Statistical Analysis of Models for Punching Resistance Ensuring

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Abstract:
In this paper the statistical assessment of the models that ensure the safety of the reinforced concrete slabs is focused. The investigation results in choosing the best model as the one that can aspire to become normative for European Union after 2020. Authors dispose with a sufficient number of data yielded from experimental tests. Having input geometrical and physical parameters of each experiment at hand, the corresponding theoretical value is computed by using three formulas provided by three models involving the same inputs. Case by case, the ratio between measured and theoretical value reveals the safety immediately. This ratio stands as the one parametric dimensionless statistical variable which is analysed afterwards. Due to statistical parameters evaluation and in accordance with engineers a new model was suggested, statistically verified and nominated as the normative one.

Keywords: *data mining, reinforcement structures punching reliability, Shapiro-Wilk test, Tuckey's fence, quartiles, coefficient of variance*

1. Introduction

There are various type of column system involved in constructions. In Fig. 1 there are three types of columns-slab junction performed, flat slabs, downstand beam support and locally supported slabs in the place of column - ceiling junction. Even though there exist some drawbacks as softness of the system resulting in great deflections of the slab, heavy shear load of the slab nearby the column often resulting in punching; the flat slabs are very often used right because the nice flat lower ceiling, because of simple casing, reinforcement by nets enabled, unified concrete mixture and short building time of the construction.

Concrete reinforced flat slabs are used very frequently in civil engineering. Punching is the most often failure of them. Unfortunately, due to its brittle character it is very sudden and very dangerous. Brittle character means that the failure spreads from the initial crack place very quickly towards all directions causing a progressive collapse. Indeed, the reinforced concrete flat slabs have to be built up with respect to prior investigation based significantly on experimental results. The investigation aiming to the constructions failure avoiding, together with economical reasoning involved, resulted in normative prescriptions ensuring the their further safety. Several models were built up, introduced and implemented in various countries of Europe. Recently there exists an effort to involve the prescription for building up the flat slabs as to ensure the maximal safety of the designed constructions. Nowadays there were three models introduced that compete to become normative.

1.1. Flat Slab Reliability Models

As detected from experiments, each failure of flat slab involves so called critical crack. That is why so called Critical shear crack theory was developed involving all physical parameters of concrete as well as the geometry of the structure. Some models ensuring safety of these constructions employ this theory.

We have verified three models their match with experimental data set and safety:

The first model is fully empirical. It was set up in 1990 in Model Code, [7]. Only the statistical observation of the experimental data were taken into account. The formula (1) performs the shear stress dependence on the reinforcement ratio ρ and on the concrete compressive stress f_{ck}, formula (1).

Later the nonlinear elasticity theory was employed in the investigation and the Critical shear crack theory was developed. The theory was introduced by Muttoni and Schwartz in [6] and upgraded by Mutttoni in [3], resulting in Model Code 2010 normative form (2), [4]. The model is more complex as it inter alia includes greater number of geometrical and physical parameters, e.g. longitudinal shear reinforcement, and magnitude of the aggregate. Lately, due to some simplification effort, the third model EC (2017) was developed, represented by (5) [5]. Afterwards it was included to the second generation of Euro Code of the second generation, EC2.

Each model is represented by unique formula quantifying the shear stress resistance $V_{Rd,c}$; for better clarity sake, some auxiliary forms are provided.

Model EC2 (2004)

$$V_{Rd,c} = \frac{C_{Rk,c}}{\gamma_c} k (100 \rho f_{ck})^{\frac{1}{3}} u_1 d \tag{1}$$

with
- $C_{Rk,c}[MPa]$ empirical factor
- $\gamma_c[-]$ partial safety factor
- $k[-]$ size effect factor, $k = 1 + (200[mm]/d[mm])^{\frac{1}{2}}$
- $d[m]$ effective depth of the slab, i.e. the vertical distance from the bottom of the slab up to the reinforcement placement
- $u_1[m]$ basic control perimeter at the distance $2d$ from the axis of the column

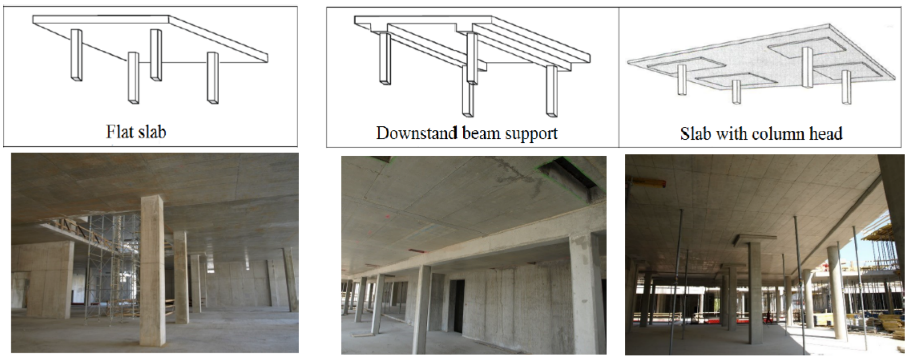

Fig. 1. Columns without (left), with down-stand beam (middle) with a local upper support (right), [2]

Fig. 2. Critical crack of flat slab, [2]

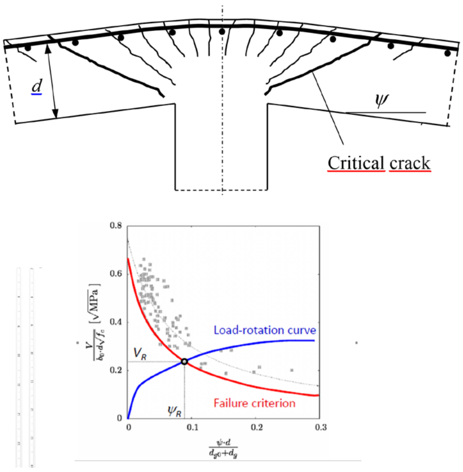

Fig. 3. Theory of Critical Shear Crack, [1]

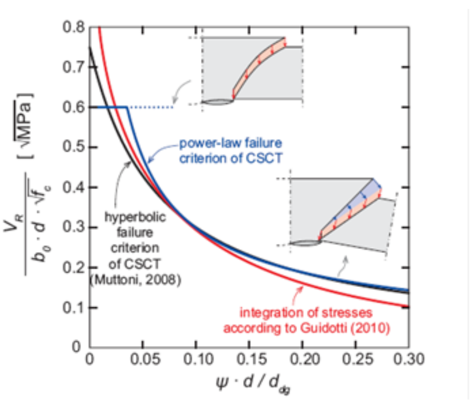

Fig. 4. Graphical performance of failure criterion as stipulated by different models

$$\rho = (\rho_x \rho_y)^{\frac{1}{2}} \tag{2}$$

with

- ρ_x and $\rho_y[-]$ reinforcement ratios in x and y direction respectively:

$$\rho_x = \frac{A_{sx}}{d_x b}$$

$$\rho_y = \frac{A_{sy}}{d_y b}$$

with

- A_{sx} and $A_{sy}[m^2]$ areas of reinforcement in x and y direction respectively
- $b[m]$ longitude of specimen

Model MC (2010)

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{3}$$

$$k_\psi = \frac{1}{1.5 + 0.9 k_{dg} \psi d}$$
$$\psi = \frac{r_s}{d} \frac{f_y d}{E_s} \left(\frac{m_{Sd}}{m_{Rd}}\right)^{\frac{3}{2}} \tag{4}$$

with

- $d_v[m]$ effective depth of the slab, usually $d_v = d$ $b_0[m]$ the length of control perimeter at the distance $d_v/2$
- $k_{dg}[m]$ factor involving maximal aggregate magnitude $d_g[mm]$: $k_{dg} = \frac{32}{16+d_g}$
- $r_s[m]$ distance from axis of the column to the line of contraflexure of radial bending momentums
- $f_{yd}[MPa]$ yielded strength of principal reinforcement
- $m_{Sd}[m^{-1}]$ average design bending capacity per unit length
- $m_{Rd}[m^{-1}]$ average design bending capacity per unit length

Model EC2 (2017)

$$V_{Rd,c} = \frac{b_0 d_v}{\gamma_c} \min\left\{k_b\left(100\rho f_{ck} \frac{d_g}{a_v}\right)^{\frac{1}{3}}, 0.6\sqrt{f_{ck}}\right\} \tag{5}$$

$$k_b = \max\left\{1, \sqrt{8\mu \frac{d}{b_0}}\right\} \tag{6}$$

with $a_v[m]$ being shear span ($\geq 2.5d$) , geometric average of shear spans in both orthogonal directions, μ parameter accounting shear force and bending momentums in the shear region, in case of indoor column without unbalanced momentum it is set to 8. Accordingly, for normal weight concrete it is taken $d_g = 32mm$.

These three models performed by formulas for $V_{Rd,c}$ computation, compete for becoming normative in the on-coming unified European norm that is intended to be valid from 2020.

2. Statistical Analysis

2.1. Description of Data Set

The focused data set was withdrawn form a large database with more than 600 experimental results gathered through the decades, initiated in 1938 in RTWH Aachen and collected by Carsten Siburg. The data of shear resistance were recorded together with several input information, i.e. geometrical and physical characteristics of tested samples. As time lapsed, the set of input parameters had been standardized. However, some of older data are not usable in our investigation as they lack some essential input information. We have used as much data as possible, different number of data for particular model, as the models required different input parameters. The database is still being enhanced by new experimental data from European countries. Such tests are realized in our university, as well.

Primary idea was to compare three models mentioned above. Comparison of match between theoretical and experimental values and the level of safety was decisive. For this sake the statistical analysis has been carried out. As reasoned above, we disposed with 404 items of EC2004, 385 items of Model Code 2010 and 385 items of Euro Code 2017 within the statistical analysis.

Several characteristics within the data can be analysed within the investigation. From the engineering point of view, three influencing input parameters were selected and focused more precisely:

- effective depth $d[m]$, with values $d \in [50, 660.9]$
- reinforcement ratio $\rho[-]$, with values $\rho \in [0.0025, 0.0702]$
- cylindrical stiffness of concrete f_{ck}, with values $f_{ck} \in [9.282, 119]$

In this paper we provide the more detailed analysis with regard to cylindrical concrete compressive stress f_{ck}.

All values (3 theoretical and 1 experimental) are included in the database. In accordance with civil engineering practice, not differences, but the ratios between experimental value V_{test} and corresponding theoretical value $V_{Rd,c}$ (V_{model}) will be treated, namely in three cases of $V_{Rd,c}$: that yielded form (1), from (3) or from (5), respectively. Thus the ratio

$$\frac{V_{test}}{V_{Rd,c}} = x_i$$

stands as the statistical variable to be handled. The item $\frac{V_{test}}{V_{Rd,c}}$ enhances the five-dimensional vector $(d, \rho, f_{ck}, V_{Rd,c}, V_{test})$ belonging to each model, by one. Moreover, it is worth to note that the ratio above 1 means safety, under 1 means failure. The statistical investigation and data mining is exerted.

2.2. Primary Statistical Analysis

Primary statistical analysis involves graphical analysis and computation of basic characteristics of location and variation. The aim of graphical analysis is a synoptic comparison of the three models performed. Graphical analysis involves histograms depicting the probability distribution of ratio values, box-and-whiskers diagrams demonstrating the overall distribution of data set, quartile distribution as well and detecting the outstanding data. We have found out that all of three data sets corresponding to the three models include some outstanding data. Afterwards, the Tuckey's fence test (7) detects and omits these outstanding values (outliers) from the further consideration.

$$[Q_1 - k(Q_3 - Q_1), Q_3 + k(Q_3 - Q_1))] \qquad (7)$$

with

- Q_1, Q_3 the first (lower) and the third (upper) quartile respectively
- k coefficient of outlying, usually $k = 1.5$ for outliers, $k = 3$ for far out values.

From Figs. 5, 6 it is evident that model EC2 (2017) is not sufficiently safe. The Tab. 1 affirms that even the mean value is below one. Histograms refer to the normality of data distribution in all three cases both before and after excluding the outstanding data. From the point of view of civil engineering practice it is interesting to trace the outstanding data with regard to the particular input parameters. In Fig. 5 we provide an example of such an approach. It is apparent that the most outstanding data are situated in the interval of the most used values of f_{ck}, i.e. $[20, 40]$. The normality is affirmed by Shapiro-Wilk test, see Chapter 2.3.

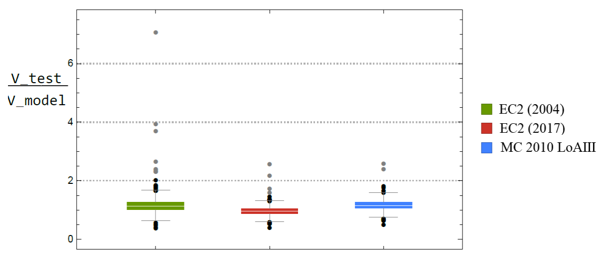

Fig. 5. Box plot of ratio experimental/theoretical value of punching resistance EC2 (2004), EC2(2017), Model Code (2010), original data

Statistical characteristics

- mean

$$\hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

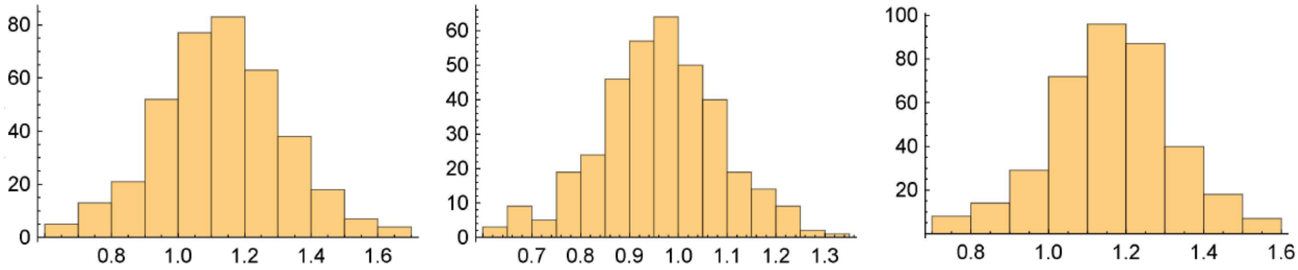

Fig. 6. Histograms of ratio experimental/theoretical value of punching resistance EC2 (2004), EC2 (2017), Model Code (2010), outstanding data excluded

model	median	average	variance	variation coefficient	5 0.05 quantile	$\mathbf{P(x_i \leq 1)}$
EC2(2004)	1.12054	1.12842	0.186157	0.164971	0.809558	0.238845
EC2(2017)	0.958613	0.96237	0.124694	0.12957	0.760439	0.627072
MC(2010)	1.15752	1.16523	0.160081	0.137381	0.884772	0.137466

Tab. 1. Basic statistical characteristics of three focused models

- standard deviation

$$\hat{\sigma} = s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- variation coefficient

$$V_x = \frac{\bar{x}}{s_x}$$

- k quantile q_k

$$P(X \leq q_k) = k$$

where

$$x_i = \frac{V_{test}}{V_{model}}$$

with n number of measurements.

For the civil engineers, 0.05 quantile is interesting, as well as safe and unsafe zone split of data set. Herein safe zone involves all data $x_i \geq 1$. Even from the primary statistical analysis it is evident that although EC2 (2017) is least safe (more than 62% of data falls to unsafe zone), low value of standard deviation refers to low degree of data set variability. Indeed, large number of data fall below zero very narrowly. That indicates a new proposal to modify perspective normative model EC2 (2017) in sense of increasing mean i.e. to ensure more safeness.

2.3. Subinterval Analysis

In engineering practice, absolutely prevalently used concrete is the one of concrete compressive stress value f_{ck} within the interval $[20, 40]$. That is why we have paid the special attention to data within this interval.

That is why we carried out a subinterval analysis and compare the three models piecewisely, as well. By such an analysis also the suggestion can be reasoned, how EC2 (2017) should be eventually improved.

On each of the subinterval it is performed the complex primary statics, normality of data verification included. Moreover, on each subinterval test have been carried out, detecting whether samples originate from the same distribution. In case of normally distributed data sets and equality of variance (in statistical sense) the parametric test ANOVA was used, otherwise nonparametric Kruskal-Wallis test was employed. The results are gathered in Table 2. Graphical performance of statistical characteristics are performed in Figs. 7-9.

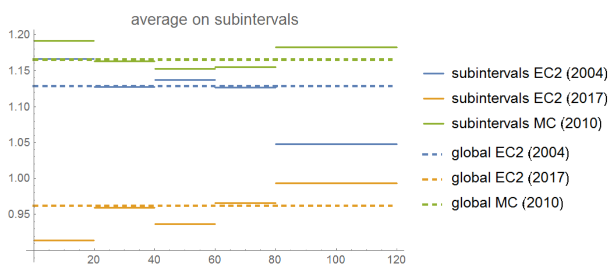

Fig. 7. Subinterval analysis. Global average (dashed line) and subinterval (continuous line) average on particular subintervals

It is apparent from Table 2 and from Fig. 7 and 8 that the subintervals are different as far as distribution concerned. Even in the most important subinterval of f_{ck}, i.e. the most frequently used qualitative type of concrete the difference is statistically meaningful. The average of model EC2(2004) and MC (2010) are on the side of safety, but overestimated generally and in each subinterval, as well. On the other hand, the model EC2 (2017) generally falls into the unsafe region,

model	median	average	variance	variation coefficient	0.05 quantile	$P(x_i \leq 1)$
EC2(2004)	1.12054	1.12842	0.186157	0.164971	0.809558	0.238845
EC2(2017)	0.958613	0.96237	0.124694	0.12957	0.760439	0.627072
MC(2010)	1.15752	1.16523	0.160081	0.137381	0.884772	0.137466

Tab. 2. Basic statistical characteristics of three focused models

subinterval	normality	variance	test	distribution
≤ 20	yes	equal	nonparametric	different
$20 - 40$	yes	not equal	nonparametric	different
$40 - 60$	yes	not equal	nonparametric	different
$60 - 80$	yes	equal	parametric	different
$80 - 120$	yes	equal	parametric	different

Tab. 3. Subintervals statistical comparison of three models

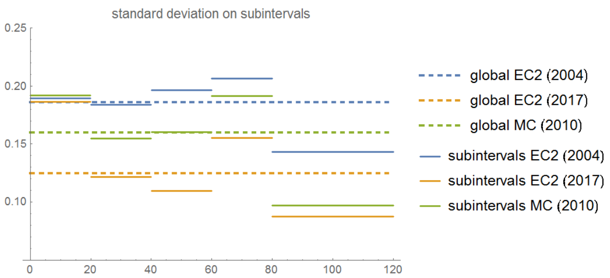

Fig. 8. Subinterval analysis. Global standard deviation (dashed line) and subinterval (continuous line) variance on particular subintervals

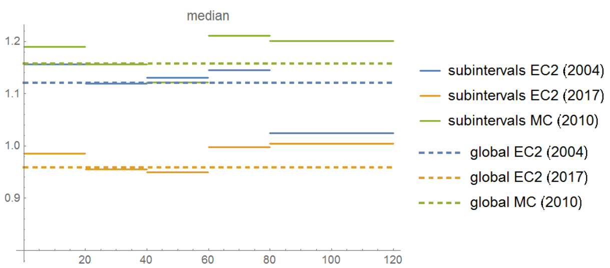

Fig. 9. Subinterval analysis. Global median (dashed line) and subinterval (continuous line) median on particular subintervals

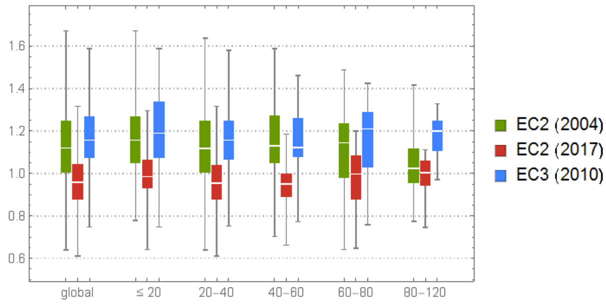

Fig. 10. Subinterval analysis. Box whisker plots

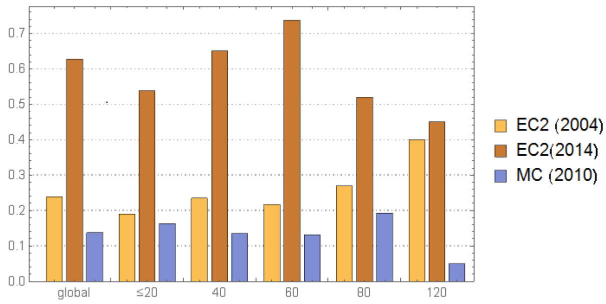

Fig. 11. Amount of data below 1, i.e. on the side of unsafety, in particular three cases

slightly underestimated, therefore, roughly spoken, it should be refused. The underestimation was additionally verified by recent experiment in the laboratory of our university.

Partial Conclusion

Regarding the lower variance both in global and interval-wise sense, see Fig. 10 we have suggested an improvement of model EC2 (2017) represented by (5). The suggestion consists in certain modification of the normative formula that increases the ratio of experimental-to-model value up to the safety side with keeping variance almost unchanged. After such modification the newly arisen model will be again subjected to the statistical analysis targeting in stipulating the best of all four models as the normative for the future.

2.4. Time Dependent Analysis

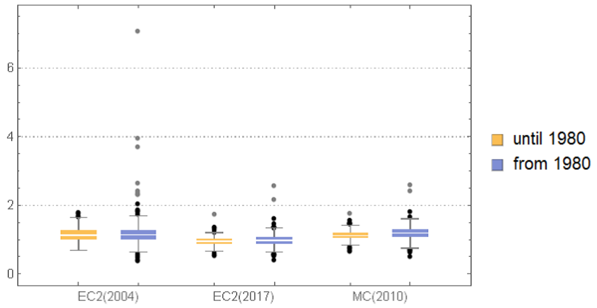

Fig. 12. Time split of entire data set to two sets: until 1980(left), after 1980 (right). Outliers emphasised

Due to the change in technologies and lab measurement equipment and instruments and with regard the fact that the time and location item is involved in the global database, a time dependent statistical analysis is justified and enabled. We carried out the time dependent analysis. Then in accordance with engineers' request we have split the data to two time subintervals for future additional observation. We have found out a peculiar fact. In contrary to our presumption, and it can be seen in Fig. 12, there are higher number outstanding data and quite a higher number of far outliers in newer (after 1980) group of measurements. After more detailed look we could see this surprising fact is probably caused by a systematic error in one of the lab in 1984. Namely, in case of EC2 (2004) more than one half of outliers came from unique lab yielded in 1984, EC2 (2017) more than one third of the outliers come from the same lab and the same year, for MC (2010) almost one half of the outliers came the same lab and the same year, see Fig 13.

A glance to the Table 4 gives us the idea of data quality within the two time dependent sets - though the mean value is greater, in case of the set after 1980, the dispersion is greater, too. Accordingly, the data was distributed more narrowly before than after 1980.

3. Conclusion

The importance of the statistical analysis of data and data mining in engineering practice is indisputable. In case of reliability of punching resistance investigation including three models initially, inspired us to a completely new solution. As the result, instead of plain choosing the best of three provided models, an idea of new model creation arose which is still in progress.

Another fact contributing to the improvement of final result was detected during the analysis. Technical problems in one of the laboratories involved in the database caused mistaken measurements in one year. The wrong data were disclosed within the time dependent analysis. In order to increase the preciseness of results the data should be cut from the further analysis.

Validation of potential normative models is influencing factor of future normative formulas utilization in civil building practice. It contributes to the higher safety of construction having economical optimality in mind.

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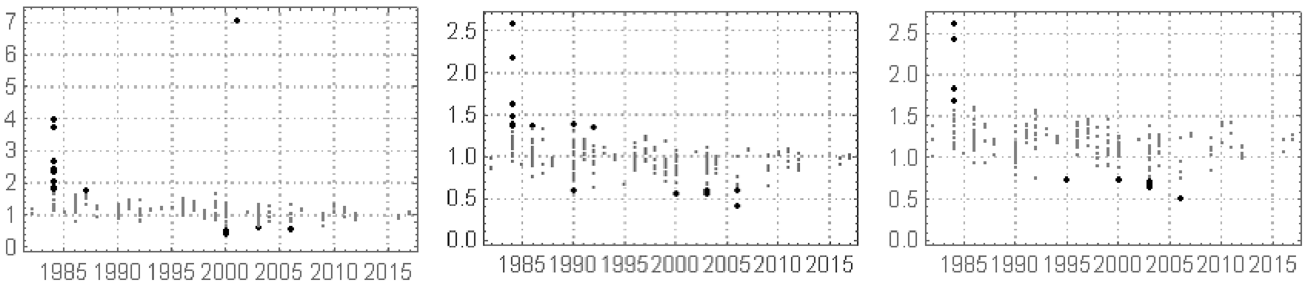

Fig. 13. Particular models outliers on the data set collected after 1980

EC2(2004)	median	average	variance	variation coefficient	0.05 quantile	$P(x_i \leq 1)$
until 1980	1.2043	1.3277	0.179722	0.158658	0.817335	0.228571
from 1980	1.13008	1.1392	0.201077	0.176507	0.7794788	0.232877
EC2(2017)	**median**	**average**	**variance**	**variation coefficient**	**0.05 quantile**	$P(x_i \leq 1)$
until 1980	0.94732	0.942543	0.108319	0.114922	0.761974	0.734848
from 1980	0.972726	0.974977	0.136169	0.139664	0.7551	0.57561
MC(2010)	**median**	**average**	**variance**	**variation coefficient**	**0.05 quantile**	$P(x_i \leq 1)$
until 1980	1.11726	1.12244	0.124926	0.111298	0.906012	0.164063
from 1980	1.19188	1.18848	0.168826	0.142052	0.884772	0.123223

Tab. 4. Basic statistical characteristics of three focused models

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