Journal of Polish Safety and Reliability Association
Summer Safety and Reliability Seminars, *Volume 8, Number 2, 2017*

Kołowrocki Krzysztof
ORCID ID: 0000-0002-4836-4976

Soszyńska-Budny Joanna
ORCID ID: 0000-0003-1525-9392
Maritime University, Gdynia, Poland

Critical infrastructure operation process

Keywords

critical infrastructure, operation, prediction, climate-weather change

Abstract

The operation process of the critical infrastructure is considered and its operation states are introduced. The semi-Markov process is used to construct a general probabilistic model of the critical infrastructure operation process.

1. Introduction

The operation process of the critical infrastructure is considered and its operation states are introduced. The semi-Markov process is used to construct a general probabilistic model of the critical infrastructure operation process. To build this model, the vector of probabilities of the critical infrastructure operation process staying at the initial operation states, the matrix of probabilities of the critical infrastructure operation process transitions between the operation states, the matrix of conditional distribution functions and the matrix of conditional density functions of the critical infrastructure operation process conditional sojourn times at the operation states are defined. To describe the critical infrastructure operation process conditional sojourn times at the particular operation states, the uniform distribution, the triangular distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull distribution and the chimney distribution are suggested.

The operation process of a critical infrastructure is very complex and often it is difficult to analyze these critical infrastructure safety with respect to changing in time its operation process states and operating environment conditions that are essential in this analysis. The complexity of the critical infrastructure operation process and its influence on changing in time the critical infrastructure structure and its components' safety parameters are essential in critical infrastructure safety analysis and protection. Usually, the critical infrastructure environment have either an explicit or an implicit strong influence on the critical infrastructure operation process. As a rule, some of the environmental events together with the infrastructure operation conditions define a set of different operation states of the critical infrastructure in which the critical infrastructure change its safety structure and its components safety parameters. In this report, we propose a convenient tool for analyzing this problem applying the semi-Markov model [Ferreira, Pacheco, 2007], [Glynn, Hass 2006], [Grabski, 2002], [Kolowrocki, 2014], [Limnios, Oprisan, 2005], [Mercier, 2008] of the critical infrastructure operation process, both without including critical infrastructure environment threats and with including them into this model.

2. Critical infrastructure operation process-modelling

2.1. Semi-Markov model of critical infrastructure operation process

We assume that the critical infrastructure during its operation process is taking $\nu, \nu \in N$, different operation states $z_1, z_2, \ldots, z_\nu$. Further, we define the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_\nu\}$. Moreover, we assume that the critical infrastructure operation process $Z(t)$ is a semi-Markov process [Grabski, 2002], [Limnios, 2005], [Mercier, 2008], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011] with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l$. Under these assumptions, the critical infrastructure operation process may be described by:

- the vector of the initial probabilities $p_b(0) = P(Z(0) = z_b)$, $b = 1,2,...,v$, of the critical infrastructure operation process $Z(t)$ staying at particular operation states at the moment $t = 0$

$$[p_b(0)]_{1\times v} = [p_1(0), p_2(0),..., p_v(0)]; \tag{1}$$

- the matrix of probabilities p_{bl}, $b, l = 1,2,...,v$, of the critical infrastructure operation process $Z(t)$ transitions between the operation states z_b and z_l

$$[p_{bl}]_{v\times v} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \cdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix}, \tag{2}$$

where by formal agreement

$$p_{bb} = 0 \text{ for } b = 1,2,...,v;$$

- the matrix of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $b, l = 1,2,...,v$, of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states

$$[H_{bl}(t)]_{v\times v} = \begin{bmatrix} H_{11}(t) & H_{12}(t) & \ldots & H_{1v}(t) \\ H_{21}(t) & H_{22}(t) & \ldots & H_{2v}(t) \\ \ldots \\ H_{v1}(t) & H_{v2}(t) & \ldots & H_{vv}(t) \end{bmatrix} \tag{3}$$

where by formal agreement

$$H_{bb}(t) = 0 \text{ for } b = 1,2,...,v.$$

We introduce the matrix of the conditional density functions of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states corresponding to the conditional distribution functions $H_{bl}(t)$

$$[h_{bl}(t)]_{v\times v} = \begin{bmatrix} h_{11}(t) & h_{12}(t) & \ldots & h_{1v}(t) \\ h_{21}(t) & h_{22}(t) & \ldots & h_{2v}(t) \\ \ldots \\ h_{v1}(t) & h_{v2}(t) & \ldots & h_{vv}(t) \end{bmatrix},$$

where

$$h_{bl}(t) = \frac{d}{dt}[H_{bl}(t)] \text{ for } b, l = 1,2,...,v, \ b \neq l,$$

and by formal agreement

$$h_{bb}(t) = 0 \text{ for } b = 1,2,...,v.$$

We assume that the suitable and typical distributions suitable to describe the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl}, $b, l = 1,2,...,v$, $b \neq l$, in the particular operation states are defined in [Kołowrocki, Soszyńska-Budny, 2011] and [EU-CIRCLE Report D2.1-GMU2, 2016).

3. Critical infrastructure operation process – prediction

3.1. Prediction of critical infrastructure operation process characteristics

Assuming that we have identified the unknown parameters of the critical infrastructure operation process semi-Markov model:

– the initial probabilities $p_b(0)$, $b = 1,2,...,v$, of the critical infrastructure operation process staying at the particular state z_b at the moment $t = 0$;

– the probabilities p_{bl}, $b, l = 1,2,...,v$, $b \neq l$, of the critical infrastructure operation process transitions from the operation state z_b into the operation state z_l;

– the distributions of the critical infrastructure operation process conditional sojourn times θ_{bl}, $b, l = 1,2,...,v$, $b \neq l$, at the particular operation states and their mean values $M_{bl} = E[\theta_{bl}]$, $b, l = 1,2,...,v$, $b \neq l$;

we can predict this process basic characteristics.

As the mean values of the conditional sojourn times θ_{bl} are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int_0^\infty t h_{bl}(t) dt, \tag{5}$$

$b, l = 1,2,...,v$, $b \neq l$,

then for the distinguished distributions (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], the mean values of the system operation process $Z(t)$

conditional sojourn times θ_{bl}, $b,l = 1,2,...,v$, $b \neq l$ at the particular operation states are respectively given by [Kołowrocki, Soszyńska-Budny, 2011]:

- for the uniform distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl}}{2}; \quad (6)$$

- for the triangular distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3}; \quad (7)$$

- for the double trapezium distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3} + \frac{w_{bl}(y_{bl})^2 - q_{bl}(x_{bl})^2}{2} + \frac{w_{bl} + q_{bl}}{6}[(x_{bl}z_{bl} - y_{bl}z_{bl}) + \frac{x_{bl}y_{bl}(x_{bl} + y_{bl})}{y_{bl} - x_{bl}}] - \frac{(x_{bl})^3 q_{bl} + (y_{bl})^3 w_{bl}}{3(y_{bl} - x_{bl})}; \quad (8)$$

- for the quasi-trapezium distribution

$$M_{bl} = E[\theta_{bl}] = \frac{q_{bl}}{2}[(z_{bl}^1)^2 - (x_{bl})^2] - \frac{A_{bl} - q_{bl}}{6}[2(z_{bl}^1)^2 - 5x_{bl}z_{bl}^1 - (x_{bl})^2] + \frac{A_{bl}}{2}[(z_{bl}^2)^2 - (z_{bl}^1)^2] + \frac{w_{bl}}{2}[(y_{bl})^2 - (z_{bl}^2)^2] - \frac{w_{bl} - A_{bl}}{6}[2(z_{bl}^2)^2 - 5y_{bl}z_{bl}^2 - (y_{bl})^2]; \quad (9)$$

- for the exponential distribution

$$M_{bl} = E[\theta_{bl}] = x_{bl} + \frac{1}{\alpha_{bl}}; \quad (10)$$

- for the Weibull distribution

$$M_{bl} = E[\theta_{bl}] = x_{bl} + \alpha_{bl}^{-\frac{1}{\beta_{bl}}} \Gamma(1 + \frac{1}{\beta_{bl}}), \quad (11)$$

where

$$\Gamma(u) = \int_0^{+\infty} t^{u-1} e^{-t} dt, \ u > 0,$$

is the gamma function;

- for the chimney distribution

$$M_{bl} = E[\theta_{bl}] = \frac{1}{2}[A_{bl}(x_{bl} + z_{bl}^1) + C_{bl}(z_{bl}^1 + z_{bl}^2) + D_{bl}(z_{bl}^2 + y_{bl})]. \quad (12)$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,...,v$, of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,...,v$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ b = 1,2,...,v. \quad (13)$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,...,v$, at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ b = 1,2,...,v, \quad (14)$$

where M_{bl} are defined by the formula (5) in a case of any distribution of sojourn times θ_{bl} and by the formulae (2.6)-(2.12) in the cases of particular defined respectively by (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b), \ t \in < 0, +\infty), \ b = 1,2,...,v,$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,...,v, \quad (15)$$

where M_b, $b = 1,2,...,v$, are given by (2.14), while the steady probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (16)$$

In the case of a periodic system operation process, the limit transient probabilities p_b, $b = 1,2,...,v$, at the operation states defined by (15), are the long term proportions of the system operation process $Z(t)$ sojourn times at the particular operation states z_b, $b = 1,2,...,v$.
Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,v$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta, \ b = 1,2,...,v, \qquad (17)$$

where p_b are given by (15).

4. Conclusions

The probabilistic model of the critical infrastructure operation process presented in this paper is the basis for further considerations in particular tasks of the EU-CIRCLE project. This model will be developed in order to construct the integrated model of critical infrastructure Safety (IMCIS) Related to Its Operation Process – IMCIS Model 1.

Acknowledgements

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/

References

Barbu V., Limnios N., Empirical estimation for discrete-time semi-Markov processes with applications in reliability. Journal of Nonparametric Statistics, Vol. 18, No. 7-8, 483-498, 2006

Collet J., Some remarks on rare-event approximation. IEEE Transactions on Reliability 45, 106-108, 1996

EU-CIRCLE Report D2.1-GMU2, Modelling outside dependences influence on Critical Infrastructure Safety (CIS) – Modelling Critical Infrastructure Operation Process (CIOP) including Operating Environment Threats (OET), 2016

EU-CIRCLE Report D2.1-GMU3, Modelling outside dependences influence on Critical Infrastructure Safety (CIS) – Modelling Climate-Weather Change Process (C-WCP) including Extreme Weather Hazards (EWH), 2016

EU-CIRCLE Report D2.1-GMU4, Modelling outside dependences influence on Critical Infrastructure Safety (CIS) - Designing Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) by linking CIOP and C-WCP models, 2016

Kołowrocki K., Safety of critical infrastructures – An overall approach, Keynote Speech. International Conference on Safety and Reliability - KONBiN 2013a

Kołowrocki K., Reliability of Large and Complex Systems, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014b

Kołowrocki K., Soszyńska J., A general model of industrial systems operation processes related to their environment and infrastructure. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 2, Vol. 2, 223-226, 2008

Kołowrocki K., Soszyńska J., Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 3, Vol. 1, 2, 211-222, 2009d

Kolowrocki K., Soszynska J., Testing uniformity of statistical data sets coming from complex systems operation processes. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 1,123-132, 2010b

Kołowrocki K., Soszyńska-Budny J., Reliability and Safety of Complex Technical Systems and Processes: Modeling - Identification - Prediction - Optimization, London, Dordrecht, Heildeberg, New York, Springer, 2011

Kołowrocki K., Soszyńska-Budny J., Introduction to safety analysis of critical infrastructures. Proc. International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering - QR2MSE-2012, Chendgu, China, 1-6, 2012a

Limnios N., Oprisan G., Semi-Markov Processes and Reliability. Birkhauser, Boston, 2005

Limnios N., Ouhbi B., Sadek A., Empirical estimator of stationary distribution for semi-Markov processes. Communications in Statistics-Theory and Methods, Vol. 34, No. 4, 987-995 12, 2005

Rice J. A., Mathematical statistics and data analysis. Duxbury. Thomson Brooks/Cole. University of California. Berkeley, 2007