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Bayesian approach to shipping reliability and safety

Keywords

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Abstract

In a Bayesian approach, there are two main sources of information about parameters of interest such as prior beliefs or the prior distribution of the parameter and the likelihood of observing the data given our expectations about the parameter. The prior distribution may be based on previous studies, literature reviews or expert opinions and indicates how we believe the parameter would behave if we had no data upon which to base our judgments. In case where we have less data, the prior has greater influence. The maximum likelihood estimate predominates only when we have a lot of data. The posterior distribution is the result of combining the prior distribution and the likelihood. In the paper the examples of using Bayes approach to shipping operational reliability and safety is presented.

1. Introduction

It is necessary to use all available information, historical, objective or subjective, when making decisions under uncertainty. This is especially true when the consequences of the decisions can lead to a collision of ships.

Decisions made in the areas of safety and reliability have great impact on the humans and environment. Very often the information on which such decisions have to be based are only partially relevant, incomplete or even outdated and subjectively interpreted.

Theory Bayesian methods are central to modelling behaviour under uncertainty. Analysing ship safety we maximize an objective function conditional on available information, and if more information becomes available we update decisions using Bayes rule. Bayesian methods apply this paradigm to the navigator as a decision maker, [16].

2. Bayesian methodology in used for reliability evaluation

Usually the reliability of a technical system is expressed mathematically in terms of probability distribution. The describing of its life time is done by failure time distribution which depend on at least one unknown parameter. Those parameters must be estimated based on observed life tested data, [4], [11]. At shipping there are several random physical causes which separately or collectively may be responsible for the failure. Many of them changed at time so their numerical characteristics are not constant but are described by time dependent processes. It is why the parameters involved in failure should be treated as random variables. What's more they behave according to unknown probability distribution.

Suppose that the conditional failure time distribution $F(t|\theta)$ depends on a random parameter θ whose probability distribution $G(\theta)$ is unknown. The unconditional failure time distribution is given by formula, [22]

$$F_G(t)=\int F(t|\theta)\, dG(\theta). \qquad (1)$$

We have to consider the estimation of G in reliability models when a priori information about the parameter θ is specified in the form of an initial guess, G_0 , of G.

Utilizing the concepts of Dirichlet process priors on G, a Bayes estimate $F\Box_G$ of F_G may be obtained based on k observed lifetimes from F_G. Then an estimate $\hat{G}_k$ of G is found from $F\Box_G$ using a linear programming approach. For the Weibull failure time distribution $F(t|\theta)$ with random scale parameter θ , the effect of using the estimated prior $\hat{G}_k$ in Bayes

estimation of reliability is studied by Monte Carlo simulations.
Bayesian analysis considers population parameters to be random, not fixed
Life data information, or subjective judgment, is used to determine a prior distribution for these population parameters. Knowing of the parameters is necessary when making decisions under uncertainty especially if the consequences of the decisions can have a significant risks to environment , life or implicate great financial louses, as it is in sea transport.
Statistical analysis generally restricting the information used in an analysis to that obtained from a current set of clearly relevant data. Prior knowledge is used to make the choice of a population model which "fit" to the data. Chosen model is later checked against the data for reasonableness, [19].
At Bayes approach at the first step we use old information, or subjective judgments, to construct a prior distribution model for these parameters.
Such model expresses initial assessment of how likely various values of the unknown parameters are. We then use Bayes formula to revise this starting assessment, deriving the posterior distribution model for the population model parameters. Parameter estimates are calculated directly from the posterior distribution. Since the unknown parameters are considered random, not fixed then credibility intervals are legitimate probability statements about these parameters.
Parametric Bayesian prior models are chosen because of their flexibility and mathematical convenience. The conjugate priors are a natural and popular choice of Bayesian prior distribution models.

2.1. Bayesian approach

The essential for Bayes model is Bayes Theorem, [10]. We start from notations partition of a Sample Space.
A partition of a sample space Ω is a collection of mutually disjoint and collectively exhaustive events H_i.
That is $H_i \cap H_j = \varnothing$ whenever $i \neq j$, and $\cup_i H_i = \Omega$, where the union includes all of the sets H_i.

Law of Total Probability.

Let $\{H_i;\ i=1,2,\ldots\}$ be a countable infinite partition of Ω, then for any event D

$$P(D) = \sum_i P(D \mid H_i) P(H_i) \tag{2}$$

where sum is taken over i = 1, 2, ...
The total probability of D has been expressed as a sum of probabilities of disjoint sets.

Bayes Theorem.

Let H_i form a finite partition of Ω, then for any element H_j of partition

$$P(H_j \mid D) = \frac{P(D \mid H_j) P(H_j)}{P(D)}. \tag{3}$$

If we know the probabilities $P(H_i)$ of all of the partition sets, and we know all of the conditional probabilities $P(D|H_i)$, then we know the probabilities of each of the events involved in the law of total probability, formula (2), than we can evaluate $P(D)$.

Thus we can find any of the "reverse" conditional probabilities $P(H_j|D)$.

2.2. Bayesian hypothesis testing

Lets assume that we have two complimentary hypotheses, H_0 and H_1. Letting D stand for the observed data, Bayes theorem then becomes:

$$P(H_0 \mid D) = \frac{P(H_0) \cdot P(D \mid H_0)}{P(H_0) \cdot P(D \mid H_0) + P(H_1) \cdot P(D \mid H_1)}\ , \tag{5}$$

and

$$P(H_1 \mid D) = \frac{P(H_1) \cdot P(D \mid H_1)}{P(H_0) \cdot P(D \mid H_0) + P(H_1) \cdot P(D \mid H_1)}. \tag{6}$$

The $P(H_0|D)$ and $P(H_1|D)$ are posterior probabilities, the probability that the H_0 is true given. The $p(H_0)$, $p(H_1)$ are prior probabilities, the probability that the H_0 or the alternative is true prior to considering the new data. The $P(D|H_0)$ and $P(D|H_1)$ are the likelihoods, the probabilities of the data given one or the other hypothesis.
That is,

$$\frac{P(H_1 \mid D)}{P(H_0 \mid D)} = \frac{P(H_1)}{P(H_0)} \cdot \frac{P(D \mid H_1)}{P(D \mid H_0)} \tag{7}$$

In classical hypothesis testing, we considers only $P(D|H_0)$, it means the probability of obtaining sample data as or more discrepant with null hypothesis than are those on hand, that is, for the obtained significance level, p, and if that p is small enough, we reject the null hypothesis and asserts the alternative hypothesis. In classical hypothesis testing we do not estimate the probability that the null hypothesis is true. Using Bayesian methodology such estimation is done and if that probability is sufficiently small the null hypothesis is rejected, in favor of the alternative hypothesis. The level of small which is sufficiently small depends on an informed consideration of the relative seriousness of making one sort of error (rejecting H_0) versus another sort of error (retaining H_0).

Example 1

Suppose that we are interested in testing the two hypotheses about the acceptable distance to another ship according to a collision risk. H_0: $\mu = 4$ nautical miles versus H_1: $\mu = 4{,}5$ nautical miles. If we consider the two hypotheses equally likely, and dismiss all other possible values of μ, then the prior probability of the null is 0,5 and the prior probability of the alternative is also 0,5.

Let assume that we obtain a sample from the population of navigators and the acceptable distance is normally distributed with a standard deviation of 0,3, so the standard error of the mean is 0,3/10 = 0,03. The obtained sample mean is 4,12.

Computing for each hypothesis $z_i = \frac{m - \mu_i}{\sigma}$ we have for H_0 the z = 0,1 and for H_1 z = -0,31667.

The likelihood $P(D|H_0)$ is obtained by finding the height of the standard normal curve at z and dividing by 2 (since there are two hypotheses),

$$P(D|H_0) = 0{,}673096.$$

In the same way we obtain the likelihood

$$P(D|H_1) = 0{,}326904.$$

The

$$\begin{aligned} P(D) &= P(H_0) \times P(D|H_0) + P(H_1) \times P(D|H_1) \\ &= 0{,}5 \cdot 0{,}184135 + 0{,}5 \cdot 0{,}089429 = 0{,}136782 \end{aligned}.$$

so, the posterior probabilities formulas (5), (6) are:

$P(H_0 | D) = 0{,}6731$, and $P(H_1 | D) = 0{,}3269$.

The ratio, formula (7), of prior odds was 1 but the posterior odds ratio equals

$$\frac{0{,}6731}{0{,}3269} = 2{,}059$$

and it is the same as the likelihood ratio .

That is, after gathering the data we know that H_1 is more than 2 times more likely than is $H_\varnothing$.

Multiplying the prior odds ratio by the likelihood ratio gives us the posterior odds. If we assume that the prior probabilities are equal we will make the mistake.

Bayes formula provides the mathematical tool that combines prior knowledge with current data to produce a posterior distribution.

We can transform the formula, in terms of probability density function models formula (1), then it takes the form:

$$h(\lambda | x) = \frac{f(x|\lambda)h(\lambda)}{\int_0^\infty f(x|\lambda)h(\lambda)d\lambda}$$

where $f(x|\lambda)$ is the likelihood function (called conjugate distributions), for the observed data x given the unknown parameter λ, $h(\lambda)$ is the prior distribution model for λ (the conjugate prior) and $h(\lambda | x)$ is the posterior distribution model for λ given that the data x have been observed.

We called $f(x|\lambda)$ the conjugate distributions and $h(\lambda)$ the conjugate prior if $h(\lambda | x)$ and $h(\lambda)$ were belong to the same probability distribution family.

If sampling have an exponentially distributed population then the Gamma model is a conjugate prior for the failure rate λ.

The gamma, exponential conjugate pair is used in Bayesian system reliability applications, in many cases.

2.3. Bayesian methodology for reliability model

We can take into consideration different types of uncertainty sources in shipping analysis like, [14][18][3]:

- physical uncertainty or inherent variability, quantified by a probability distribution estimated from observed data, ship reliability, human error;
- statistical uncertainty, which refers to the uncertainty in the statistical distribution parameters of the random variables;
- modeling uncertainty, which includes uncertainty in both probabilistic and system analysis models,

Operational reliability

Ongoing monitoring is required to remain vigilant to navigation situation changes that may lead to increase of collision risk. Vessels at sea may be in the position of being the stand on vessel or the give

way vessel. At no time should any vessel actually navigate its way into a collision. The rule is that the ship on the left must give way, [21], [23].
In efforts to quantify online changes in ships movements and their positions, we face data issues falling in two categories observation errors caused by lack of information according action planning by navigator and insufficient data because there is no information about navigators subjective level of risk acceptance. Hierarchical Bayesian analysis, provide the flexibility to deal with such complex data and a mechanism for the integration of multiple sources of error.
The area of observation can take any shape and take different values of the collision probabilities. It is important for decision making process to use the random map of potential collision hazards [6], [11].

Modeling collision risk area

With respect to ship-to-ship collisions, the three different collision scenarios should be examined separately namely, [28]:

- Head-on collision, in which two vessels collide on a straight leg of a fairway as a result of two-way traffic on the fairway;
- Collision, in which two vessels moving in an opposite direction on the same fairway collide on a turn of the fairway as a result of one of the vessels neglecting or missing the turn (error of omission) and thus coming into contact with the other vessel;
- Crossing collision, in which two vessels using different fairways collide at the fairway crossing.

The shape and dimensions of dynamic constrains of the ship domain, as a collision risk area, depends on assumed safety conditions.
In paper we assume that it is ellipse $E(x_i, y_i)$ whose parameters depend on the motion vector of the ship. In this area is determined two-dimensional cut Gaussian probability measure $f_i(x,y)$ which specifies the location of the ship s_i, by formulas

$$p(\mathbf{w}) = \frac{1}{2\pi |\boldsymbol{\Sigma}|^{1/2}} \cdot \exp\left[-\frac{1}{2}(\mathbf{w}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{w}-\boldsymbol{\mu}) \right] \quad (8)$$

$$f_i(x,y) = \frac{p(\mathbf{w})\chi_{E(x_i,y_i)}}{\iint_{E(x_i,y_i)} p(\mathbf{w})d\mathbf{w}}$$

where

- $\mathbf{w} = [x, y]$,
- d_i – distance to the ship s_i,
- $\chi_{E(x_i,y_i)}$ - indicator function of ellipse $E(x_i, y_i)$,
- φ_i – angle between axis OX and bearing of an ship s_i from the own ship,
- $\boldsymbol{\mu}$ is the mean vector, $\boldsymbol{\mu}_i = [d_i \cos(\varphi_i), d_i \sin(\varphi_i)]$,
- $\boldsymbol{\Sigma}$ is the covariance matrix, for the Gaussian density.

The general measure of all collision risk areas could be defined as,(8), if we take the same importance to each ship,

$$g(x,y) = \sum_{i=1}^{n} \frac{1}{n} \cdot f_i(x,y).$$

The assessment of the navigational situation is the subjective due to the navigator's relative risk attitude [29], [15]. Thus, we define the acceptable level of hazard as related to sufficient time (making and doing decisions) to avoid potential hazard situation between the own and target ships dependent on navigator's attitude.
If not all ships have the same level of importance, according to collision hazard, we can use the more general model such that:

- ***I*** – the set of own ships, ***I***={1,2,3,…, i},
- ***J*** – the set of target ships, $\boldsymbol{J} = \{1,2,3,\ldots, j\}$, where *j* is the number of target ships on the considered area,
- ***R*** **–** the set of danger type**,** $\boldsymbol{R} = \{1,2,3,\ldots,r\}$
- $a(i,r) \equiv a_i^r \in \Re^+$ is the number describing the *i*-th own ship's safety time needing according to collision hazard of *r*-th type,
- $b(j,r) \equiv b_j^r \in \Re^+$ is the number describing the j-th target ship's "danger supply" time of the collision hazard of r-th type,
- the $\mu: J \times R \longrightarrow \{0,1\}$ is describing the relation between the *j-th* target ships on the area and the *r-th* type of danger, $\mu(j,r) \equiv \mu_j^r = 1$ when *j-th* target ship is a risk source of *r-th* type, and the other hand $\mu_j^r = 0$;
- $M = [\mu_j^r]_{J\times R}$ – the matrix of *r-th* type of threatening objects from *j-th* target ship,
- $\Psi(j)$ - is discreet random variable

describing the time for "acceptable level of hazards" (*r-th* type) and for target ship *j* with distribution function (i.e. the random variable describing the sufficient time to avoid the potential hazard situation of *r-th* type with target ship *j*):

$$P(\Psi(j)=\psi(j,l,r))=p(j,l,r)=p_j^{l,r},$$

where

$$i\in I,\ j\in J,\ l=1,2,3,\ldots,a(i,r),\ r\in R,$$

$$\forall_{j\in J}\forall_{r\in R}\ p(j,l,r)\geq 0 \text{ and } \forall_{j\in J}\forall_{r\in R}\ \sum_{l\subset U_j^r} p(j,l,r)=1;$$

$$\forall_{i\in I}\forall_{r\in R}\ p(i,l,r)\geq 0$$

- $\lambda: I\times J\times R\rightarrow\Re^+$ - measure the effects of *r-th* type danger from *j-th* target ship for *i-th* own ship, where $\lambda(i,j,r)\equiv\lambda_{i,j}^r\in\Re^+$ is the number of the cost of effect, $i\in I$, …,
- $g: I\times J\rightarrow\Re^+$ - significance of the effects, where $g(i,j)\equiv g_i^j\in\Re^+$ is the number describing the the strength of interaction between *i-th* own ship and *j-th* target ship,
- $x: I\times J\times R\rightarrow\Re^+$ - the measure, where $x(i,j,r)\equiv x_{i,j}^r\in\Re^+$ is describing the time to the *r-th* type risk, when the *j-th* target ship is considered, for *i*-th own ship.

An estimator which minimizes the Bayes risk over all estimators, [6],

$$r_\pi(\partial)=\int R(s,\partial)\pi(s)ds$$

is called a generalized Bayes estimator with respect to a weighting function $\pi(s)$.

2.4. Bayesian estimation of the mean of a normally distributed variable with known variance

Let assume that the general form of a mixture distribution is , [20], [9]

$$f(x)=\sum_{k=1}^{c}p_k f_k(x;\theta_k)$$

where p_k is the probability that an observation will come from the *k*th component (the so-called *k*th mixing proportion), *c* is the number of components, $f_k(x;\theta_k)$ is the distribution of the *k*th component, and θ_k is the vector of parameters describing the *k*th. Note that the p_k must be non negative and sum to 1.

If we assume that sample of *n* data centre points of ships domains have arisen independently from a normal distribution with unit variance and unknown mean θ then the likelihood function for θ is, [7], [12], [31], [32]

$$l(\theta\mid x_1,\ldots,x_n)=(2\pi)^{-1/2}\exp\left[-\frac{1}{2}\sum(x_i-\theta)^2\right]$$

and $\hat{\theta}=\sum x_i/n$, the sample mean.

The maximum likelihood estimator of the variance σ^2 of a normal distribution $N(\mu,\sigma^2)$ is

$$1/n\sum(x_i-\bar{x})^2.$$

However this estimator is biased, [34]. It is common to use the unbiased estimator

$$1/(n-1)\sum(x_i-\bar{x})^2.$$

We believe a single data point *x* comes from a normal distribution with unknown mean θ and known variance α. Suppose our prior distribution for θ is $\theta\sim N(\theta_0,\alpha_0)$, with θ_0 and α_0 known. Then

$$p(\theta\mid x)=p(x\mid\theta)p(\theta)$$
$$=\exp\left\{-\frac{1}{2}\theta^2(1/\alpha_0+1/\alpha)+\theta(\theta_0/\alpha_0+x/\alpha)\right\}$$

If we consider the following reparameterisation.

$$\alpha_1=(\alpha_0^{-1}+\alpha^{-1})^{-1}$$

and

$$\theta_1=\alpha_1(\theta_0/\alpha_0+x/\alpha)$$

then

$$p(\theta\mid x)=\frac{1}{\sqrt{2\pi\alpha_1}}\exp\left\{-\frac{1}{2}(\theta-\theta_1)^2/\alpha_1\right\}$$

This is a normal distribution $N(\theta_1,\alpha_1)$. This means that the normal prior distribution has been updated to yield a normal posterior distribution. Given a normal prior for the mean, and data arising from a normal distribution we can obtain the posterior simply by

computing the updated parameters. Moreover the updating of the parameters is not chaotic.
When there are *n* data points, with the same situation as above, the posterior is again normal, now with updated parameter values

$$\alpha_1 = \left(1/\alpha_0 + n/\alpha\right)^{-1}$$

and

$$\theta_1 = \alpha_1\left(\theta_0/\alpha_0 + \bar{x}n/\alpha\right)$$

We wish to fit a normal mixture distribution

$$f(x) = \sum_{k=1}^{c} p_k f_k(x; \mu_k, \sigma_k)$$

where μ_k is the mean of the *i*th component and σ_k is the standard deviation of the *k*th component. Suppose for the moment that we knew the values of the μ_k and the σ_k. Then, for a given value of *x*, the probability that it arose from the *k*th class would be

$$P(k \mid x) = \frac{p_k f_k(x; \mu_k, \sigma_k)}{f(x)} \tag{9}$$

From this, we could then estimate the values of the p_k, μ_k and σ_k as

$$\hat{p}_k = \frac{1}{n}\sum_{i=1}^{n} P(k \mid x_i)$$

$$\hat{\mu}_k = \frac{1}{n\hat{p}_k}\sum_{i=1}^{n} P(k \mid x_i)x_i$$

$$\hat{\sigma}_k = \frac{1}{n\hat{p}_k}\sum_{i=1}^{n} P(k \mid x_i)(x_i - \hat{\mu}_k)^2$$

where the summations are over the *n* points in the data set.
This set of equations leads to an obvious iterative procedure. We pick starting values for the μ_k and σ_k, plug them into formula (9) to yield estimates $\hat{P}(k \mid x)$ and use it to obtain estimates $\hat{p}_k$, $\hat{\mu}_k$, $\hat{\sigma}_k$ and then iterate back using the updated estimates of μ_k and σ_k, cycling round until some convergence criterion has been satisfied.

3. The hazards map

There is a concept of the risk map based on two dimensional probabilities measures, [15], [24].
It is introduced for the navigational situation (i.e., number and location of ships and their vectors courses) which dynamically changes in time. We take into account the velocity vectors of all units in the relevant area.
Map of the hazards of collisions is closely linked to navigation situation (ie, number and location of ships and their vectors courses).
In this article we use two-dimensional multimodal density distributions which is a mixture of two-dimensional normal density functions.
Because of lack of information of ship course changes for each ship a fuzzy define collision domains should be described, [33], [24].
The kernel will be determined by a set of probability measures as two dimensional normal density functions (where (m_1,m_2) is equal to the position of a ship),
As the set centered for which the probability of finding the vessel is equal to 1-alpha. Where alpha is approved, an acceptable level of error arising from uncertain or incomplete information. In any case it is a function of S-type due to the diameters of the core domain.
Membership function depends on the degree of conformity of an individual's position at time t + dt calculated on the basis of information about an individual's position at the time t and the parameters of its motion vector.
The database will complement the area for which the probability of finding the unit will be beta (alpha> beta)
Risk map allows for prioritization of threats of conflict for individuals on supervised sea area. Adopted domain kernel fuzzy shape (elliptical, circular, polygonal) determines the analytic form of membership function. Membership function is dependent on the argument which is the length of the radius vector of a point. The verification of navigational situation at intervals of time dt, the length of which depends on the dynamics of navigational situation and the risk indices calculated in the previous step.
This requires the use Baeys approach where the distribution of positions of individuals at time t + dt and thus the risk map is determined a priori at the time that then at time t + dt verified a posterior on the basis of current information about the position and motion vectors of individuals.
Risk map allows for prioritization of threats of conflict for individuals on supervised waters. Adopted domain kernel fuzzy shape (elliptical,

circular, polygonal) determines the analytic form of membership function.
According to Rasch Model we can classified the ships into separate classes.
The Rasch Model (RM), due to the work of Rost, [1], [25], contains both, latent trait and latent class variables. We assume that the RM does not hold for the entire population of target ships, but does so within subpopulations of individuals which are not known before hand. The probability that the navigator at ship *i-th* react at collision situation with *j*-th correctly is:

$$P\left(X_{ij}=1\mid\theta_i,\phi_i,\beta_j\right)=\frac{\exp\left(\theta_i-\beta_{\phi_i j}\right)}{1+\exp\left(\theta_i-\beta_{\phi_i j}\right)},$$

where
θ_i - is the *i*-th ship's ability,
φ_i - indicates which latent group of the ship *i* belongs to,
$\beta_{\varphi ji}$ - denotes the situation *j*'s difficulty which depends on group variable φ.

Suppose there are G classes, number of classes is not less than 2, the unconditional probability that the ship *i* react at collision situation *j* correctly is:

$$P\left(X_{ij}=1\mid\theta_i,\beta_{\phi_i j}\right)=\sum_{g=1}^{G}\pi_g\frac{\exp\left(\theta_i-\beta_{\phi_i j}\right)}{1+\exp\left(\theta_i-\beta_{\phi_i j}\right)},$$

where
- πg - probability that the ship belongs to class g,
- $\Sigma_g \pi_g = 1$, and $0 < \pi_g < 1$,
- $\Sigma_j \beta_j = 0$ or $E(\theta) = 0$ for all classes.

The classification for each group is related to the preliminary assessment based on the type of unit and its technical and operating parameters. And then updated based on a factor of unpredictability of individual behaviour
The naive Bayes classification is a relatively simple method for classifying ships, according to collision hazard, based on the false assumption that all of the units, in this case ships in the analyzed area, are independent of each other. Even though this assumption is false, this model is done to achieve fundamental understanding concerning the effectiveness of the naive Bayes as compared to other methods, and to find a way of improving upon the performance of this classifier.
There have been many studies which have used the naive Bayesian classification method, and it was first used in a published paper in 1966 for a medical study on computer-assisted diagnosis. Hand and Yu in 2001 reviewed past uses of the naive Bayesian method for classification. Using theoretical and real data situations, they showed that the naive Bayes is not an excessively inaccurate method because of its false assumption that all of the variables, which in my project are occurrences of words, are independent.
The Naive Bayesian Classifiers could help as to answer the question which classification is the most probable for this new instance if we have a look at the training data.

Example 2
An instance of a ship could be (course, size, change of course, type of domain). An Naive Bayes System could calculate values for the following two classifications "collision hazard" and "no collision hazard" according to the available training data.
Using formulas (2), (3) we obtain

$$\begin{aligned} v &= \max_{v_j\in V} P(v_j \mid a_1, a_2 \ldots a_n) \\ &= \max_{v_j\in V} \frac{P(a_1, a_2 \ldots a_n \mid v_j) P(v_j)}{P(a_1, a_2 \ldots a_n)} \\ &= \max_{v_j\in V} \prod_i P(a_i \mid v_j) P(v_j) \end{aligned}$$

where
- X is a set of instances $(a_1, a_2, \ldots, a_n)$
- V is a set of classifications v_j, j=1,2

Naive Bayes assumption leads to the following algorithm:
- for each target value v_j estimate $P(v_j)$
- for each attribute value a_i of each attribute a estimate $P(a_i \mid v_j)$
- $v = \max_{vj\in V} P(v_j) \prod_{a_i\in x} P(a_i \mid v_j)$.

3.1. Hierarchical Bayes model

In a Bayesian approach, our two main sources of information about parameters of interest (θ) are our prior beliefs or the prior distribution of the parameter (Pr[θ]) and the likelihood of observing the data given our expectations about the parameter (Pr[y|θ]), [2], [27].

$$Pr[\theta|y] = Pr[y|\theta] * Pr[\theta],$$

Choice of the prior distribution is critical as it essentially indicates how we believe the parameter would behave if we had not sufficient information to make the decision. The posterior distribution is a combination of two probabilities.

In a hierarchical or mixed Bayesian model we specify a distribution for how risk is distributed across a group of individuals and also varies across higher levels of organization by specifying an additional set of parameters. It can help account for irregular groupings and autocorrelation.
In describing the likelihood, the risk for each area is transformed to a log scale (making relationships additive rather than multiplicative) and is set equal to an intercept term and two random effects, one non-spatial the other spatial, [13], [2].
The spatially structured component is described as a conditional autoregressive Gaussian process where the conditional distribution of each parameter is normal. We can use the matrix of neighbors that defines the neighborhood structure. The non-spatial component of the model is defined at normally distributed.
We can use Bayesian network to assess the risk indices for individual units, and calculate the safety level of conflict in a given sea area, [27]. A Bayesian network is an acyclic directed graph consisting of encoding a domain's variables (nodes) , the probabilistic influences among them (arcs) and the joint probability distribution over these variables.

3.2. Bayes and MCMC

The Bayesian approach implies the calculation of complicated multidimensional integrals. A class of numerical procedures, such as Markov Chain Monte Carlo (MCMC) techniques, were revolutionalized the Bayesian approach because the integral is approximated by Monte-Carlo sampling, [26]. There are two major classes of MCMC techniques: Gibbs sampling and Metropolis-Hastings sampling, [30].

Simulation approach

The classification for each group is related to the preliminary assessment based on the type of unit and its technical and operating parameters. And then updated based on a factor of unpredictability of individual behavior.
As for how to approach the information is represented by a multidimensional time series, prediction by the naive method.

General algorithm

1. The calculation of collision risk index to individuals at time t.
2. Determination of the risk index. The weighted average risk indices with the weights assigned based on relative increases in risk indices (see the time series).
3. The classification of the situation to the appropriate category of supervision (appointment time step).
4. The calculation of risk indices for the next collision time t + dt, the distribution of apriori.
5. Verification at the time t + dt hazard indices based on updated information about the navigational situation.
6. Determination of membership function parameters for each unit based on the compatibility factor priori and aposteriori distribution, location of the individual.
7. Go to Step 2 or alarm about the threat.

4. Conclusion

The essence of The Empirical Bayes approach is to use two types of information that must be taken into account for estimating the safety of ships at analyzed area. The first type of information is found in navigational situation and weather conditions. The second type of information is derived from the history of collision occurrences.
The Bayesian procedures is used to obtain as accurate a posterior distribution as possible, and then use this distribution to calculate risk measures or failure rate (probability characteristic) estimates with credibility intervals.
Advantages of using Bayes Methodology**:**
- uses prior information,
- less new testing may be needed to confirm a desired probability characteristic at a given confidence,
- confidence intervals are intervals for the (random) probability characteristic ,

Disadvantages of using Bayes Methodology:
- prior information may not be accurate so choice of prior may not be correct,
- not acceptance validity of prior data or expert judgments,
- different approaches can give different results.

The navigational conditions are relevant for collision scenarios. The presented method is able to take into account arbitrary navigational conditions. The ship traffic is divided into traffic streams which help as to group ships by type according to collision hazard. There is possible to use presented methods and models for calculating the ship safety in different navigational situation.

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