Planning Task-Based Motions of Multi-Link Manipulator Models Prone to Vibration

Elżbieta JARZĘBOWSKA
Warsaw University of Technology, Department of Power and Aeronautical Engineering, 00-665 Warsaw, Nowowiejska 24, Poland, elajarz@meil.pw.edu.pl

Krzysztof AUGUSTYNEK
University of Bielsko-Biala, Department of Mechanical Engineering Fundamentals, 43-309 Bielsko-Biala, Willowa 2, Poland, kaugustynek@ath.bielsko.pl

Andrzej URBAŚ
University of Bielsko-Biala, Department of Mechanical Engineering Fundamentals, 43-309 Bielsko-Biala, Willowa 2, Poland, aurbas@ath.bielsko.pl

Abstract

A new method of designing tip-point trajectories for industrial manipulators with structural flexibility and friction in their supports and joints is presented in the paper. The method is based upon the formulation of the reference dynamics model of the manipulator model. The reference dynamics models are derived off-line for desired trajectories, i.e. trajectory patterns are delivered in constraint equation forms. A library of reference dynamics models can be generated off-line and serve for verification of desired motions feasibility, ability of reaching required velocities and accelerations in order to obtain the desired trajectories, and estimate power needed for desired motions execution. It enables off-line tests of a system model behavior when it moves a desired trajectory. Also, the reference dynamics enables verification of vibrations that may accompany desired motions, since at the stage of defining a desired trajectory, e.g. for performing a servicing task, a system behavior is unknown. The paper is dedicated to reference dynamics analysis, for a multi-link manipulator model, for which friction in joints is modeled by the LuGre model, the manipulator links are rigid but light and prone to vibrations. The results of the reference dynamics analysis are presented in a series of simulation studies.

Keywords: dynamics, GPME algorithm, manipulator, LuGre friction model

1. Introduction

Industrial manipulators are dedicated to variety of services and motions they perform are pre-specified, i.e. task-based the most often. These task-based motions usually refer to tip-point trajectories for industrial manipulators or some specified points on them. Consequently, they require system links to move some specified trajectories, velocities, accelerations and jerks. This link kinematics may be not known when a desired trajectory is specified. It may be tested ad hock when a system moves, i.e. experimentally, or systematically off-line.

Usually, trajectory generation performed for industrial manipulators consists of deciding on the velocity, acceleration and jerk profiles along the planned path as a function of time. The generated profiles must not exceed limitations on velocity, acceleration and jerks for a given system design, and in most cases must satisfy certain performance criteria. Usually, trajectory planning can be done on-line and off-line; see the problem overview in [1] and references there.

The paper motivation was to develop a new systematic method of designing tip-point trajectories for industrial manipulators with additional testing of these trajectories with respect to velocities, accelerations and jerks for any point on the system. Also, structural flexibility, damping and friction in the model supports and joints may be added to the presented method. The method is based upon the formulation of the reference dynamics model of the manipulator model [2, 3]. It is the constrained dynamics including task-based constraints, e.g. a desired trajectory formulated as a position constraint. The reference dynamics models are derived off-line for desired trajectories, i.e. trajectory patterns are delivered in constraint equation forms. The reference dynamics derivation is computationally automated so the desired motion can be obtained systematically [4]. A library of reference dynamics models can be generated off-line and serve for verification of desired motions feasibility, ability of reaching required velocities and accelerations in order to obtain the desired trajectories, and estimate power needed for desired motions execution. This systematic method provides what a typical trajectory planner does. Based upon the reference dynamics the construction of tracking controllers is available with minimum on-line and off-line computations.

The paper contribution is then to develop an automatic method of generation reference dynamics [4], which enables off-line tests of a system model behavior when it moves a desired trajectory. Also, the reference dynamics, based upon a dedicated method of constrained dynamics generation referred to as the generalized programmed motion equations (GPME) [2,3], enables verification of vibrations that may accompany desired motions, since at the stage of defining a desired trajectory, e.g. for performing a servicing task, a system behavior is unknown. The paper is dedicated to reference dynamics analysis, for a multi-link manipulator model, for which friction in joints is modeled by the LuGre model, the manipulator links are rigid but light and prone to vibrations. The results of the reference dynamics analysis are presented in a series of simulation studies.

The paper is organized as follows. In section 1 the manipulator model is presented. It comprises basic manipulator properties including revolute and translational joints, friction and a task-based trajectory for its tip-point. Section 2 develops the reference dynamics for the manipulator and section 3 illustrates simulation tests related to task-based trajectory generation. The paper ends with conclusions and a list of references.

2. Mathematical model

The model of a manipulator built of four rigid links ($n_l = 4$) is presented in Fig. 1. The first and third links are driven by driving torques – rigid $\mathbf{t}_{dr}^{(1)}$ and flexible $\mathbf{t}_{dr}^{(3)}$. Motion of the manipulator is limited by programmed constraints, depending on the assumed trajectory of an end-effector E .

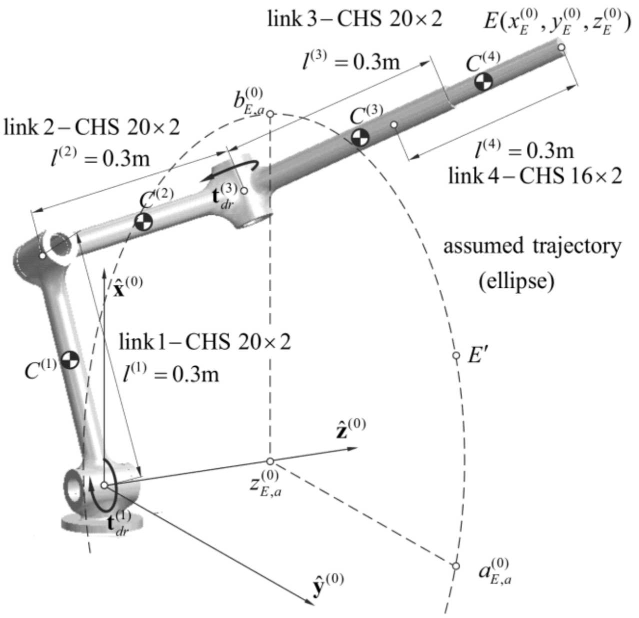
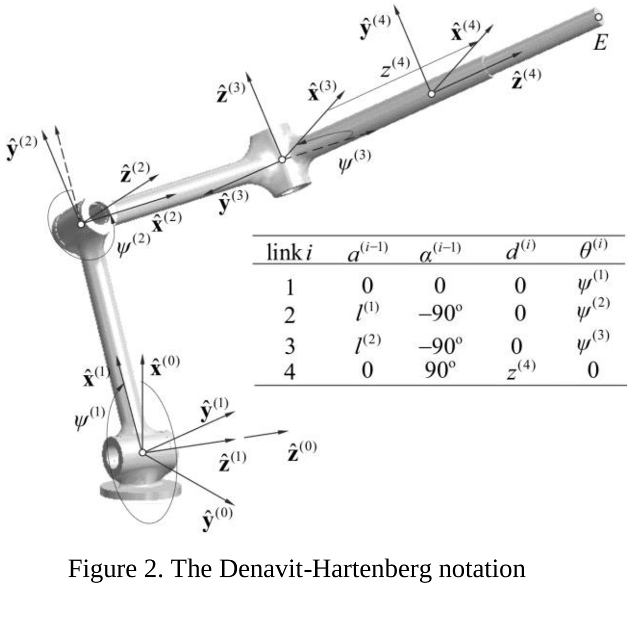


Figure 1. Model of manipulator

The Denavit-Hartenberg notation based on the joint coordinates and homogeneous transformation matrices formalism is used to describe the geometry of the manipulator (Fig. 2).



link i	$a^{(i-1)}$	$\alpha^{(i-1)}$	$d^{(i)}$	$\theta^{(i)}$
1	0	0	0	$\psi^{(1)}$
2	$l^{(1)}$	–90°	0	$\psi^{(2)}$
3	$l^{(2)}$	–90°	0	$\psi^{(3)}$
4	0	90°	$z^{(4)}$	0

Figure 2. The Denavit-Hartenberg notation

Dynamics of the analyzed manipulator is described by means of the vector of the generalized coordinates in the following form:

$$\mathbf{q} = \left(q_j\right)_{j=1,\dots,4} = \begin{bmatrix} \psi^{(1)} & \psi^{(2)} & \psi^{(3)} & z^{(4)} \end{bmatrix}^T . \tag{1}$$

This vector contains dependent and independent coordinates:

$$\mathbf{q} = \left(q_i\right)_{i \in i_{i_c} \cup i_{d_c}} \in \left\{ \mathbf{q}_{i_c}, \mathbf{q}_{d_c} \right\}, \tag{2}$$

where: i_{i_c}, i_{d_c} – index for independent and dependent coordinates, respectively.

In the analyzed manipulator it is assumed that:

$$i_{i_c} \in \{1,3\} \rightarrow \mathbf{q}_{i_c} = \begin{bmatrix} \psi^{(1)} & \psi^{(3)} \end{bmatrix}^T , \tag{3.1}$$

$$i_{d_c} \in \{2,4\} \rightarrow \mathbf{q}_{d_c} = \begin{bmatrix} \psi^{(2)} & z^{(4)} \end{bmatrix}^T . \tag{3.2}$$

The homogeneous transformation matrices from link local coordinate systems to the reference system are determined by the following formulas:

$$\mathbf{T}^{(l)}\Big|_{l=1,\dots,n_l} = \mathbf{T}^{(l-1)}\tilde{\mathbf{T}}^{(l)}, \tag{4}$$

where: $\mathbf{T}^{(0)} = \mathbf{I}$, $\mathbf{I}$ - identity matrix.

The GPME take the form [2,4]:

$$\left.\frac{\partial R_1}{\partial \dot{q}_i}\right|_{i \in i_{i_c}} + \sum_{j \in i_{d_c}} \frac{\partial R_1}{\partial \dot{q}_j} \frac{\partial \dot{q}_j}{\partial \dot{q}_i} = 0, \tag{5}$$

where:

$$R_1 = \dot{E}_k - 2\sum_{i=1}^{n_{dof}} \frac{\partial E_k}{\partial q_i}\dot{q}_i + \sum_{i=1}^{n_{dof}} \frac{\partial E_p}{\partial q_i}\dot{q}_i - \sum_{j=1}^{n_{dof}} Q_{f,j}\dot{q}_j ,$$

$$E_k = \sum_{l=1}^{n_l} E_k^{(l)}, \quad E_k^{(l)} = \frac{1}{2}\operatorname{tr}\left\{ \dot{\mathbf{T}}^{(l)}\mathbf{H}^{(l)}\left(\dot{\mathbf{T}}^{(l)}\right)^T \right\}, \quad E_p = \sum_{l=1}^{n_l} E_p^{(l)}, \quad E_p^{(l)} = m^{(l)} g \mathbf{J}_1 \mathbf{T}^{(l)} \mathbf{r}_{C^{(l)}}^{(l)},$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \mathbf{J}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The friction phenomenon in the manipulator joints is taken into account. The LuGre friction model is adopted, i.e.:

$$\dot{z}^{(j)} = \mathrm{LuGre}\left(t, \dot{q}_j, z^{(j)}\right), \tag{6}$$

where:

$$\mathrm{LuGre}\left(t, \dot{q}_j, z^{(j)}\right) = \dot{q}_j \left(1 - \frac{\sigma_0^{(j)} z^{(j)} \operatorname{sgn}(\dot{q}_j)}{\mu_k^{(j)} + \left(\mu_s^{(j)} - \mu_k^{(j)}\right) \exp\left(- \left(\frac{\dot{q}_j}{\dot{q}_{S,j}} \right)^2 \right)} \right),$$

$\mu_\alpha^{(j)}\big|_{\alpha \in \{s,k\}}$ – static and kinetic friction coefficients,

$\dot{q}_{S,j}$ – the Stribeck velocity,

$z^{(j)}$ – average deflection of bristles which model flexibility of contact surfaces.

The friction coefficients are calculated as follows:

$$\mu^{(j)} = \sigma_0^{(j)} z^{(j)} + \sigma_1^{(j)} \dot{z}^{(j)} + \sigma_2^{(j)} \dot{q}_j, \tag{7}$$

where: $\sigma_\alpha^{(j)}\big|_{\alpha \in \{0,1,2\}}$ – stiffness, damping and viscous damping coefficients.

The friction torques and force are determined in the following way:

$$t_f^{(j)} = \mu^{(j)} f_n^{(j)} \frac{d^{(j)}}{2}, \tag{8.1}$$

$$f_f^{(j)} = \mu^{(j)} f_n^{(j)}, \tag{8.2}$$

where: $f_n^{(j)} = \begin{cases} \sqrt{\left(f_x^{(j)}\right)^2 + \left(f_y^{(j)}\right)^2} & \text{for a revolute joint} \\ \sqrt{\left(f_{xz}^{(j)}\right)^2 + \left(f_{yz}^{(j)}\right)^2} & \text{for a prismatic joint} \end{cases}$

$f_{xz}^{(j)}, f_{yz}^{(j)}$ – normal forces acting in the prismatic joint in the planes $\hat{\mathbf{x}}^{(j)}\hat{\mathbf{z}}^{(j)}, \hat{\mathbf{y}}^{(j)}\hat{\mathbf{z}}^{(j)}$,

$f_x^{(j)}, f_y^{(j)}$ – joint forces determined from the Newton-Euler recursive algorithm,

$d^{(j)}$ – diameter of the revolute joint.

The end-effector moves along an elliptical trajectory designed in a plane parallel to the $x^{(0)}y^{(0)}$ plane and has to maintain a constant coordinate value $z_{E,a}^{(0)}$. Thus, the dynamics' equations are supplemented by the programmed constraints in the form:

$$\Phi_1 \equiv 0 \Rightarrow \left(\frac{x_E^{(0)}}{a_{E,a}^{(0)}}\right)^2 + \left(\frac{y_E^{(0)}}{b_{E,a}^{(0)}}\right)^2 - 1 = 0\,, \tag{9.1}$$

$$\Phi_2 \equiv 0 \Rightarrow z_E^{(0)} - z_{E,a}^{(0)} = 0\,, \tag{9.2}$$

where: $x_E^{(0)} = \mathbf{J}_1\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}$, $y_E^{(0)} = \mathbf{J}_2\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}$, $z_E^{(0)} = \mathbf{J}_3\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}$.

According to the GPME algorithm [2], the constraint equations are differentiated:

$$\dot{\Phi}_1 \equiv 0 \Rightarrow \mathbf{u}\dot{\mathbf{q}} = 0, \tag{10.1}$$

$$\dot{\Phi}_2 \equiv 0 \Rightarrow \mathbf{C}_3\dot{\mathbf{q}} = 0, \tag{10.2}$$

$$\ddot{\Phi}_1 \equiv 0 \Rightarrow \mathbf{u}\ddot{\mathbf{q}} + v = 0, \tag{10.3}$$

$$\ddot{\Phi}_2 \equiv 0 \Rightarrow \mathbf{C}_3\ddot{\mathbf{q}} + \mathbf{d}_3 = 0, \tag{10.4}$$

where:

$$\mathbf{C} = \begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\\ \mathbf{C}_3\end{bmatrix} = \left(c_{ij}\right)_{\substack{i=1,2,3\\ j=1,\dots,4}} = \mathbf{J}\left[\mathbf{T}_1^{(4)}\mathbf{r}_E^{(4)} \cdots \mathbf{T}_4^{(4)}\mathbf{r}_E^{(4)}\right],$$

$$\mathbf{d} = (d_i)_{i=1,\dots,3} = \mathbf{J}\left(\left(\sum_{i=1}^{n_{dof}}\sum_{j=1}^{n_{dof}}\mathbf{T}_{ij}^{(4)}\dot{q}_i\dot{q}_j\right)\mathbf{r}_E^{(4)}\right),$$

$$\mathbf{u} = (u_j)_{j=1,\dots,4} = \frac{1}{\left(a_{E,a}^{(0)}\right)^2}\mathbf{J}_1\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}\mathbf{C}_1 + \frac{1}{\left(b_{E,a}^{(0)}\right)^2}\mathbf{J}_2\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}\mathbf{C}_2,$$

$$v = \frac{1}{\left(a_{E,a}^{(0)}\right)^2}\left((\mathbf{C}_1\dot{\mathbf{q}})^2 + \mathbf{J}_1\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}d_1\right) + \frac{1}{\left(b_{E,a}^{(0)}\right)^2}\left((\mathbf{C}_2\dot{\mathbf{q}})^2 + \mathbf{J}_2\mathbf{T}^{(4)}\mathbf{r}_E^{(4)}d_2\right).$$

Vector of dependent velocities determined from equations (10.1) and (10.2) are:

$$\dot{\mathbf{q}}_{d_c} = -\mathbf{K}_{d_c}^{-1}\mathbf{K}_{i_c}\dot{\mathbf{q}}_{i_c}\,, \tag{11}$$

where: $\mathbf{K}_{d_c} = \begin{bmatrix}u_2 & u_4\\ c_{12} & c_{14}\end{bmatrix}$, $\mathbf{K}_{i_c} = \begin{bmatrix}u_1 & u_3\\ c_{11} & c_{13}\end{bmatrix}$.

The GPME equations supplemented by the equations of programmed constraints take the form:

$$\begin{bmatrix} \mathbf{M}_i\big|_{i\in i_{i_c}} + \sum_{j\in i_{d_c}} \mathbf{M}_j \frac{\partial \dot{q}_j}{\partial \dot{q}_i} \\ \mathbf{u} \\ \mathbf{C}_3 \\ \end{bmatrix} \ddot{\mathbf{q}} = \begin{bmatrix} \mathbf{h}_i + \mathbf{Q}_i + \sum_{j\in i_c \cup i_{d_c}} \dot{q}_j \frac{\partial Q_j}{\partial \dot{q}_i} + \sum_{k\in i_{d_c}} \left(h_k + Q_k + \sum_{j\in i_c \cup i_{d_c}} \dot{q}_j \frac{\partial Q_j}{\partial \dot{q}_k} \right) \frac{\partial \dot{q}_k}{\partial \dot{q}_i} \\ -\nu \\ -\mathbf{d}_3 \\ \end{bmatrix}, \quad (12)$$

where:

$$\mathbf{M} = \sum_{l=1}^{n_l} \mathbf{M}^{(l)}, \ \mathbf{M}^{(l)} = \left(m_{ij}^{(l)}\right)_{i,j=1,\dots,n_{dof}^{(l)}}, \ m_{ij}^{(l)} = \mathrm{tr}\left\{\mathbf{T}_i^{(l)} \mathbf{H}^{(l)} \left(\mathbf{T}_j^{(l)}\right)^T\right\},$$

$$\mathbf{h} = \sum_{l=1}^{n_l} \mathbf{h}^{(l)}, \ \mathbf{h}^{(l)} = \left(h_i^{(l)}\right)_{i=1,\dots,n_{dof}^{(l)}}, \ \mathbf{T}_i^{(l)} = \frac{\partial \mathbf{T}^{(l)}}{\partial q_i^{(l)}}, \ \mathbf{T}_{i,j}^{(l)} = \frac{\partial^2 \mathbf{T}^{(l)}}{\partial q_i^{(l)} \partial q_j^{(l)}},$$

$$h_i^{(l)} = \sum_{m=1}^{n_{dof}^{(l)}} \sum_{n=1}^{n_{dof}^{(l)}} \mathrm{tr}\left\{\mathbf{T}_m^{(l)} \mathbf{H}^{(l)} \left(\mathbf{T}_{m,n}^{(l)}\right)^T\right\} \dot{q}_m^{(l)} \dot{q}_n^{(l)} + 2\sum_{m=1}^{n_{dof}^{(l)}} \sum_{n=1}^{n_{dof}^{(l)}} \mathrm{tr}\left\{\mathbf{T}_m^{(l)} \mathbf{H}^{(l)} \left(\mathbf{T}_{i,n}^{(l)}\right)^T\right\} \dot{q}_m^{(l)} \dot{q}_n^{(l)},$$

$$\mathbf{Q} = -\left(\mathbf{g} + \mathbf{t} + \mathbf{f}\right), \ \mathbf{g} = \sum_{l=1}^{n_l} \mathbf{g}^{(l)}, \ \mathbf{g}^{(l)} = \left(g_i^{(l)}\right)_{i=1,\dots,n_{dof}^{(l)}}, \ g_i^{(l)} = m^{(l)} g \mathbf{J}_1 \mathbf{T}_i^{(l)} \mathbf{r}_{C^{(l)}}^{(l)},$$

$$\mathbf{t} = \begin{bmatrix} t_{dr}^{(1)} & 0 & t_{dr}^{(3)} & 0 \end{bmatrix}^T, \ t_{dr}^{(1)} = \mathrm{const}, \ t_{dr}^{(3)} = -s_{dr}^{(3)} \left(\psi_a^{(3)} - \psi^{(3)}\right), \frac{\partial \dot{\mathbf{q}}_{d_c}}{\partial \dot{\mathbf{q}}_{i_c}} = -\mathbf{K}_{d_c}^{-1} \mathbf{K}_{i_c},$$

$\mathbf{f} = \begin{bmatrix} t_f^{(1)} & t_f^{(2)} & t_f^{(3)} & f_f^{(4)} \end{bmatrix}^T$, $\mathbf{H}$ - pseudo-inertia matrix, g - the acceleration of gravity.

3. Simulation study results – programmed motion of the manipulator end-effector

Data regarding the initial configuration of the manipulator and parameters related to the LuGre friction model are presented in Table 1. In simulations, it is assumed that constant torque $t_{dr}^{(1)} = 20\,\mathrm{Nm}$ is applied to link 1 and motion of link 3 is enforced by the flexible drive with stiffness $s_{dr}^{(3)} = 10^4 \frac{\mathrm{Nm}}{\mathrm{rad}}$, which rotates it by $90°$. Dynamics equations of motion of the manipulator are integrated using the Runge-Kutta 4th order scheme with the constant time step $h = 10^{-4}\,\mathrm{s}$. Two cases are analyzed in simulation studies: with and without friction in the joints.

Table 1. Parameters used in the simulations

Parameter	Symbol	Link 1	Link 2	Link 3	Link 4
Initial configuration	$\mathbf{q}\|_{t=0\,\mathrm{s}}$	0°	270°	0°	0.3 m
Static friction coefficient	$\mu_s^{(j)}$	0.1	0.1	0.1	0.1
Kinetic friction coefficient	$\mu_k^{(j)}$	0.2	0.2	0.2	0.2
Stiffness coefficients of the bristle	$\sigma_0^{(j)}$	$2.5\frac{\mathrm{Nm}}{\mathrm{rad}}$	$2.5\frac{\mathrm{Nm}}{\mathrm{rad}}$	$2.5\frac{\mathrm{Nm}}{\mathrm{rad}}$	$25\frac{\mathrm{N}}{\mathrm{m}}$
Damping coefficients of the bristle	$\sigma_1^{(j)}$	$\sqrt{2.5}\frac{\mathrm{Nms}}{\mathrm{rad}}$	$\sqrt{2.5}\frac{\mathrm{Nms}}{\mathrm{rad}}$	$\sqrt{2.5}\frac{\mathrm{Nms}}{\mathrm{rad}}$	$\sqrt{25}\frac{\mathrm{Ns}}{\mathrm{m}}$
Viscous damping coefficients	$\sigma_2^{(j)}$	0	0	0	0
The Stribeck velocity	$\dot{q}_{S,j}$	$0.175\frac{\mathrm{rad}}{\mathrm{s}}$	$0.175\frac{\mathrm{rad}}{\mathrm{s}}$	$0.175\frac{\mathrm{rad}}{\mathrm{s}}$	$0.001\frac{\mathrm{m}}{\mathrm{s}}$

Fig. 3 and 4 show time courses of the configuration variables and those ones which result from assumed programmed constraints.

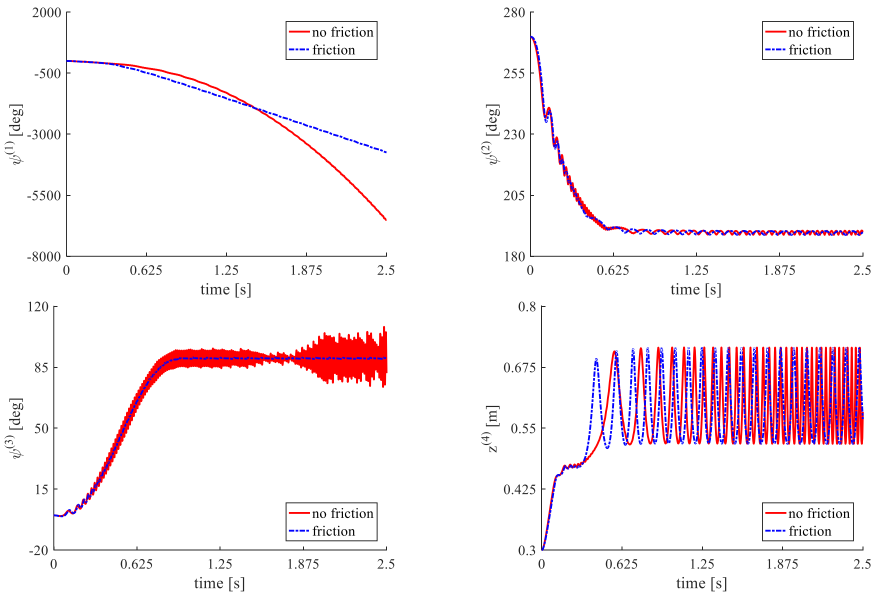


Figure 3. Time courses of the joint coordinates

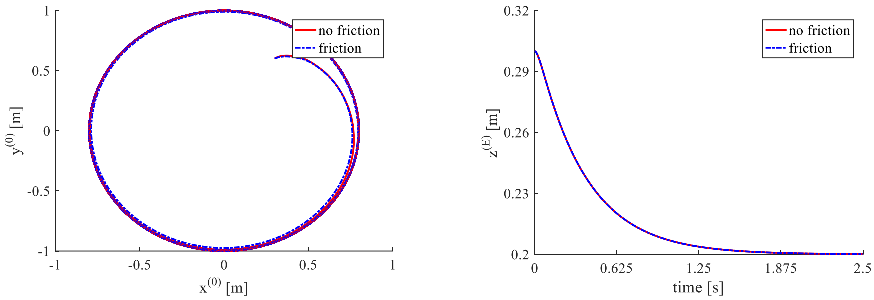


Figure 4. The trajectory and *z* coordinate time courses of the end-effector

It can be observed, that presence of friction in the joints, when programmed motion is analyzed, has a significant influence on time courses of the joint coordinates. The friction reduces vibrations appearing in the time course of the angle of rotation of link 3 and introduces additional resistance to the motion of links 1 and 4.

4. Conclusions

The paper presents a computational algorithm for generating the reference dynamics model of the multilink spatial manipulator with friction effects in the joints. Friction is modelled by means of the LuGre model. The methodology presented in the paper is based on the GPME algorithm and it can be easily extended to any open-loop kinematic chains. The numerical simulations results show that friction has a significant influence on dynamics of the system which is subjected to the programmed constraints. In the case of the analysed manipulator, motion performed by the end-effector satisfies the assumed programmed constraints.

References

1. Z. Shiller, *On-line and off-line Trajectory Planning*, in Motion and Operation Planning of Robotic Systems. Background and Practical Approaches, G. Carbone, F. Gomez-Bravo, Eds, Springer 2015.
2. E. Jarzębowska, *Dynamics Modeling of Nonholonomic Mechanical Systems: Theory and Applications*, Nonlinear Analysis – Theory Methods and Applications, **63**(5-7) (2006) 185 – 197.
3. E. Jarzębowska, *Quasi-coordinates Based Dynamics Modeling and Control Design for Nonholonomic Systems*, Theory and Applications, Nonlinear Analysis – Theory methods and Applications, **71**(12) (2009) 118 – 131.
4. E. Jarzębowska, K. Augustynek, A. Urbaś, *Computational Reference Dynamical Model of a Multibody System with First Order Constraints*, Proceedings of the ASME 2017 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, IDETC2017, August 6-9, 2017, Cleveland, Ohio, USA.