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INVESTIGATION OF THE DYNAMICS OF SINGULAR PROTECTED SYSTEMS

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Abstract. The analysis of asymptotic representations of the systems protected from harmful influences is carried out. Various types of general models of the "man-machine-environment" with protection are considered. Each of them adequately describes some of the practically important qualities of the object, and they all together describe the object in terms of its safe operation. The dynamic properties of complex ergonomic systems, presented in the form of systems of differential equations with a small parameter at the derivative are investigated The methods of reducing the impact on the person of harmful factors are theoretically substantiated. The dynamic protection response speed is considered to be significantly greater than the harmful factor production rate.

Numerical solution of the general problem and the analytical solution for autonomous case is obtained for harmful effects. By using asymptotic the system of equations has been solved in closed form not only for autonomous case, but also for parameters smoothly changing in time. The estimates of the cost of protection was obtained for the various cost-functionals and for different functions in the right-hand side of the equation describing the dynamics of protection. To assess the accuracy of model calculations and for graphic representation of the results mathematical package MAXIMA is used.

Key words: "Man-Machine-Environment" model, non-linear system, singular equations, asymptotic, linearization.

INTRODUCTION

It is known [1], that safety and efficiency are conflicting criteria, because they compete for the same resources. Their union in the single criterion is possible only in the super-system [1, 2]. This approach allowed us to consider the model of the "man-machine-environment-protection" as a well-known general model of competition of two factors – the safety and efficacy [3].

It runs a large number of processes with different time scales. The hierarchy of these times is such that they differ by many orders of magnitude [4]. Usually, various problems of physics and engineering are modeled by means of differential or algebraic equations. And almost always it turns out that they have a high order, and when it comes to systems, they are of large dimensions. To overcome this problem the two diametrically opposite approaches are known.

The essence of the first lies in the fact that if their characteristic elements are similar in the system, they can be considered equal in a first approximation. And we use that symmetry, considering small deviations in the subsequent approximations.

The second is used when the individual elements of the system are very different in their characteristics. In this case, we introduce small parameters representing their attitude and conduct an asymptotic reduction of dimension, i.e., reducing the number of degrees of freedom. The use of asymptotic methods are not always stipulated specifically, and sometimes not even realized in modelling. So, in engineering practice it is extremely widespread to model systems with one degree of freedom. It is clear that the use of such models involves an asymptotic reduction of dimension. If the system in question consists of sets of similar elements, the asymptotic approach does not lead to a reduction of dimension, but rather to increase it. This method is applied to a very important class of models in which discrete systems are replaced by continuous, that is, as in our case, a system of differential equations. As a rule, the solution based on the asymptotic method cannot be expressed in a finite form, but only with the help of some series [4]. It turns out that the perturbation series are not necessarily converge. For example, it often happens that you can use the infinite series that diverges, but have the value in a certain sense. A typical situation is as follows: a function can be expanded in a series of functions and approximation, given by the first few terms of the series. It serves the better, the closer is the independent variable, or a parameter to a certain limit value. In many cases, the values of terms at first decrease rapidly, but then again begin to increase. In mathematical literature such series are called asymptotic series [5].

Here we first consider the overall system "Man-Machine-Environment" under this approach. The input to this system is the information from the superior system (targets, instructions, etc.); the exit of such system is the result of work and a lot of other factors that are harmful to the environment and personnel.

In operation the system changes its internal state. The element "man" has three functional parts: the part, that controls the "machine"; the object of the external environment and the impact of an object from the "machine".

"Machine" element performs basic technological functions: impact on the subject of work and change the parameters of the environment.

The paper discusses different types of common models of "human-machine environment", each of which adequately describes some of the practically important quality of the object, and all together they describe the object in terms of its safe operation [6]. We get further details and results in the well-known and, as well as, some new models of subsystems [7]. This work is devoted to the quantitative analysis of an important model of a system with protection of a human from the harmful effects of the external environment and the impact of sub-system called the "machine".

THE ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

In works [8, 9] a model of dynamic system describing the situation where primary subsystem "produces" a harmful factor, and second sub-system called “protection”, tries to reduce it completely, or at a reasonable price. As the base model – the basis for modification – a system of ordinary differential equations was taken. It describes fundamental laws of competition [10], and also known in ecology as a model of coexistence of species [11 – 15].

We first introduce the basic assumptions, directly following from everyday experience. They are evident so, do not require additional justification but only need to be formalized. A more detailed discussion of these issues are delivered in [8].

We call Bioinfluence U of the harmful factor an increasing function of time t and the intensity of the factor u. In the first approximation, it can be written as an integral

$$U = \int_0^T u\ (t)dt \cdot$$

It also fits the additive property [6].

The following axiom are true:

a) autocumulative;

b) mutual cumulative;

c) intensity of bioinfluence u fits:

– in a regular situation $\frac{\partial}{\partial t}u \le 0$;

– in critical situations (positive feedback) $\frac{\partial}{\partial t}u > 0$.

Protection factor $z(t)$ may be controlled adaptively or programmatically, depending on the value of $u(t)$.

The cost of protecting C=$C(z)$ is natural to consider as a monotonically increasing function of its intensity.

In [6-8], we have conducted a formal description of the model that is under study here.

Suppose there are two types of internal system states: production factors (including the production of harmful substances) U and the impact of protective factors Z. Let f and g be smooth functions, monotonically increasing in both arguments, such that $f(0,Z) = g(U,0) = 0,\ \forall U,Z$. Then it is natural to consider that $U' = f(U,Z)$; $Z' = g(U,Z)$. This is the most common model of dynamics of the system with protection. However, to obtain meaningful results, it must be detailed.

Suffice general case of such a model of the system can be represented as:

$$\begin{cases} u'(t) = \alpha u(t) - \beta z(t)u(t) \\ z'(t) = F(u(t), z(t)) \end{cases}, \qquad (1)$$

with the constrains $u \ge 0,\ z \ge z_c$, where z_c is fixed (stationary) protection value.

The function $F(u, z)$ can take quite arbitrary form [8]. The most common of them are the following three:

1) $F(u(t),\ z(t)) = \gamma u(t)$;

2) $F(u(t), z(t)) = \gamma u(t) - \delta z(t)$;

3) $F(u(t), z(t)) = \gamma_1 u(t) + \gamma_2 u^2(t) - \delta_1 z(t) - \delta_2 z^2(t)$.

Solution of the system of differential equations (1) is not always possible to find analytically. Therefore, to find protection functions and the harmful effects some numerical methods for solving systems of differential equations are used. For the system (1) it is necessary to investigate the stability at different values of the parameters of the protection subsystem

$$F(u(t),\ z(t)).$$

It is also necessary to evaluate the cost of protection for different functions F. In [8] (1) is assumed to be autonomous, and the bifurcation parameters are not dependent on time.

OBJECTIVES

In this paper, in contrast to [8], it is assumed that the parameters of equation (1) depends on the time and takes into account the effect of "boundary layer" [5] near $t = 0$.

Based on the results obtained in the course of the study, an analysis of bifurcations for protection is made, ie, we find a scenario of possible loss of stability [16] and the effectiveness of protection.

THE MAIN RESULTS OF THE RESEARCH

1. Methods for studying the stability of models

The linearization theorem establishes a connection of the phase portrait of the nonlinear system (1) in the neighborhood of a fixed point with the phase portrait of its linearization [16, 17].

In general, if a nonlinear system $\dot{y} = Y(y)$ has a simple fixed point $y = 0$, then in the neighborhood of the origin, the phase portraits of this system and its linearization qualitatively equivalent, unless a fixed point of the linearized system is not the center [17].

Application of the theorem on linearization similarly is considered in the analysis of environmental models just the same way as for competition in economic systems [18-20]. We conclude that the studied in this paper system has a stationary point (0, 0) of the "saddle" type.

2. The problem of fast and slow variables.

Dynamical systems include a large number of processes with different time scales. Given the time hierarchy process reduces the number of differential equations. "Very slow" variables do not change on time scales of these processes and can be regarded as constant parameters. For "fast" variables there can be written algebraic equations for their steady-state values instead of differential equations. The "fast" variables reach their stationary values almost instantly if compared to the "slow" [18]. This difference leads to a singularity by parameter in the second of the equations (1). Note that the asymptotic solution itself obtained in [8], is singularly in *t*. Because of this, the protection features and hazards for the second term of the asymptotic approximation have the form [8]:

$$z(t)=\frac{1}{\beta t}+\varepsilon\frac{1}{2\beta}(\alpha-\frac{2\ln t}{t^2\delta})+\varepsilon^2\frac{1}{12\beta\delta^2t^3}(36-$$
$$-6\alpha\delta t^2+\alpha^2\delta^2t^4-12\ln t+12\ln^2 t);$$
$$u(t)=\frac{\delta}{\beta\gamma t}+\varepsilon\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta-2\ln t-2)+\varepsilon^2\frac{1}{12\beta\delta\gamma t^3}\times$$
$$\times(24-6\alpha\delta t^2+\alpha^2\delta^2t^4+12\ln t+12\ln^2 t).$$

Therefore, the solution obtained in [8] was adequate only far from the starting point. In this paper we use a generalized asymptotic representation, which takes into account the effect of "boundary layer" in the vicinity of the starting point.

3. Research Algorithm

We use an iterative algorithm with the following three steps.

A) We find, if possible, an analytical solution of the system (1) using the functions included in the standard MAXIMA package, which is distributed on the basis of General Public License. If a solution cannot be found in a general way, then we use numerical methods (in the default package there used sufficiently universal method of Adams [19, 21]) or the asymptotic method proposed below.

B) After a solution of (1) was found, analyze the behavior of the hazard function, at what times, if any, its values exceed its protection features, that is a system of dynamic protection comes to operation.

By finding these intervals we decide:

- to increase the protection against harmful factors that lead to an increased cost of the protection system;
- leave the system without modification;
- if the intensity of harmful factors does not exceed an opportunity of fixed protection, the overall cost of the protection system can be reduced by reducing both fixed and dynamic protection.

C) Selecting the solution and repeat the steps A) - C) until you go beyond the limitations (the time of the system work or its value).

4. Analytical study of the model

Consider a system of differential equations (1) with a small parameter ε:

$$\begin{cases}u'(t)=\varepsilon\alpha u(t)-\beta z(t)u(t)\\ \varepsilon z'(t)=\gamma u(t)-\delta z(t)\end{cases}. \qquad (2)$$

The difference of this system from the previously considered is a quasi-stationary harm $u(t)$. Let us solve the system (2) with the asymptotic method by finding a series of terms with $\varepsilon^0,\varepsilon^1,\varepsilon^2$.

To begin, write out the system (2), taking into account the dependence of the functions $u(t,\varepsilon)$ and $z(t,\varepsilon)$ in time and small parameter.

We solve the system (2) for the main asymptotic term - with ε^0 (zero approach).

Write the asymptotic for the functions $u(t,\varepsilon)$ and $z(t,\varepsilon)$ as follows:

$$u(t,\varepsilon)=u_0(t)+O(\varepsilon),\quad z(t,\varepsilon)=z_0(t)+O(\varepsilon).$$

The system (2) for the zero-order approximation takes the form :

$$\begin{cases}u_0'(t)=-\beta u_0(t)z_0(t)\\ 0=\gamma u_0(t)-\delta z_0(t)\end{cases}. \qquad (3)$$

After substitution $u_0(t)=\frac{\delta}{\gamma}z_0(t)$, with stationary protection $z_c=0$ we get:

$$z_0'(t)=-\beta z_0^2(t).\quad z_0(t)=\frac{1}{\beta t},\quad u_0(t)=\frac{\delta}{\beta\gamma t}.$$

The resulting singularity at t=0 indicates the impossibility of such a solution for the Cauchy problem at t_0 =0.

Protection functions and hazard at zero approximation for $z_c>0$ have the form:

$$z(t)=\frac{1}{\beta t+1/z_c},\quad u(t)=\frac{\delta}{\gamma(\beta t+1/z_c)}.$$

Similarly, we solve the system (2) with regard to the term ε^1.

So, we write asymptotic for $u(t,\varepsilon)$ and $z(t,\varepsilon)$. The system (2) for the first approximation takes the form:

$$\begin{cases}u_0'(t)+\varepsilon u_1'(t)=\varepsilon a u_0(t)-\beta u_0(t)z_0(t)-\\ -\varepsilon\beta(u_1(t)z_0(t)+u_0(t)z_1(t))\\ \varepsilon z_0'(t)=\gamma u_0(t)+\varepsilon\gamma u_1(t)-\delta z_0(t)-\varepsilon\delta z_1(t)\end{cases}. \qquad (4)$$

The terms with the factor ε in powers of 2 and higher form the remainder term $O(\varepsilon^2)$.

The obtained system for $z_1(t)$ and $u_1(t)$ has the form:

$$\begin{cases} u_1'(t) = \alpha\dfrac{\delta}{\beta\gamma t} - \beta(u_1(t)\dfrac{1}{\beta t} + \dfrac{\delta}{\beta\gamma t}z_1(t)) \\ -\dfrac{1}{\beta t^2} = \gamma u_1(t) - \delta z_1(t) \end{cases} \qquad (5)$$

After replacement $u_1(t) = \dfrac{1}{\gamma}(\delta z_1(t) - \dfrac{1}{\beta t^2})$ we solve the equation:

$$\frac{1}{\gamma}(\delta z_1'(t) + \frac{2}{\beta t^3}) = \alpha\frac{\delta}{\beta\gamma t} - $$
$$-\beta(\frac{1}{\gamma}(\delta\, z_1(t) - \frac{1}{\beta t^2})\frac{1}{\beta t} + \frac{\delta}{\beta\gamma t}z_1(t)).$$

As a result, the function $z_1(t)$ is found and, with its help, also the function $u_1(t)$, that are the first terms of the asymptotics.

$$z_1(t) = \frac{1}{2\beta}(\alpha - \frac{2\ln t}{t^2\delta}),$$
$$u_1(t) = \frac{1}{2\beta\gamma t^2}(t^2\alpha\delta - 2\ln t - 2).$$

Then the resulting functions of protection and hazard intensities for the first approximation take the form :

$$z(t) = \frac{1}{\beta t} + \varepsilon\frac{1}{2\beta}(\alpha - \frac{2\ln t}{t^2\delta}),$$
$$u(t) = \frac{\delta}{\beta\gamma t} + \varepsilon\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta - 2\ln t - 2).$$

Similarly to [8] a decision based on the second term of the asymptotics is given above.

5. Selection of the coefficients and the number of expansion terms

We solve the system (2) with the asymptotic method for ε^0, ε^1. It will be shown that, in this study, the first two members of the series will be sufficient to obtain a good approximation.

We present an algorithm for constructing an asymptotic solution of the problem (2) is similar to [18], under the assumption that the function on the right side is sufficiently smooth. We will look for it in the form of the asymptotic expansion

$$z(t,\varepsilon) = \bar{z}(t,\varepsilon) + \Pi z(\tau_0,\varepsilon)$$

where $\bar{z}(t,\varepsilon) = \bar{z}_0(t) + \varepsilon\bar{z}_1(t)\ldots$ is the so-called regular series:

$$\Pi z(\tau,\varepsilon) = \Pi_0 z(\tau) + \varepsilon\Pi_1 z(\tau) + \ldots,$$

that describes boundary layer in the neighborhood of t=0 ($\tau = t/\varepsilon$).

We choose the coefficients of system (2) from the physical meaning of the problem :

α=0.5, β=5, γ=2, δ=1, ε=0.0001.

Also, we define the initial conditions at t=0 for numerical and asymptotic solutions as $z_c = z^0 < u^0$, because the system has to emerge from the fixed protection value. Let z^0=2, u^0=3 and T <10 – the time interval for the system. In the first approach the parameters of the system (2) are constant.

6. The asymptotic solution of the problem

We show that for $\beta \in (0, 10)$, the asymptotic solution built for the first two terms of the expansion, is little different from a sufficiently accurate numerical one.

Using the procedure described in [8, 22, 23], we obtain the graph of the solution of problem (2). The results are shown in the figures below. For a better representation of the system behavior near the boundary layer, we draw the schedule not on the whole range of T, but only at the beginning of its section .

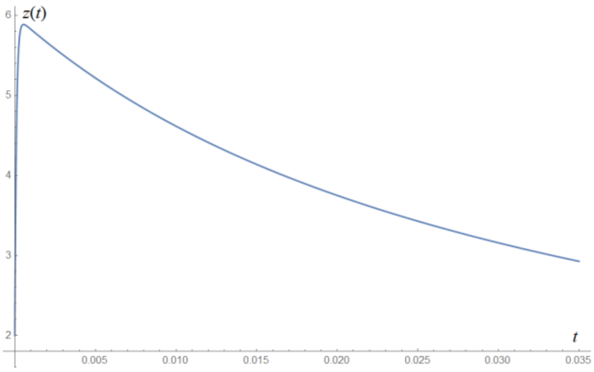

Fig. 1. The schedule of the first approximation (the first two terms of the series) and regular part of the asymptotic behavior at the boundary layer for protection function $z(t, \varepsilon)$

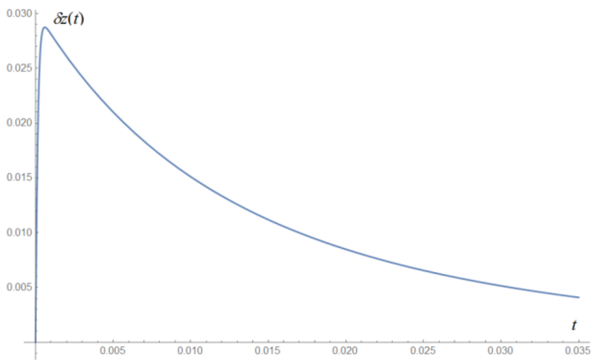

Fig. 2. Schedule of regular member of the zero error and the border of the asymptotic solutions for the protection function with respect to the numerical one

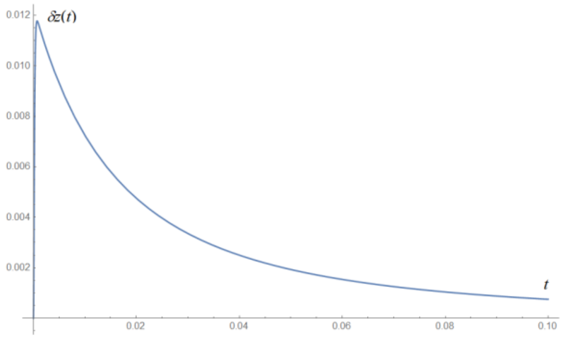

Fig. 3. Graph of error of the first approximation of regular and border of the asymptotic solutions for the protection function with respect to its numerical solution.

As can be seen from the graphs, the error decreases with increasing number of terms in the expansion in powers of *ε*.

7. Estimates of the cost of protection

We use the function

$$C(T)=\int_0^T c(z-z_0)dt+C_0\ , \qquad (6)$$

where: C_0 – the cost of fixed protection; z_0 – the value of fixed protection; $c(z)$ – cost function, which can take the form of a), b) and c) below.

We integrate, taking T=6.5 (the time during which the necessary protection of the system will take a value less than z_0) and write down the results:

а) $c(z)=z$, C=1270;

б) $c(z)=z^2$, C=1744;

в) $c(z)=z\ln z$, C=1421.

A disadvantage of the cost function (6) is that the formula did not account for the protection increases with the increasing reaction rate *β*. Therefore it is suggested the following clarification:

$$c(z)=\int_0^t\left(z(\tau)-z_c+K\max\left[0,z'(\tau)\right]\right)d\tau+C_0, \qquad (7)$$

wherein the coefficient K is selected from considerations of the reaction speed value contribution in the total cost. In the experiments K was chosen, such that the contribution rate of charge and other factors was equivalent ($K=0.01$). But as *β* increases, the integration period is reduced without limit together with the integral value. This does not fit the actual conditions.

Therefore the task of optimizing the cost function that depends not on a one-time operation of the system but also on all the contingencies that can happen for the entire life span of the system as well as the actual cost of purchasing the system.

Then the cost function takes the form:

$$S(\beta,\varepsilon)=n\cdot c\big(z(t)\big)+\varphi(\varepsilon,\beta), \qquad (8)$$

where: $c(z(t))$ is a function (7); $\phi(\varepsilon,\beta)$ is a function of the purchase price of protection system.

A coefficient n is numerically equal to the number of emergency situations in which the protection system goes from a stationary mode and for each adverse factor It is calculated using the formula $n = T \times N$, where T is the average life-term of the protection system, N – the average number of emergencies in a year.

This makes it possible to calculate the minimum cost of the whole system and say what speed parameters β and ε we need to buy it. The problem is reduced to one-dimensional optimization of $S(\beta,\varepsilon)$ for small ε. Here is an example for $n = 10$. The function in (8) is chosen in the form:

$$\varphi(\varepsilon,\beta)=\frac{\beta}{2\varepsilon}.$$

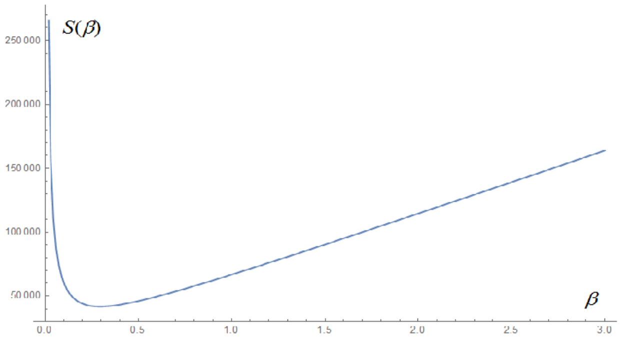

Fig. 4. Schedule of value S on the parameter β at $n=10$

The minimum is achieved when $\beta\approx 0.294$ and the value of the cost function $S(\beta)\approx 41792$.

CONCLUSIONS

In this paper there are first obtained or improved the following results and methods:

1. For the first time there proposed a dynamic model of the system with protection from harmful factors taking into account the great difference in order of specific operating times and speeds of the subsystems.

2. For the “singular” differential equations of this model there improved and applied the method of asymptotic expansion in small parameter of the solution taking in view of the phenomenon of boundary layer.

3. The method [8] got its further development, which allowed to determine the total cost, depended on the intensity of dynamic protection functions. It uses previously obtained analytical expressions and the variety of cost functions for the specific cases of protection.

The paper also proposes for practical application the approach that saves the total cost of the protection [6]. For this purpose were studied times and the system states when the intensity of harmful factor *u(t)* does not exceed the threshold of dynamic protection action and hence the value z_0 of static protection may be redundant.

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