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APPLICATION OF THE MAXIMA DISTRIBUTION TO THE EVALUATION OF THE VARIABILITY OF FLOOD RISKS IN THE ODRA RIVER BASIN ON THE BASIS OF QUARTERLY MAXIMA OF DAILY WATER LEVELS

The method of assessing the flood risk variability has been presented based on maximum value distributions. Hydrological data were used in the form of daily water levels for the period 1981–2013. The collected data originate from the Malczyce Hydrological Station on the 300th km of the Odra River. To assess the risk of flooding based on the collected data quarterly highs were selected. As a measure of flood risk, the authors suggested the probability of exceeding the emergency level in the studied section of the river. This risk was calculated using the theoretical distribution function of the quarterly heights of the water level. The study used the Gumbel distribution. A special attention was paid to the possibility of using the presented solutions for an integrated flood risk management process in accordance with applicable national and European legislation.

1. INTRODUCTION

For several years, we have witnessed catastrophic floods destroying Europe and, consequently, Poland. They contribute to social, economic and cultural losses. Long-term forecasts force the authorities of the EU countries to prepare for the strengthening of these phenomena. In response to this question, the European Union took a number of measures to prepare the member states to combat floods. Given this, preventive measures that are economically and socially justified have been transposed into EU legislation in the field of water management [1] and the Flood Risk Management Directive [2]. Poland, as

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a member state of the European Union and a country that often suffers from the effects of floods, has transferred these provisions to its national legislation, i.e., the Water Law. One of the regions most affected by flooding over the past few years has been the territory of the Odra River Basin.

The overall objective of this article is to present flood risk assessment methods based on extreme distributions of selected hydrological characteristics. In addition, the authors set a goal to study the variability of flood risk using the above methods using the example of a selected location in the Mid Odra River Water Region. An additional objective of this article is to present the possibilities of applying the presented methods of flood risk assessment as support for the widely understood process of flood risk management, which is an integral part of the implementation of EU legislation [3, 4].

2. MATERIAL AND METHODS

The main objective of the project is to assess the variability of flood risk using the maximum value distributions, chosen paradigm and method of research, which will be implemented, are described below.

Choice of theoretical background. Based on numerous scientific publications and reliable data from many studies, the authors adopted theories and information from such sources to carry out the dynamics of flood risks for a selected area of the Odra River Basin to calculate the risk using theoretical distribution function of the quarterly water level maxima.

Choice of the method. The data collected via desk research included many results of scientific research in a field of water flood and environmental management for the years 1981–2013. The project is based on quantitative research which supplies figures and data.

3. GENERAL CHARACTERISTICS OF FLOOD EVENTS IN THE ODRA RIVER BASIN WITH SPECIAL EMPHASIS ON THE MID ODRA RIVER WATER REGION

The Odra River Basin covers a total area of 118 861 km², 118 015 km² of which located in Poland, which is 38% of the country. The territory of the basin covers the south-western, western and north-western parts of Poland and from an administrative perspective, it is located in provinces of Silesia, Opole, Łódź, Lower Silesia and Kujawy-Pomerania, Wielkopolska, Lubuskie, West Pomerania and Pomerania [5].

Based on the analysis of historical floods, it should be noted that in the area of the Odra River basin floods occur mainly in the summer (from May to October). The main cause of river floods in the Upper and Mid Odra River Water Region was extensive precipitation which caused the largest floods. Thaws occurred in the water areas of Warta River, the Lower Odra River and the West Seaside, especially in the tributaries of the major rivers of the region much more often than in the Upper and Middle Odra River Water Regions. Stormy rains contributed to the formation of the so-called flash floods, especially in mountain tributaries of larger rivers, causing high damages and frequent fatalities. Thaws and embolic floods appeared often in the water regions of Warta River and Central Odra River. In the case of water region of Lower Odra River and the Western Coast, flooding from the sea (the phenomenon of a backwater) and embolic fluvial floods often occurred. Some of the biggest floods in the Odra River basin occurred in the years: 1903, 1979, 1997 and 2010.

In the water region of the Central Odra River existed a high level of integrated flood risk in the watershed area of the Odra River from Gliwice Canal to the Lusatian Neisse River and the watersheds of the Bóbr, the Nysa Kłodzka and the Kaczawa Rivers.

4. FLOOD RISK MANAGEMENT. AN OPERATIONAL TOOL TO REDUCE THE SOCIO-ECONOMIC CONSEQUENCES

Throughout the Odra River basin, the problem is the incoherent system of hydrological and meteorological coverage, especially the one that dedicated to the watershed areas with high sensitivity to the flood risk, serving the public with a forecasting and warning function of an impending threat. In addition, in terms of planning, preparing and conducting rescue operations, as well as reconstructing activities after flood damage, an inefficient system related to flood risk management has been identified. The main problem concerning Poland is insufficient social awareness of the flood risk and insufficient knowledge of flood risk reduction methods at the preparatory stage, during a flood and post flood repairs.

The main strategic objective of the flood risk management is to limit and to inhibit the growth in the number of negative consequences of flood on human health and life, the environment, cultural heritage and economic activities, primarily through a series of non-technical measures aimed at reducing the flood risk, and decreasing the vulnerability of special flood risk areas as well as measures to strengthen all elements of flood risk management.

The flood risk management plan for the Odra River basin [5] describes the basics of effective flood risk management. Conclusions drawn from the prepared plan will also serve as the basis for creating a catalog of good practices in the field of flood protection and affect the development of the industry, the future property management structure, as well as investment methodology and support for prioritizing actions in the form of a catalog of legal, economic and educational tools [6–8].

According to the Article 10, paragraph 1 of the Directive 2007/60/EC of the European Parliament and of the Council of 23.10.2007 [2] on the assessment and management of flood risk (Official Journal of the European Union L. 288/27 of 6.11.2007), member states shall make public the preliminary flood risk assessment, flood hazard maps, flood risk maps and flood risk management plans. Considering paragraph 2 of Article 10 of the Floods Directive requires encouraging stakeholders to actively participate in the development, review and updating of the flood risk management plans [2].

In turn, the Polish legislator in Article 119, paragraph 3a of the Water Law dated 07.18.2001 [9] imposed on the President of the National Water Management Authority an obligation to ensure public participation in the preparation or updating of the flood risk management plans in the basin on the basis and in order specified in the Act of 3 October 2008 on the provision of information about the environment and its protection, public participation in environmental protection and environmental impact assessments [10]. The numerical map of flood hazard and a flood risk map were used for the analysis of the spatial distribution of the flood hazards as well as for the analysis of risk and losses from floods. The analysis was performed with 94 rivers. For the Odra River basin area, Świna River, Szczecin Lagoon and 6 coastal stretches.

It should be emphasized that the social, environmental and economic costs caused by flood events are very high, and the costs of counteracting the potential effects of floods are probably less, hence one of the basic environmental protection principles, i.e., the precautionary principle has a logical application not only in the economic reality but also in a wide range of environmental risk management and the implementation of these risk limitation methods. The passage of two flood waves on the Odra River brought losses to the local authorities of Lower Silesia in the amount of more than 33.5 million EUR. The province's infrastructure suffered the largest losses. According to preliminary estimates, the cost of repairing roads, sewer and hydrological equipment could even reach 30.7 million €.

The methods for estimating the flood risk presented in this article constitute an applicable tool that effectively supports the process of flood risk management and limits potential consequences. This tool for estimating flood risks in the studied areas is the perfect complement to the previously used methods of forecasting and the research results presented later in this article show the dynamic nature of the flood risk within the adopted periodical scope of the research.

5. MODELS OF EXTREME VALUES. SELECTED ISSUES

5.1. APPLICATIONS OF EXTREME VALUE THEORY IN HYDROLOGY AND RELATED FIELDS

A very rich and comprehensive bibliography on the theory of extreme value distributions and their applications consisted of more than 1100 positions at the beginning of the 21st century. Such extensive literature indicates a great interest in this field of science as well as its widespread use. Therefore, in this chapter, only selected elements will be presented that according to the authors, have had a significant impact on the development of the theory and that is closely related to the issues raised in this article.

Yurtal et al. compared the method of maximum likelihood to weighted method of moments for estimating the parameters of hydrological data distributions probability obtained from measuring stations on the Ceyhan river in southern Turkey [11]. Nachabe and Paynter conducted research using the generalized distribution of extreme values on hydrological data from the selected lakes in the south-west of Florida [12].

5.2. EMPIRICAL DISTRIBUTION VISUALIZATION TECHNIQUES

An empirical distribution function $\hat{F}_n(x)$ will be used to graphically represent the empirical distribution, a one-dimensional random variable X for data $x_1, ..., x_n,$ and x is the number x_i that is less than or equal to 0. Hence,

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i \le n} I(x_i \le x) \tag{1}$$

where in the indicator function is defined by $I(x_i \le x) = 1$ when $y \le x$ and 0 otherwise. Empirical distribution functions are particularly useful for the presentation of samples with small numbers.

Data from the sample $x_1, ..., x_n$ ordered from smallest to largest are marked by

$$x_{1:n} \le \cdots \le x_{n:n}$$

We get $\hat{F}_n\left(x_{i:n}\right) = i/n$ when $x_{i:n}$ is not a point repeating many times in the sample. It should be noted also that $\hat{F}_n$ is constant between consecutive ordered values of the variable. This article will deal mainly with situations where each observation x_i is generated from a population sharing a common distribution function F and empirical distribution function $\hat{F}_n$ is approximately equal to the theoretical distribution function F [13].

5.3. MAXIMA

We assume that the y_i observations are the maxima, i.e., that,

$$y_i = \max\left\{x_{i1}, ..., x_{im}\right\}, \quad i = 1, ..., n \tag{2}$$

where x_{ij} may not be observable. In the case where x_{ij} are observable, the selection of certain maxima from certain sets with m number of elements is a form of a selection of the upper extreme values from a data set. This method is called the block method or the Gumbel method [13].

The block maxima method requires defining the time horizon (the block) and calculating the maxima of the tested variable for the specified horizon. The most commonly used blocks of one year, half a year, quarter, month or smaller size depending on the research needs. For data in the form of hydrometric parameters blocks of the mentioned above size are used. The block size cannot be too small to prevent the occurrence of the relationship between the maximum values of the adjacent blocks of time. The 10 day period is considered the minimum limit value of the time block size for which the independence of neighboring maxima can be accepted [14].

At this point, namely one more fact deserves attention, that the observations y_i are the embodiments of the random variable M_m defined by the formula:

$$M_m = \max\left\{X_1, \ldots, X_m\right\} \tag{3}$$

5.4. PROBABILISTIC MODELS OF EXTREME VALUES

In the analysis of hydrometric data maxima, it is suggested to apply first the Gumbel distribution, which is one of the three types of extreme value distributions [15]. The IACWD (US Interagency Advisory Committee on Water Data – Hydrology Subcommittee) [16] report recommends the Persona III distribution with the log-normal transformation for long-term data to predict flood events as well as the log-normal distribution.

According to the statement on the types of extreme values distributions [17], the distributions of extreme values are one of three data distributions by the following formulas:

- Gumbel (EV0 or type I)

$$G_0(x) = \exp(-\mathrm{e}^{-x}), \quad -\infty < x < \infty \tag{4a}$$

- Frechet (EV1 or type II)

$$G_1(x) = \exp(-x^{-a}), \quad \text{for } \alpha > 0,\ x > 0 \tag{4b}$$

- Weibull (EV2 or type III)

$$G_2(x) = \exp\left(-(-x)^{\alpha}\right), \quad \text{for } \alpha > 0,\ x \le 0 \tag{4c}$$

The family of the maxima limits distributions functions specified in Eq. (6) can be extended by introducing parameterization to distribution function's patterns. The location parameter μ and the scale parameter σ are used. Transformation based on the addition of the above-mentioned parameters is carried out according to the following theorem.

Theorem 1. If the random variable X has the distribution function F, then the random variable $(\mu + \sigma X)$ has the distribution function $F_{\mu,\sigma}(x) = F\left(\frac{x-\mu}{\sigma}\right)$ (cf. [25]).

By virtue of this theorem, the distribution functions given in Eq. (4), after parameterization, take the form:

- model EV0

$$G_{0,\mu,\sigma}(x) = \exp\left(-\mathrm{e}^{-(x-\mu)/\sigma}\right) \tag{5a}$$

- models EV1 and EV2

$$G_{i,\alpha,\mu,\sigma}(x) = G_{i,\alpha}\left(\frac{x-\mu}{\sigma}\right), \quad i = 1, 2 \tag{5b}$$

The above parameterization of the family of distributions functions of extreme values significantly expands the spectrum of possibilities associated with modeling the distributions extreme values of various random variables. Using the parameterized distribution functions of extreme values, one can very precisely choose a theoretical distribution function which satisfactorily describes the distribution of these extreme characteristics value. Concluding, it should be mentioned that all distribution functions are presented in Eqs. (4) and (5) as members of one family of distributions described by the formula [16]

$$G_{\gamma,\mu,\sigma}(x) = \begin{cases} \exp\left(-\left(1+\gamma\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\gamma}\right) & \text{if } \gamma \neq 0 \\ \exp\left(-\exp\left(\frac{x-\mu}{\sigma}\right)\right) & \text{if } \gamma = 0 \end{cases} \tag{6}$$

5.5. ESTIMATION METHODS AND TESTS OF SIGNIFICANCE

Of the many estimation methods used for the distributions of extreme values, in this article, the highest reliability method is used to estimate the parameters of extreme value distributions from the distributions family described with Eq. (6). This method gives effective results in specific cases. These cases coincide with the cases considered in this study [18]. The γ parameter estimator exists for the $\gamma > -1$, and for $\gamma > -0.5$, the variance has an asymptotically normal distribution [19].

To verify the hypothesis concerning the compliance of the studied empirical distributions with the selected distributions of extreme values from the family expressed with Eq. (6), the following compliance tests were applied: the χ^2 test, Kolmogorov–Smirnov test, and Anderson–Darling test.

6. FLOOD RISK MODELING IN THE ODRA RIVER BASIN

6.1. HYDROLOGICAL DATA

In this article, one of the hydrometric parameter, called the state of water is used [20]. Daily water levels measured at the hydrological stations in Malczyce were collected in the period from 01.01.1981 to 31.12.2013 which gives the sample of $n = 12\ 053$. The test period was divided into 3 periods each 11 years long. Period I spans over the years 1981–1991, period II over the years 1992–2002 period, and period III – over the years 2003–2013. Using the Gumbel method for each of the three fixed periods quarterly, daily water level maxima collected in the Malczyce hydrological station were selected. The selected quarterly maxima in each of the three periods for which Eq. (2) is

$$y_i = \max\left\{x_{i1}, ..., x_{i91}\right\}, \quad i = 1, ..., 44 \tag{7}$$

are collected in Table 1.

Table 1

Quarterly daily water level maxima from the three 11-year long summer periods [cm]

1981–1991				1992–2002				2003–2013			
212	114	458	386	314	156	788	404	400	432	423	349
416	156	256	206	500	264	290	436	376	294	364	532
246	230	590	300	230	312	348	156	286	734	274	240
374	228	548	190	114	336	340	402	212	446	509	302
570	270	506	224	224	236	318	404	294	422	636	440
358	314	244	194	360	274	334	576	614	416	459	355
400	356	370	180	142	412	320	436	328	468	280	312
308	630	402	152	140	524	504	398	208	216	761	446
286	654	272	350	320	508	488	328	230	552	796	493
414	370	260	390	436	396	160	352	614	346	565	627
344	322	260	160	290	326	340	242	320	274	581	248

6.2. EMPIRICAL AND THEORETICAL QUARTERLY MAXIMA PROBABILITY DISTRIBUTIONS

Based on the empirical distribution function expressed with Eq. (1), empirical distributions for the quarterly daily water level maxima for the periods I, II and III are presented in Figs. 1–3, respectively.

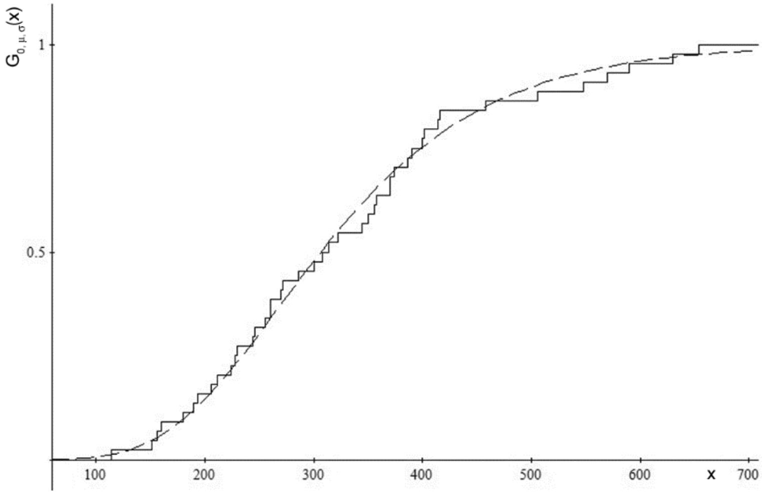

Fig. 1. The distribution function of the quarterly daily water level maxima for the 1981–1991 period (solid line) and theoretical distribution function of the probability distribution for that period expressed with Eq. (8) (dashed line)

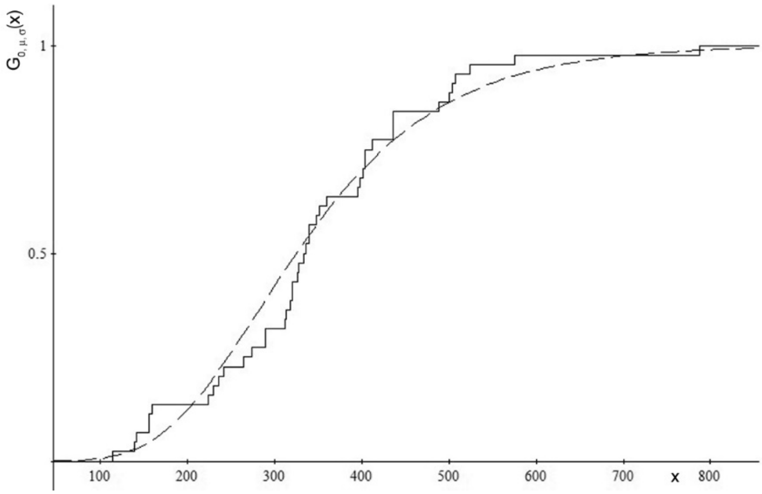

Fig. 2. The distribution function of the quarterly daily water level maxima for the 1992–2002 period (solid line) and theoretical distribution function of the probability distribution for that period expressed with Eq. (8) (dashed line)

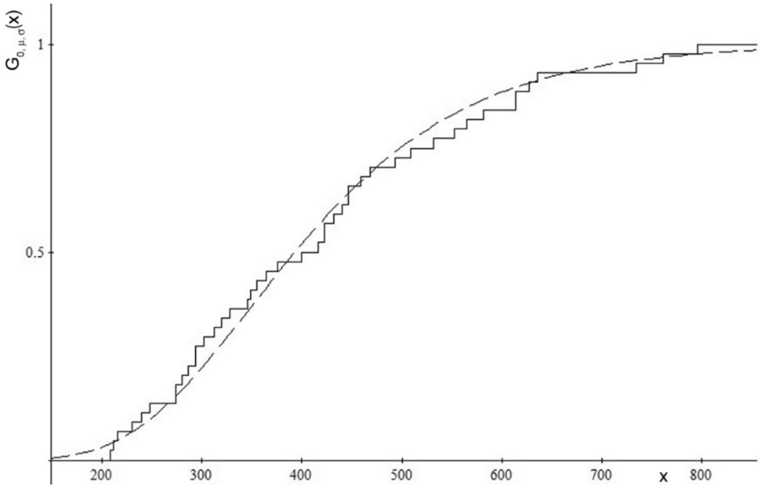

Fig. 3. The distribution function of the quarterly daily water level maxima for the 2003–2013 period (solid line) and theoretical distribution function of the probability distribution for that period expressed with Eq. (8) (dashed line)

Using the highest likelihood method, the parameters of theoretical distributions from the family were estimated, being expressed by the formula optimally suited to the empirical distributions of quarterly daily water level maxima for each of the three studied periods. The values of the μ position and σ scale parameter estimates are presented in Table 2.

Table 2

Parameter estimates of μ and σ for the studied periods

Periods	Estimator values
1981–1991	$\hat{\mu}_{\mathrm{I}} = 268.3, \ \hat{\rho}_{\mathrm{I}} = 104.3$
1992–2002	$\hat{\mu}_{\mathrm{II}} = 283.6, \ \hat{\rho}_{\mathrm{II}} = 113.2$
2003–2013	$\hat{\mu}_{\mathrm{III}} = 349.1, \ \hat{\rho}_{\mathrm{III}} = 119.8$

Below the patterns of distribution are given for the distributions of the quarterly daily water level maxima for each of the three studied periods with the estimates of the parameters from Table 2:

- for the period 1981–1991

$$G_{0,\mu,\sigma}(x) = \exp\left(-\mathrm{e}^{-(x-268.3)/104.3}\right) \tag{8a}$$

- for the period 1992–2002

$$G_{0,\mu,\sigma}(x) = \exp\left(-\mathrm{e}^{-(x-283.6)/113.2}\right) \tag{8b}$$

- for the period 2003–2013

$$G_{0,\mu,\sigma}(x) = \exp\left(-e^{-(x-349.1)/119.8}\right) \tag{8c}$$

In Figures 1–3 one can see a very good match of the assumed theoretical distributions to empirical distributions. In order to confirm this result, three of the most commonly used test of compliance were made: χ^2, Kolmogorov–Smirnov, and Anderson –Darling tests. The results of the tests, given as p-values in Table 3, confirm very high compatibility of the empirical distributions with those calculated theoretically.

Table 3

p-values of the tests for the compatibility of the probability distributions for quarterly daily water level maxima in the studied periods

Period	Test		
	χ^2	Kolmogorov–Smirnov	Anderson–Darling
1981–1991	0.7574	0.9761	0.993
1992–2002	0.5292	0.8155	0.7253
2003–2013	0.7691	0.9614	0.9439

Such high compatibility of experimental data with the results of calculations allows the distribution functions of the theoretical distributions for the respective periods for modeling the flood risk in these periods.

7. FLOOD RISK MODELING IN THE ODRA RIVER BASIN IN LOWER SILESIA

The measure of flood risks in this area was defined as the probability of exceeding a certain water level (u) by the maximum daily state over the time horizon predefined in the study. In this study, the time horizon, on the basis of which the maxima of daily water levels were calculated, has a length of a quarter. The water level for which the flood risk will be calculated is the state of emergency for the hydrological stations in Malczyce and is at u_0 = 600 cm. Table 4 shows the results of the calculations of the probability exceeding the emergency level by the quarterly daily water level maxima for the three studied periods.

Table 4

Measure of flood risk occurrence in the studied periods

Period	Risk measure $P(X_{\mathrm{MAX}} > u_0)$
1981–1991	0.0407
1992–2002	0.0593
2003–2013	0.1159

Analyzing the results of the risk measures in the form of probabilities, a clear upward trend of flood risks in the studied area in the time horizon can be observed in this study. The flood risk measured by the probability of exceeding the emergency level in the years 1992–2002 is 45.7% higher than in the previous test period – in the years 1981–1991. On the other hand, the flood risk in the years 2003–2013 is almost 100% higher or exactly 95.4% than in 1992–2002 [22]. The results confirm the main hypothesis of this research that at the turn of the last few years we have seen very dynamic changes in the flood risk in some parts of Lower Silesia.

8. CONCLUSIONS

In the light of the threat of flooding, whose the authors were eyewitnesses and understanding the scale and the social and economic consequences of flood disasters, the authors attempted to prepare the tools to support a preventive system of flood protection [23, 24].

The presented methods for estimating the flood risk, based on extreme distributions of selected hydrological characteristics, were presented on the example of daily hydrological data from the 1981–2013 time horizon, collected at the Malczyce hydrological station on the 300 km of the Oder river. The obtained results show a clear increase in the flood risks in the studied area.

The risk measured by the probability of exceeding the emergency levels in the years 1992–2002 is by 45.7% higher than in 1981–1991, while the risk in the years 2003–2013 is nearly by 100% higher than the risk in the previous eleven years. These results indicate a very dynamic nature of flood risk in the studied area and They also indicate effectiveness of the methods for estimating the flood risk presented in this article in the wider process of flood risk management.

The results showed that the use of maxima distributions for the flood risks would be an effective tool to support the three-step process of planning in the case of flooding in accordance with the provisions of Directive 2007/60/EC [2] by:

- having the possibility of simple and rapid assessment of the flood risk for selected areas,
- updating flood hazard maps and flood risk maps,
- classifying areas in terms of the flood risk level,
- having the ability to implement tools presented nationally and internationally.

Because of the socio-economic importance of these studies and achievement of satisfactory results in this article, research on the presented issue will be continued in the next research cycle.

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