} \right. \tag{8}$$

where

$$\begin{aligned} F_{01} &= K''_{x\alpha}\, K'''_{x\alpha} - K'''_{x\alpha}\, K''_{x\alpha} \\ F'_4 &= K''_{x\alpha}\, K_{x\alpha} - K''_{x\alpha}\, K_{x\alpha} \end{aligned} \tag{9}$$

Functions (9) and it combination have practical application (JAROSZEWICZ, ZORYJ 1997).

In equation (8) considered, that $K''(l,l)$ and $K'''(l,l) = 1$, which results from Cauchy's function.

$$F'_4 = 0 \left| \begin{array}{l} x = x_1\, l \\ \alpha = 0 \end{array} \right. \tag{10}$$

This corresponds articulated supported position the right end of the beam. In the general case Cauchy's function can be create in power series relative variable x or relative square of the frequency parameter ω^2.

In this case the second method is used, what gives the following form of the function $K(x, \alpha)$

$$K(x, \alpha) = \sum_{i=1}^{\infty} (-\omega)^{2i} K_i(x, \alpha) \tag{11}$$

where

$$K_0(x, \alpha) = f(\alpha) \int_{\alpha}^{x} \frac{(x, - s)(s - \alpha)}{f(s)} ds \tag{12}$$

$$K_i(x, \alpha) = \int_{\alpha}^{x} v(t) K_0(x, t) K_{i-1}(t, \alpha) dt, \qquad i = 1,2,3 \tag{13}$$

After introduction of signs

$$J_0(x,\ \alpha) = \int_{\alpha}^{x} \frac{(x, - s)(s - \alpha)}{f(s)} ds \tag{14}$$

$$J_1\ (x,\ \alpha) = \int_{\alpha}^{x} \mu(t) \cdot J_{i-1}(t,\ \alpha) \cdot dt \tag{15}$$

series (11) can be written as:

$$F_{01} = f^2(\alpha) = \sum_{k-0}^{\infty} (-\ \omega)^{2k} a_k \tag{16}$$

$$F_4' = f^2(\alpha) = \sum_{k=0}^{\infty} (-\ \omega)^{2k} b_k \tag{17}$$

$$a_k \sum_{i=0}^{k} (J_i'' J_{k-1}''' - J_i''' J_{k-1}'') \tag{18}$$

$$b_k \sum_{i=0}^{k} (J_i J_{k-1}'' - J_i J_{k-1}''),\ k = 1,2,3 \tag{19}$$

Determination of the basic frequency of a cantilever beam

Trough suitable transformations including expression (11) from the equation (7) we received a characteristic series:

$$\Delta \equiv \sum_{k=0}^{\infty} (-\ \omega)^{2k}\ A_k = 0,\ k = 0,1,2,3... \tag{20}$$

Three coefficients defined by formulas:

$$A_0 = 1 \tag{21}$$

$$A_1 = \int_0^l g(t) \int_0^t \frac{(t - s)^2}{f(s)}\ dsdt + M \int_0^l \frac{(l - s)^2}{f(s)}\ ds \tag{22}$$

$$A_2 = \left\{\int_0^l g(t) \int_0^t \frac{t(s-t)}{f(s)}\, dsdt\right\}\left\{\int_0^l g(t) \int_0^t \frac{s(t-s)}{f(s)}\, dsdt\right\} + M\left\{-\int_0^l g(t) J_0(l,t)\right.$$

$$\left.\left[\int_0^t \frac{(t-s)(s-l)}{f(s)}\, ds\right] dt + \int_0^l g(t)(l-t)[J_0(l,0)J_0(t,0) - J_0(l,0)J_0(t,0)]dt\right\} \tag{23}$$

With coefficients A_0, A_1, A_2 series (20) can calculate the basic vibration frequency applying Bernstein bilateral estimators (MIKIEŁADZE 1951):

$$\frac{A_0}{\sqrt{A_1^2 - 2A_0A_2}} \le \alpha\,\omega_0^2 \le \frac{2A_0}{A_1 + \sqrt{A_1^2 - 4A_0A_2}} \tag{24}$$

The particular case where concentrated M is much higher than solid beam mass:

$$\int_0^l \rho(x)\, S(x) dx << M \tag{25}$$

The characteristic equation (7) can be represented in the form:

$$1 + r_1 f(x_1)[(x - \alpha)\, J_0(x, \alpha) - J_0(x,\alpha)] = 0 \;\Big|\; \begin{matrix} x = x_1 = l \\ \alpha = 0 \end{matrix} \tag{26}$$

Received the following formula on square of the frequency appropriate replacement system with one degree of freedom:

$$\alpha\,\omega_0^2 = \left[M \int \frac{(l-t)}{f(t)}\, dt\right]^{-1} \tag{27}$$

Simple calculations selected shafts and cantilever beams shaped taper

Stiffness and mass can be described by the formulas:

$$f(x) = El_0\left(1 - \gamma\frac{x}{l}\right)^n,\; g(x) = m_0\left(1 - \gamma\frac{x}{l}\right)^m \tag{28}$$

$$\gamma = \frac{l - H}{l} \tag{29}$$

where

$$I = I_1 + H \tag{30}$$

where

E is the modulus Young's, I_0, m_0 appropriate moment of inertia and mass per unit length of the beam at base, n, m – parameters of the beam cross – section, which may take the actual ($n = 2m$). Concentrated mass M, attached on the end of beam, included by way of $\mu = \frac{M}{m_0 l}$.

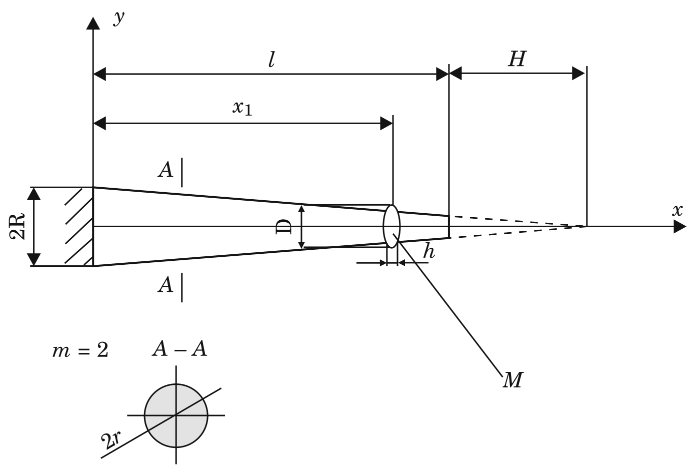


Fig. 2. Beam schematic

Considered stiffness of the beam in figure 2 under the general formula (2, 4, 5) in paper (JAROSZEWICZ, ZORYJ 1999), which found strict expression describing the function Cauchy ($n = 4$) (JAROSZEWICZ, ZORYJ 1985):

$$\phi(x, \alpha) = \frac{(x, \alpha)^3}{6 \cdot EJ_0 \cdot (1 - k \cdot x)^2 \cdot (1 - k \cdot \alpha)^2} \tag{31}$$

where

$$k = \frac{\gamma}{l} \tag{32}$$

As a first example been considered homogenous beam in the form truncated taper in fig. 2, axis x aimed along the axis taper the beginning at the center base with a radius R.

Enter the following markings: H – full height complement of taper, ρ – density of the material, γ – coefficient convergence. Stiffness and mass of taper describe formulas (29), when $n = 2$, $m = 4$, $I_0 = \frac{1}{4}\pi R^4$, $m_0 = \pi\rho R^2$. Coefficient $\gamma = 0$ corresponds for shafts in the shape of a cylinder.

For shafts in the shape of a cone with a taper $\gamma = 0$ and $\gamma = 1$ at the end of the fixed weight concentrated disc shaped small size. In this assumption: $\frac{D}{l} \leq \frac{1}{6}$ and, $\frac{h}{D} \leq \frac{1}{4}$ since they do not meet at the considered geometry – the moment of inertia of the disk and make the boundary conditions (MARTIN 1956, PANUSZKA, UHL 1983).

The case $\gamma = 1$ and $\mu \neq 0$ is a singular zero due to the stiffness of the cone at its apex full, that is, the mounting mass. Taking into account equations (29), the integrals are calculated in Expressions (22), (23) to give a series of characteristic as follows:

$$1+\left[\frac{5-4\gamma}{60}+\frac{\mu}{3(1-\gamma)}\right]\alpha\omega^2+\frac{1}{360}\left[\frac{1}{-}+\frac{\varphi_1(\gamma)}{28\gamma^8}+\mu\frac{\varphi_2(\gamma)}{\gamma^7(1-\gamma)}\right]\alpha^2\omega^4-...=0 \quad (33)$$

where

$$\varphi_1(\gamma) = 4\gamma^7 + 14\gamma^6 + 84\gamma^5 - 875\gamma^4 + 1820\gamma^3 - 1470\gamma^2 + 420\gamma + 420(I-\gamma)^4 \ln(I-\gamma) \quad (34)$$

$$\varphi_2(\gamma) = \gamma^7 + \gamma^6 + 3\gamma^5 + 15\gamma^4 - 110\gamma^3 + 150\gamma^2 - 60\gamma\, 60(I-\gamma)^3 \ln(I-\gamma) \quad (35)$$

$$\alpha = \frac{m_0 l^4}{EJ_0} \quad (36)$$

Where γ takes a small value to calculate the function (34) is preferably used instead of the exact patterns corresponding ranks.

$$\frac{\varphi_1(\gamma)}{\gamma^8} = -420 \cdot 24\left(\frac{3!}{8!}+\frac{4!}{9!}+\frac{5!}{10!}\gamma^2+...\right) \quad (37)$$

$$\frac{\varphi_2(\gamma)}{\gamma^7} = 1 - 360\left(\frac{3!}{8!} + \frac{4!}{9!} + \frac{5!}{10!}\,\gamma^2 + \ldots\right) \tag{38}$$

For the special case when γ tends to 0, the series (33) takes the form:

$$1 + \frac{1}{12}\,(1 + 1\mu)\;\alpha\omega^2 + \frac{1}{71}\,(1 + 8\mu)\alpha^2\omega^4 + \ldots = 0 \tag{39}$$

when for $\mu = 0$ has following form:

$$1 + \left(\frac{5 - 4\gamma}{60}\right)\alpha\omega^2 + \frac{1}{360}\left(\frac{1}{8} + \frac{\varphi_1(\gamma)}{28\gamma^8}\right)\alpha^2\omega^4 + \ldots = 0 \tag{40}$$

The rough estimate described in BERSTEIN, KIEROPIAN (1960) can be obtained from (33) in following form:

$$\underset{\sim}{\alpha\omega}{}_0^2 = \frac{3(1 - \gamma)}{\frac{1}{3}\,(3 - 3\gamma + \gamma^2) + \mu} \tag{41}$$

The simplest “course estimate of the” lower basic rate determined by the formula Dunkerly’a (JAROSZEWICZ et al. 2008):

$$\underline{\underline{\alpha\omega}}{}_0^2 = \frac{60(1 - \gamma)}{(5 - 4\gamma)(1 - \gamma) + 20\mu} \tag{42}$$

To the “rough estimate” above the fundamental frequency, applied mark $\overset{=}{\alpha\omega}{}_0^2$.

$$\overset{=}{\alpha\omega}{}_0^2 = \frac{3(1 - \gamma)}{\mu} \tag{43}$$

Accurate estimates of the bottom of the base rate $\underline{\alpha\omega}{}_0^2$ and top $\overline{\alpha\omega}{}_0^2$ was calculated from the formulas Bernstein (24). It should be noted that, with the first three coefficients of characteristic series may also be calculated on the other frequency estimators bilateral their approximate estimates of the third and fourth frequencies.

Discussion of results of double-sided estimators of the basic frequency

On the basis of the frequency equation (8), after solving integrals (23) and for applying estimators (24) calculations results were achieved that are introduced in table 1.

Additionally in approximate estimators were presented from the top and bottom and course estimator from the bottom, based on mathematical formulas (41, 42, 43).

Graphical dependences of appropriate estimators on the coefficient of the gamma similarity without concentrated mass ($\mu = 0$) is presented on figure 3 as well as at different ratio values of the concentrated mass to constant mass they presented on figure 4 and figure 5. From graph in figure 3 we can conclude, that difference between arithmetic means of estimators for the cylinder (that is $\gamma = 0$) amounts to the 2.4% in relation to the direct value 12.36 and for the sharp cone $\gamma = 1$ amounts to the 0.63% towards the direct value 75.73.

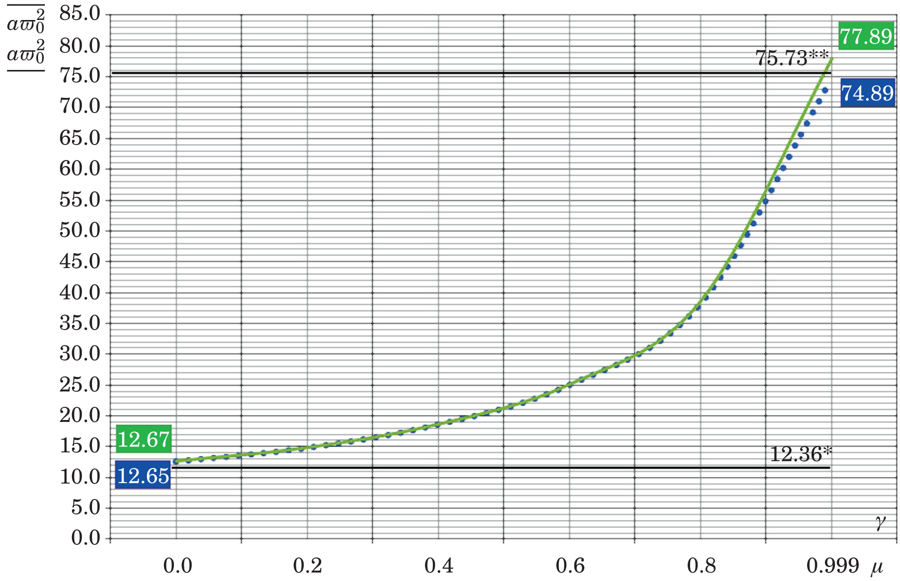
Theoretical calculations confirms experimental results received in Panuszka R. papers, Uhl T and with direct theoretical results – sharp cone 75.75. ($\gamma = 1$, and $\mu = 0$) and for the cylinder ($\gamma = 0$ and $\mu = 0$) received by Tymoshenko S.P., quoted in positions (Gogel 1976, Kukla, Szewczyk 2005). In considered shaft cases even comparatively small concentrated mass that is put on their endings (figure 4 and figure 5) lowering a great deal of the basic frequency. The rate of the γ similarity causes the increase in the frequency for beams without mass or with very small mass ($\mu \leq 0.001$) what results from graphs on figure 4. Course estimators from above and from the bottom (figure 5), are calculated on the basis of very simple formula from the table 2.3 (Jaroszewicz, Zoryj 1997) and the bottom estimators are calculated on the basis of formula (42) deduced from the Dunkerly'a formula (Bernstein, Kieropian 1960) that are giving good calculation accuracy (up to the 5%) for many practical cases which are meeting the conditions from the table 2.4 (Jaroszewicz, Zoryj 1997). Patterns from the table 2.3 (Jaroszewicz, Zoryj 1997) are giving a good approximation at $\mu \geq 5$, formula (42) is giving a close approximation to the accurate value at any given values μ. Pattern (41) can be applied, when $\gamma > 0$ and $\mu < 5$. Graph figure 5. is illustrating this process.

From graphs in figure 5 one can see, that estimators of the basic frequency at $\gamma \to 1$ are making their way to 0. At grate enough value μ ($\mu \geq 5$) curves $\mu\alpha\bar{\bar{\omega}}_0^2$ (solid lines) from figure 5 are identical with appropriate curves $\mu\alpha\underline{\underline{\omega}}_0^2$ from figure 4, how it should be.

Table 1

Results of calculation of estimators: accurate ($\underline{\alpha\omega}_0^2$, $\overline{\alpha\omega}_0^2$), approximate ($\underline{\underline{\alpha\omega}}_0^2$, $\overline{\overline{\alpha\omega}}_0^2$), rough ($\underset{\sim}{\alpha\omega}_0^2$)

V	μ	A_0	A_1	A_2	$\underline{\alpha\omega}_0^2$	$\overline{\alpha\omega}_0^2$	$\underset{\sim}{\alpha\omega}_0^2$	$\overline{\overline{\alpha\omega}}_0^2$	$\underline{\underline{\alpha\omega}}_0^2$	$\mu\underline{\alpha\omega}_0^2$	$\mu\overline{\alpha\omega}_0^2$	$\mu\overline{\overline{\alpha\omega}}_0^2$	$\underline{\underline{\alpha\omega}}_0^2$
0	1	1	0.416667	0.000347	2.404814	2.404819	1.500000	3.000000	2.400000	2.404814	2.404819	3.000000	2.400000
0.2	1	1	0.480667	0.002104	2.073828	2.073917	1323529	2.400000	2.054795	2.073828	2.073917	2.400000	2.054795
0.4	1	1	0.612222	0.002479	1.644306	1.644343	1.088710	1.800000	1.633394	1.644306	1.644343	1.800000	1.633394
0.6	1	1	0.876667	0.002949	1.145086	1.145095	0.789474	1.200000	1.140684	1.145086	1.145095	1.2000000	1.140684
0.8	1	1	1.696667	0.003753	0.590161	0.590161	0.424528	0.600000	0389391	0.590161	0.590161	0.600000	0.589391
0.99	1	1	33.350667	0.005665	0.029985	0.029985	0.022443	0.030000	0.029984	0.029985	0.029985	0.030000	0.029984
0	2.5	1	0.916667	0.000347	1.091360	1.091360	0.857143	1.200000	1.090909	2.728400	2.728401	3.000000	2.727273
0.2	2.5	1	1.111667	0.004888	0.903130	0.903137	0.724346	0.960000	0.899550	2.257825	2.257843	2.400000	2.248876
0.4	2.5	1	1.445556	0.005677	0.693663	0.693665	0.570825	0.720000	0.691776	1.734157	1.734163	1.800000	1.729439
0.6	2.5	1	2.126667	0.006850	0.470933	0.470934	0.397351	0.480000	0.470219	1.177333	1.177335	1.200000	1.175549
0.8	2.5	1	4.196667	0.008856	0.238404	0.238404	0.205950	0.240000	0.238284	0.596011	0396011	0.600000	0395711
0.99	2.5	1	83.350667	0.013391	0.011998	0.011998	0.010576	0.012000	0.011998	0.029994	0.029994	0.030000	0.029994
0	5	1	1.750000	0.000347	0.571493	0.571493	0.500000	0.600000	0371429	2.857467	2.851467	3.000000	2.857143
0.2	5	1	2.153333	0.009430	0.465344	0.465345	0.412844	0.480000	0.464396	2326718	2326723	2.400000	2321981
0.4	5	1	2.834444	0.011007	035328?	0353288	0318396	0.360000	0.352803	1.766436	1.766438	1.800000	1.764014
0.6	5	1	4.210000	0.013354	0.237709	037709	0.217391	0.240000	0.237530	1.188544	1.188545	1.200000	1.187648
0.8	5	1	8.63333	0.017371	0.119599	0.119599	0.110837	0.120000	0.119570	0397996	0.597996	0.600000	0397848
0.99	5	1	166.68400	0.026684	0.005999	0.005999	0.005621	0.006000	0.005999	0.02999?	0.029997	0.030000	0.029997
0	0	1	0.083333	0.000347	12.649111	12.668737	3.000000	–	12.000000	0.000000	0.000000	–	–
0.2	0	1	0.070000	0.000184	14.853411	14.865645	2.950820	–	14.285714	0.000000	0.000000	–	–
0.4	0	1	0.056667	0.000165	18.631297	18.662092	2.755102	–	17.647059	0.000000	0.000000	–	–
0.6	0	1	0.043333	0.000141	25.028880	25.127497	2307692	–	23.076923	0.000000	0.000000	–	–
0.8	0	1	0.030000	0.000106	38.137621	38.613054	1.451613	–	33.333333	0.000000	0.000000	–	–
0.999	0	1	0.016733	0.000050	74.534271	77.887461	0.008991	–	59.760956	0.000000	0.000000	–	–



* for the support of the fixed section, ** for the taper support sharp

Fig. 3. Independence graph of the accurate frequency estimates obtained for $\mu = 0$, based on the formula (4, 5) Values accurate by (Krishma Murty, Prabahakaran 1969)

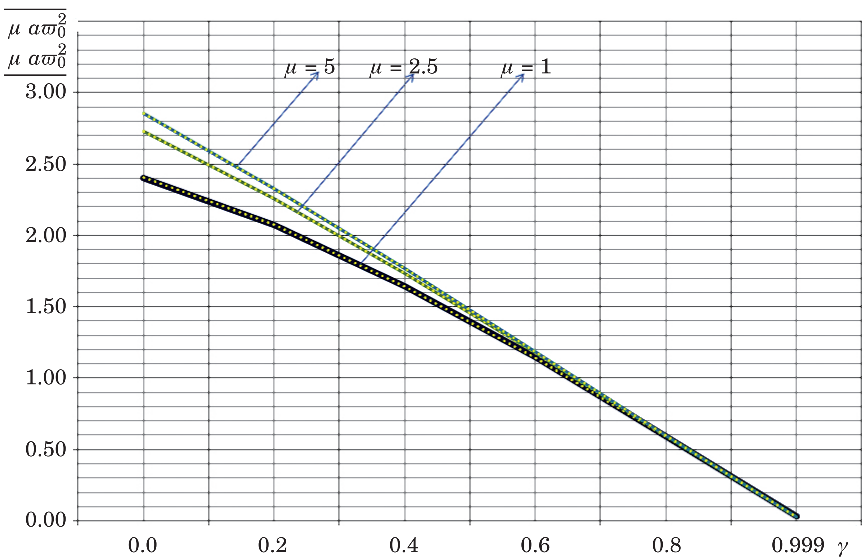


Fig. 4. Independence graph of multiplications accurate estimators trough concentrated weight ratio for continuous

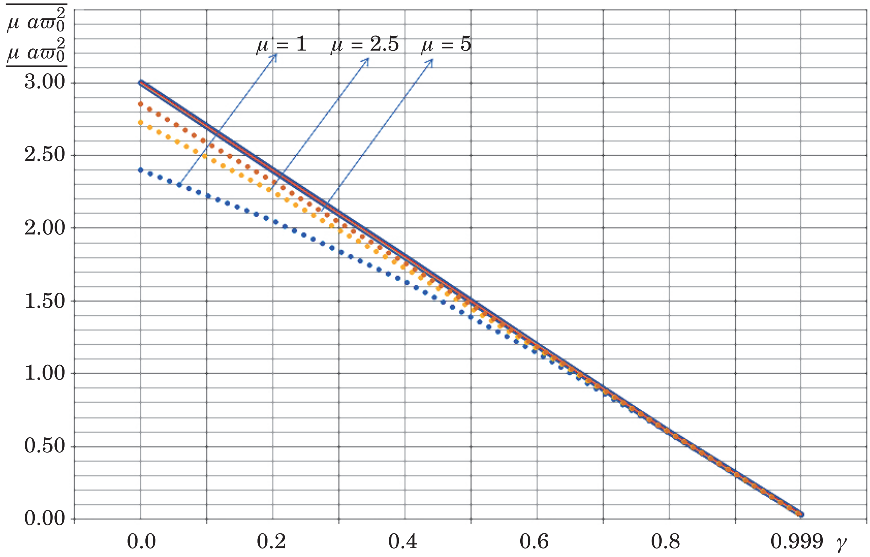


Fig. 5. Independence graph of multiplications approximate estimators trough concentrated weight ratio for continuous

Summary

In presented papers there is an analysis of applied derive formulas restrictions by Zoryj and of Jaroszewicz in theirs earlier papers. For example cone shaped crankshaft about the convergence from $\gamma = 0$ to $\gamma = 1$ with mass fastened at the end and concentrated in small disk shape. Establishing that: $D/l \leq 1/6$ and $h/D \leq 1/4$ (fig. 2).

At sharp taper without mass at the end one should apply accurate estimators and applying coarse estimators is inadmissible. In case of mass concentrated truncated cones it is possible to apply approximate estimators and coarse estimators at mass ratio above 5, because their values become levelled. The theory corresponds to the physics description of oscillation because the sharp cone at the end has a zero stiffness and even the smallest concentrated mass can't be fixed on it.

It is showed that it is possible to get a high accuracy of calculations – bring closer bottom and upper estimators and taking into account two first row rates of the characteristic A_1 and A_2, which significantly simplifies calculations and takes into account a stiffness of the crankshaft and a discrete mass interjection.

The accepted model includes the rotors class on bungs of which undersized shields are planted. In the future it seems too by intentional to expand this

method to outsize rotors planted on conical bungs in it with taking into account of the Timoshenko effect, of i.e. stresses cutting and of the inertia of the rotation of the diameter.

A high accuracy of calculations confirms base frequency calculation error to 0.63% for the sharp cone in relation to rigorous solution received from Bessela function.

Translated by AUTHORS

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