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STATISTICAL ANALYSIS OF THE RELATIONSHIP BETWEEN PARTICLE SIZE AND PARTICLE DENSITY OF RAW COAL

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Abstract: The paper presents a multidimensional analysis of mineral processing feeds consisting of different amounts of different size and density fractions. The considered feed was coal which was screened into size fractions which were subsequently separated into density fractions and their weights determined. The feed material was characterized with commonly used size and density frequency and cumulative distribution plots and next approximated with the Weibull (size) and logistic (density) mathematical functions. Having the contribution of each particle size and density fraction in the feed a two–dimensional analysis of the feed size/density properties was performed using two methods. The first one is based on the best chosen cumulative frequency function for two random variables and the second uses the so–called Morgenstern family functions. In the paper the undependability of the particles size and density was investigated using statistical approach based on the so–called χ^2 test, and the correlation between these parameters using the so–called *F*–Snedecor statistical test. In both cases it was found that particles size and density of the investigated coal particles were dependent what means that with growth of particle size its density grew too and there was correlation between them regardless of significance level assumed for the analysis.

Keywords: *approximation, coal, multidimensional analysis, statistical tests, particle size, density*

Introduction

In mineral processing operations particle properties influence separation results (Kelly and Spottiswood, 1989). In the case of coal the most important parameters are size and density. However, it is not easy to describe precisely properties of coal taking into account both properties simultaneously. Traditional methods of multidimensional statistical analysis are not always sufficient to describe well the material. Also, the data are not always complete and any kind of forecast is not precise. There is also a question regarding significance of the relation between coal particle size and density. In

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this work an attempt was undertaken to answer this question, taking coal, type 31 in Polish classification system, as an example.

Experimental

Coal, type 31 in Polish classification, called also energetic coal, contained 66% C. It originated from one of the Polish coal mines and was screened on a set of sieves of following sizes: –1.00, –3.15, –6.30, –8.00, –10.00, –12,50, –14.00, –16.00 and –20.00 mm. Then, the size fractions were additionally separated into density fractions by separation in dense media using zinc chloride aqueous solution of various densities (1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9 g/cm^3). The fractions were used as a basis for further consideration.

Coal characteristics

The size fractions of investigated coal are presented in Table 1 while frequency and cumulative size distributions are presented in Figs 1a and 1b.

Table 1. Sieve analysis of investigated coal

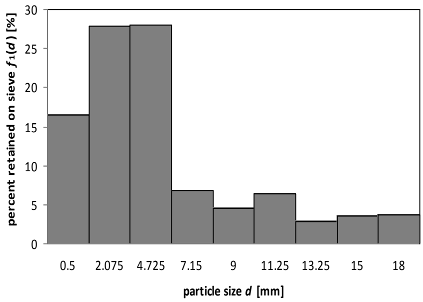
Sieve number i	Size fraction $d_{i-1} - d_i$, mm	Average particle size d_{av}, mm	Percentage retained $f_1(d_{av})$	Cumulative percent retained $F_1(d_i)$	Calculated from Eq. (1) $\Phi_1(d_i)$
1	0–1.00	0.500	16.43	16.43	16.47
2	1.00–3.15	2.075	27.80	44.23	38.56
3	3.15–6.30	4.725	28.05	72.28	66.35
4	6.30–8.00	7.150	6.76	79.04	80.46
5	8.00–10.00	9.000	4.52	83.56	87.06
6	10.00–12.50	11.250	6.42	89.98	92.14
7	12.50–14.00	13.250	2.83	92.81	94.85
8	14.00–16.00	15.000	3.54	96.35	96.56
9	16.00–20.00	18.000	3.65	100	98.22

Table 1 and Fig. 1b also show approximation of the cumulative particle size distribution with the Weibull formulae:

$$\Phi_1(d) = 1 - \exp\left(-\left(\frac{d_i}{4.33}\right)^{0.978}\right) \qquad (1)$$

where $\Phi_1(d)$ denotes cumulative percent of material weight passing a given mesh and d_i is the particle size expressed by the upper limit of size fraction. The contents 0.978

and 4.33 were found by least squared method. Figure 1a clearly shows that the distribution is asymmetrical.



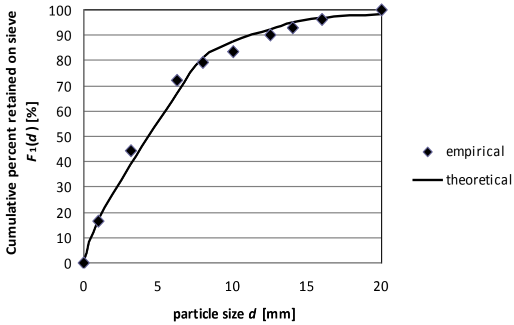


Fig. 1. Coal size distribution, a) frequency plot , b) cumulative plot

Each size fraction of coal was subjected to density analysis to determine content of different density fractions in each size fraction. The results are shown in Table 2 in the form of a matrix.

Table 2. Distribution (in weight %) of size and density fractions. The sum of all contributions is 100%

	Density fraction $\rho_{j-1} - \rho_j$, g/cm^3						
Size fraction $d_{i-1} - d_i$, mm	0–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9
0–1.00	12.32	2.93	0.47	0.12	0.05	0.40	0.13
1.00–3.15	20.15	4.84	1.25	0.42	0.45	0.39	0.30
3.15–6.30	16.47	8.53	1.38	0.46	0.44	0.42	0.35
6.30–8.00	4.19	1.74	0.32	0.12	0.11	0.12	0.16
8.00–10.00	2.14	1.48	0.34	0.20	0.12	0.07	0.17
10.00–12.50	3.06	1.89	0.64	0.27	0.22	0.18	0.16
12.50–14.00	1.25	1.18	0.14	0.04	0.10	0.06	0.06
14.00–16.00	1.66	0.48	0.84	0.39	0.09	0.03	0.05
16.00–20.00	1.23	0.92	0.79	0.27	0.08	0.10	0.26

Having contributions of all size/density fractions it becomes possible to plot particle density distribution of the coal feed by summing up the contributions, for a given density fraction, different size fraction. The results are shown in Fig. 2 and Table 3.

Table 3. Density analysis of investigated coal

$\rho_{j-1} - \rho_j$ [g/cm^3]	Average particle density ρ_{av}, mm	Fraction mass percentage $f_2(\rho_{av})$	Cumulative mass percent $F_2(\rho_j)$	Calculated from Eq. (2) $\Phi_2(\rho_j)$
0–1.3	0.65	62.47	62.47	65.85
1.3–1.4	1.35	23.99	86.46	82.68
1.4–1.5	1.45	6.18	92.64	87.25
1.5–1.6	1.55	2.29	94.93	92.33
1.6–1.7	1.65	1.66	96.59	97.08
1.7–1.8	1.75	1.75	98.34	99.72
1.8–1.9	1.85	1.66	100	100

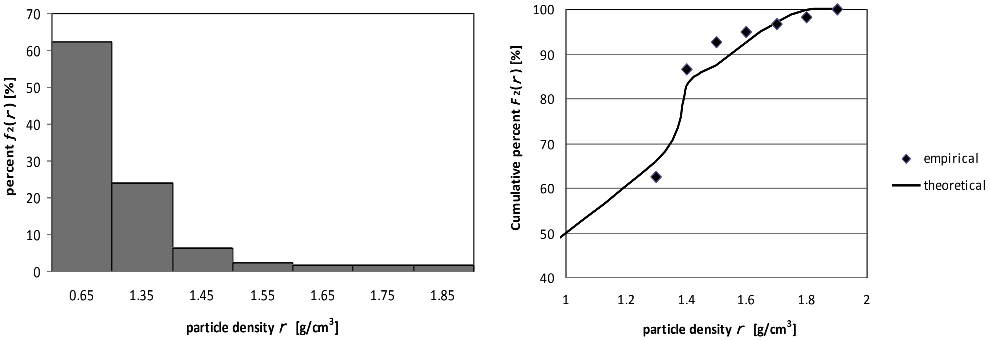


Fig. 2. Coal density fractions distribution, a) frequency plot , b) cumulative plot

The following logistic cumulative frequency function was used for approximation of the density cumulative distribution curve of investigated coal.

$$\Phi_2\left(\rho_j\right) = \left(\frac{1}{1 + 0.6616 \exp\left(-0.4687\left(\frac{\rho_j}{1.9 - \rho_j}\right)\right)}\right) \tag{2}$$

where $\Phi_2(\rho_j)$ stands for cumulative percent of material mass and ρ_j denotes upper limit of a density fraction. The results of approximation are also shown in Table 3.

Two-dimensional cumulative frequency function

To find a relationship between particle size and particle density of investigated coal a two-dimensional cumulative frequency analysis is necessary. For this purpose it is convenient to modify Table 2 with the distribution of size and density fractions into cumulative form (Table 4).

Table 4. Cumulative contribution (content in weight %) of different size d_i and density ρ_j fractions in the coal feed (in statistics terms random variable of cumulative frequency function $F_0(d_i, \rho_j)$ describing the cumulative percent of fraction of size lower or equal d_i and density lower or equal ρ_j)

	ρ_{j-1}						
ρ_j / $d_{i-1} - d_i$	0–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9
0–1.00	12.32	15.25	15.72	15.84	15.89	16.30	16.42
1.00–3.15	32.47	40.24	41.90	42.49	42.99	43.77	44.23
3.15–6.30	48.94	62.54	68.34	69.34	70.27	71.47	72.28
6.30–8.00	53.13	71.15	74.57	75.69	76.73	78.04	79.05
8.00–10.00	55.26	74.77	78.53	79.85	81.02	82.41	83.55
10.00–12.50	58.32	79.72	84.12	85.70	87.09	88.66	89.98
12.50–14.00	59.57	82.15	86.70	88.33	89.92	91.45	92.80
14.00–16.00	61.23	84.29	89.68	91.71	93.29	94.95	96.32
16.00–20.00	62.46	86.46	92.64	94.93	96.59	98.34	100

The simplest way of approximation of two-dimensional cumulative frequency function $F_0(d, \rho)$ is its presentation as the product of two cumulative frequency functions for random variables d and ρ, what means that (Fisz, 1969; Hahn and Shapiro, 1994)

$$F_3(d,\rho) = \frac{F_1(d)F_2(\rho)}{100} \tag{3}$$

The results of distribution function $F_3(d, \rho)$ are presented in Table 5.

Table 5. Results of cumulative frequency function $F_3(d,\rho) = F_1(d)F_2(\rho)$

	ρ_{j-1}						
ρ_j / $d_{i-1} - d_i$	0–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9
0–1.00	10.26	14.20	15.22	15.60	15.87	16.15	16.43
1.00–3.15	27.63	38.24	40.97	41.98	42.72	43.50	44.23
3.15–6.30	45.15	62.49	66.96	68.62	69.81	71.08	72.28
6.30–8.00	49.38	68.33	73.22	75.03	76.34	77.72	79.04
8.00–10.00	52.19	72.24	77.40	79.31	80.70	82.16	83.55
10.00–12.50	56.21	77.80	83.36	85.42	86.91	88.49	89.98
12.50–14.00	57.96	80.22	85.96	88.08	89.62	91.25	92.79
14.00–16.00	60.17	83.28	89.06	91.43	93.03	94.72	96.32
16.00–20.00	62.47	86.46	92.64	94.93	96.59	98.34	100

To find the error of approximation the mean squared error was calculated using Eq. 4 (Dobosz, 2001)

$$s_r = \sqrt{\frac{\sum_{i=1}^{k}\sum_{j=1}^{l}\left(F_3\left(d_i,\rho_j\right)-F_0\left(d_i,\rho_j\right)\right)^2}{N-2}}\sqrt{2} \tag{4}$$

where N is number of fractions (N = 63), F_3 are values of calculated cumulative frequency function based on empirical values and F_0 are values of empirical cumulative frequency function. In this case the value of s_r was equal to 1,37. The small value of the error suggests that particle size d and density ρ may be independent, but as it occurred in the further part of the paper it is not true. When instead of empirical cumulative frequency functions $F_1(d)$ and $F_2(\rho)$ the approximations $\Phi_1(d)$ and $\Phi_2(\rho)$ will be used, given by equations (1) and (2) the function $\Phi_3(d, \rho)$ will be given by

$$\Phi_3\left(d_i,\rho_j\right)=\Phi_1\left(d_i\right)\Phi_2\left(\rho_j\right) \tag{5}$$

and

$$\Phi_3\left(d_i,\rho_j\right)=\left(1-\exp\left(-\left(\frac{d}{4.33}\right)^{0.978}\right)\right)\left(\frac{1}{1+0.6616\exp\left(-0.4687\left(\frac{\rho}{1,9-\rho}\right)\right)}\right) \tag{6}$$

The results of distribution function $\Phi_3(d, \rho)$ are presented in Table 6.

Table 6. Results of cumulative frequency function $\Phi_3\left(d_i,\rho_j\right)=\Phi_1\left(d_i\right)\Phi_2\left(\rho_j\right)$

	ρ_{j-1}						
ρ_j / $d_{i-1}-d_i$	0–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9
0–1.00	10.85	13.61	14.37	15.20	15.99	16.42	16.47
1.00–3.15	25.38	31.87	33.63	35.39	37.42	38.44	38.55
3.15–6.30	43.69	54.86	57.89	61.26	64.41	66.16	66.35
6.30–8.00	52.98	66.43	70.20	74.29	78.11	80.23	80.46
8.00–10.00	57.32	71.98	75.96	80.38	84.51	86.82	87.06
10.00–12.50	60.67	76.80	80.39	85.07	89.45	91.88	92.14
12.50–14.00	62.25	78.50	82.84	87.67	92.18	94.68	94.90
14.00–16.00	63.58	79.83	84.25	89.15	93.74	96.22	96.56
16.00–20.00	64.68	81.20	85.70	90.69	95.35	97.74	98.22

Comparing the results of empirical cumulative distribution function $F_0(d_i, \rho_j)$ and distribution function $\Phi_3(d_i, \rho_j)$ obtained by approximation, the mean standard error s_r value was equal to 4.18. A higher value of s_r is obvious because the errors of approximations of $\Phi_1(d)$ and $\Phi_2(\rho)$ influence the final result.

Application of the Morgenstern family of distribution functions

Another way to approximate cumulative frequency function F_0 is by using the so–called Morgenstern family functions, which are presented by equation (Balasubramanian and Beg, 1997; Firkowicz et al., 1977; Johnson and Kotz, 1972; Niedoba, 2009; 2011a; Niedoba and Tumidajski, 2008; Scaria and Nair, 1999; Tumidajski, 1997)

$$F_4(d,\rho) = F_1(d)F_2(\rho)\Big(1+\mu\big(1-F_1(d)\big)\big(1-F_2(\rho)\big)\Big) \tag{7}$$

where μ is a fixing parameter, $\mu \in [-1, 1]$.

By using the least squared method (Firkowicz et al., 1977), by minimizing function $L(\mu) = \sum_{i=1}^{n}\sum_{j=1}^{k}[F_4(d_i,\rho_j) - F_0(d_i,\rho_j)]^2$, under condition that $\mu \in [-1, 1]$ it is possible to calculate the value of μ

$$\mu = \frac{\sum_{i=1}^{l}\sum_{j=1}^{k}\Big(F_0(d_i,\rho_j) - F_1(d_i)F_2(\rho_j)\Big)F_1(d_i)F_2(\rho_j)\big(1-F_1(d_i)\big)\big(1-F_2(\rho_j)\big)}{\sum_{i=1}^{l}\sum_{j=1}^{k}\Big(F_1^2(d_i,\rho_j)F_2^2(d_i,\rho_j)\big(1-F_1(d_i)\big)^2\big(1-F_2(\rho_j)\big)^2\Big)}. \tag{8}$$

Table 7. Results of cumulative distribution function $F_4(d, \rho)$ based on Eq. 7

	ρ_{j-1}						
ρ_j / $d_{i-1} - d_i$	0–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9
0–1.00	13.48	15.81	16.15	16.26	16.32	16.39	16.43
1.00–3.15	33.41	41.12	42.65	43.16	43.53	43.89	44.23
3.15–6.30	49.85	64.83	68.32	69.57	70.96	71.40	72.28
6.30–8.00	53.26	70.27	74.20	75.83	76.89	78.00	79.04
8.00–10.00	55.41	73.84	78.34	79.97	81.15	82.39	83.55
10.00–12.50	58.32	78.85	83.97	85.85	87.20	88.63	89.98
12.50–14.00	59.53	81.00	86.42	88.40	89.84	91.35	92.79
14.00–16.00	61.00	83.69	89.47	91.61	93.15	94.78	96.32
16.00–20.00	62.43	86.46	92.64	94.93	96.59	99.35	100

On the basis of the empirical data the value $\mu = 1$ was obtained what means that the cumulative frequency function is

$$F_4(d,\rho) = F_1(d)F_2(\rho)\Big(1 + \big(1 - F_1(d)\big)\big(1 - F_2(\rho)\big)\Big). \tag{9}$$

The results of approximation with $F_4(d, \rho)$ are presented in Table 7.

The error of approximation with $F_4(d_i, \rho_j)$ using the Morgenstern function was equal to 0.52. The application of the Morgenstern distribution function gave better results of approximation in comparison with the previous method. It suggests that there is a significant relation between particle size d_i and density ρ_j. When instead of empirical values the approximations functions $\Phi_1(d)$ and $\Phi_2(\rho)$ are applied the value of parameter μ is also equal to 1. The equation of function $\Phi_4(d, \rho)$ is given then by the following formulae

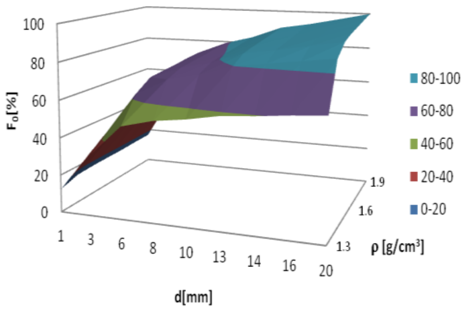
$$\Phi_4(d_i,\rho_j) = \Phi_1(d_i)\Phi_2(\rho_j)\Big(1 + \big(1 - \Phi_1(d_i)\big)\big(1 - \Phi_2(\rho_j)\big)\Big). \tag{10}$$

The values of cumulated frequency function $\Phi_4(d_i, \rho_j)$ are presented in Table 8.

Table 8. Results of cumulative frequency function $\Phi_4(d_i, \rho_j)$

	ρ_{j-1}						
ρ_j / $d_{i-1} - d_i$	0–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9
0–1.00	13.94	15.58	15.90	16.18	16.38	16.46	16.47
1.00–3.15	30.71	32.27	36.27	37.27	38.09	38.51	38.55
3.15–6.30	48.71	58.06	60.37	62.84	65.05	66.23	66.35
6.30–8.00	56.52	68.77	71.95	75.40	78.56	80.28	80.46
8.00–10.00	59.86	73.59	77.21	81.18	84.84	86.85	87.06
10.00–12.50	62.30	77.21	81.19	85.59	89.65	91.90	92.14
12.50–14.00	63.53	79.11	83.29	87.91	92.91	94.59	94.85
14.00–16.00	64.33	80.31	84.61	89.39	93.83	96.30	96.56
16.00–20.00	65.07	81.45	86.31	90.81	95.40	97.95	98.22

In this case the value of s_r is equal to 3.5. It is also smaller when compared to the approximation with function $\Phi_3(d_i, \rho_j)$. It also suggests that the particle size and density depend on each other. A comparison of all approximations is presented on Fig. 3.



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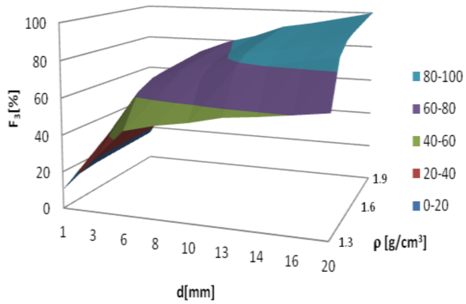

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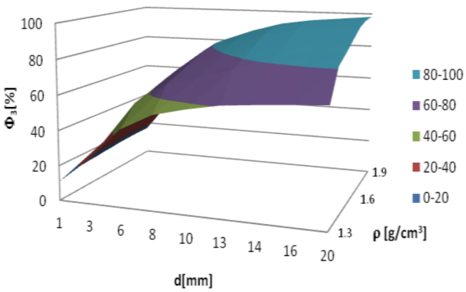

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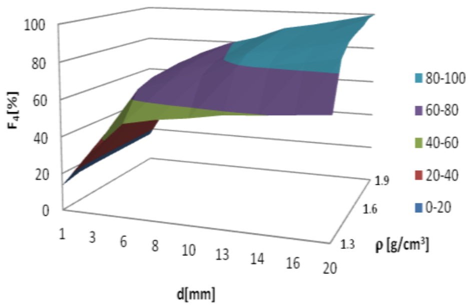

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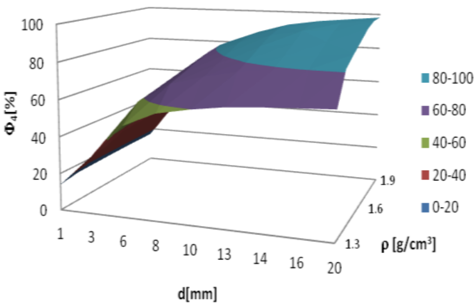

e)

Fig. 3. A comparison of cumulative functions, a) empirical cumulative frequency function $F_0(d_i, \rho_j)$; b) cumulative frequency function $F_3(d_i, \rho_j)$; c) approximated function $\Phi_3(d_i, \rho_j)$; d) cumulative frequency function $F_4(d_i, \rho_j)$; e) approximated function $\Phi_4(d_i, \rho_j)$.

Verification of statistical hypotheses

By comparing the results of approximations it can be seen that they are quite similar. It is also proved by small values of s_r. There is a question regarding a potential relation between particle size and its density. It is not possible to apply directly known statistical undependability tests to answer this question. Usually it must be assumed that all particles are cubic of length d. In this case the number of such particles N_{ij} in weight fraction ij, by density ρ_i is given by equation (11) (Saramak and Tumidajski, 2006).

$$N_{ij} = \frac{w_{ij}}{d_i^3 \rho_j} \tag{11}$$

where w_{ij} is the mass yield of fraction of size $d \in (d_{i-1}, d_i)$ and density $\rho \in (\rho_{j-1}, \rho_j)$.

The quantitative particle frequency for accepted assumptions and without the fraction of the finest particles is given by Table 9.

Table 9. Quantitative particle frequency without finest particles fraction

ρ_{j-1}, ρ_j	d_{i-1}, d_i 3.15–6.30	6.30–8.00	8.00–10.00	10.00–12.50	12.50–14.00	14.00–16.00	16.00–20.00	$n_{.j} = \sum_{i=1}^{5} N_{ij}$
1.4–1.5	910	160	79	56	11	36	16	1268
1.5–1.6	303	54	45	22	4	15	5	448
1.6–1.7	254	46	26	17	7	4	1	355
1.7–1.8	231	48	14	13	3	1	2	312
1.8–1.9	185	69	28	11	3	1	4	301
$n_{i.} = \sum_{j=1}^{7} N_{ij}$	1883	377	192	119	28	57	28	2684

where $n_{i.}$ is number of particles from i^{th} density fraction and $n_{.j}$ is number of particles from j^{th} size fraction.

To check the dependability of the random variables d and ρ the undependability test χ^2 was used (Sobczyk, 2001), where

$$\chi^2 = \sum_{i=1}^{5} \sum_{j=1}^{7} \frac{\left(n_{ij} - np_{ij}\right)^2}{np_{ij}} \tag{12}$$

and

$$p_{ij} = \frac{n_{i.} \cdot n_{.j}}{n^2} \tag{13}$$

where n is number of all particles and n_{ij} is number of particles in fraction (i, j) (according to Table 9), p_{ij} is theoretical probability for particles to be in certain fraction (i, j). The statistics is based on χ^2 distribution function with $(j - 1)(i - 1)$ degrees of freedom (Sobczyk, 2001), where *j* is a number of density fractions and *i* number of particle size fractions.

Due to the fact that the sample is large the statistics $U = \sqrt{2\chi^2} - \sqrt{2(i-1)(j-1)-1}$ can be applied, which is described by normal cumulative frequency function N(0, 1) (Sobczyk, 2001).

On the basis of the empirical data *U* = 3.7 and the array values, given in normal cumulative frequency table (Sobczyk, 2001), $u_{\frac{\alpha}{2}} = 1.96$ for α=0.05 and $u_{\frac{\alpha}{2}} = 2.58$ for α = 0.01. In both cases the inequality $U > u_{\frac{\alpha}{2}}$ is fulfilled what means that the hypothesis about undependability between random variables *d* and ρ should be rejected. Let's check this conclusion on the basis of correlative relations between these variables, by means of equations (14) and (15) (Sobczyk, 2001).

$$\eta^2_{d|\rho} = \frac{\sum_i \left(\bar{d}(\bar{\rho}_i) - \bar{d}\right)^2 n_{i.}}{\sum_j \left(\bar{d}_j - \bar{d}\right) n_{.j}} \tag{14}$$

$$\eta^2_{\rho|d} = \frac{\sum_j \left(\bar{\rho}(\bar{d}_j) - \bar{\rho}\right)^2 n_{.j}}{\sum_i \left(\bar{\rho}_i - \bar{\rho}\right) n_{i.}} \tag{15}$$

where $\eta^2_{d|\rho}$ and $\eta^2_{\rho|d} \in [0, 1]$ are correlative relations. In case when the relation between investigated features is strong these values are close to 1, in case of lack of relation the values are close to 0; $\bar{\rho}$ – mean value of random variable *P*; $\bar{d}$ – mean value of random variable *D*; $\bar{\rho}(\bar{d}_j)$ – mean value of random variable *P* by condition that $d = \bar{d}_j$; $\bar{d}(\bar{\rho}_i)$ – mean value of random variable *D* by condition that $\rho = \bar{\rho}_i$.

The values of $\bar{\rho}(\bar{d}_j)$ and $\bar{d}(\bar{\rho}_i)$ were presented in Tables 10 and 11.

Table 10. The conditional mean values of particle density ρ (for chosen i^{th} particle size fraction)

$\bar{d}_j$	6.30	8.00	10.00	12.50	14.00	16.00	20.00
$\bar{\rho}(\bar{d}_j)$	1.62	1.65	1.63	1.61	1.64	1.55	1.6

Table 11. The conditional mean values of particle size *d* (for chosen j^{th} particle density fraction)

$\bar{\rho}_i$	1.5	1.6	1.7	1.8	1.9
$\bar{d}(\bar{\rho}_i)$	7.53	7.72	7.39	7.18	7.55

The mean values of random variables are $\bar{d} = 7.5$ and $\bar{\rho} = 1.62$ and the values of correlative relations are $\eta^2_{d|\rho} = 0.0056$ and $\eta^2_{\rho|d} = 0.0135$. To verify if there is correlation between random variables D and P the following statistical hypotheses can be checked:

$$H_0\colon \eta^2_{d|\rho} = 0 \text{ and } H_0\colon \eta^2_{\rho|d} = 0$$

The tests for these hypotheses are (Sobczyk, 2001), respectively:

$$G_I = \frac{\eta^2_{\rho|d}}{1-\eta^2_{\rho|d}} \cdot \frac{n-i}{i-1} \text{ and } G_{II} = \frac{\eta^2_{d|\rho}}{1-\eta^2_{d|\rho}} \cdot \frac{n-j}{j-1} \tag{16}$$

and are called the Snedecor tests (Sobczyk, 2011) which are described by the F-Snedecor cumulative frequency function with, respectively, (i – 1, n – i) and (j – 1, n – j) statistical degrees of freedom, where i is number of fractions for random variable d and j is number of fractions of random variable ρ. The values of calculated statistics and their respective array values are presented in Table 12.

Table 12. Values of F statistics compared with array values G_α

G_I	G_{II}	$G_{I\alpha}$, α = 0.05	$G_{II\alpha}$, α = 0.05	$G_{I\alpha}$, α = 0.01	$G_{II\alpha}$, α = 0.01
6.10	3.77	2.10	2.37	2.80	3.32

Because for both levels of significance level α the inequalities $G_I > G_{I\alpha}$ and $G_{II} > G_{II\alpha}$ occurred, the statistical hypotheses H_0 should be rejected so there is basis to state that between random variables ρ and d there is correlation. It can be then assumed that there is relation between these variables. In quantitative methods only the fractions of larger density were taken into consideration because for smaller fractions the number of particles would be too high (in millions) what would cause the use of the tests impossible. To take them into account in such investigation as our the fractions of small sizes and densities should be numerous and of small range. Similar cumulative frequency functions were obtained assuming that the particles have cubic shape. On the basis of the results of applied statistical methods it can concluded that for particles of larger sizes and densities there is a correlation between their density and size.

Conclusions

On the basis of investigation results the following conclusions can be made:

1. Only multidimensional statistical analysis of coal characteristics may give sufficiently full information about relations and influences caused by its certain fea-

tures. Apart from classical approach to this issue it is proposed to use many techniques to perform such analyzes, like applications of the Morgenstern family functions. The results of approximation proved that this is good way to get good description of the investigated material.
2. Apart from particle size and density taken into consideration in this paper as random variables also other coal features may be considered as ash contents or sulfur contents which are also very important from the coal quality point of view.
3. The obtained results may suggest that there is correlation between particle size and density of coal. The application of commonly used statistical tests proved that this hypothesis can be accepted.
4. Author suggests also to apply other multidimensional techniques as kriging method which was already applied to these purposes in other publications (Niedoba, 2010; 2011b).

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References

BALASUBRAMANIAN K., BEG M.I., 1997, *Concominants of order statistics in Morgenstern type bivariate exponential distribution*, Journal of Applied Statistical Science, 54(4), 233–242.

DOBOSZ M., 2001, *Statystyczna analiza wyników badań*, Akademicka Oficyna Wydawnicza Exit, Warszawa.

FIRKOWICZ S. et al., 1977, *Metody statystyczne w sterowaniu jakością*, PWN, Warszawa.

FISZ M., 1969, *Rachunek prawdopodobieństwa i statystyka matematyczna*, PWN, Warszawa.

HAHN G.J., SHAPIRO S.S., 1994, *Statistical Models in Engineering*, John Wiley, New York.

JOHNSON N.L., KOTZ S., 1972, *Distributions in Statistics, Continuous Multivariate Distributions*, John Wiley, New York.

KELLY E.G., SPOTTISWOOD D.J., 1989, *The Theory of Electrostatic Separations, a Review*, Part I. *Fundamentals*, Minerals Engineering 2/1, 33–46.

NIEDOBA T., 2009, *Wielowymiarowe rozkłady charakterystyk materiałów uziarnionych przy zastosowaniu nieparametrycznej aproksymacji funkcji gęstości rozkładów brzegowych*, Górnictwo i Geoinżynieria, 4, 235–244.

NIEDOBA T., 2010, *Application of kriging in approximations of grained materials characteristics distribution functions*, in Proceedings of the XXV International Mineral Processing Congress, Brisbane, 3321–3326.

NIEDOBA T., 2011a, *Three-dimensional distribution of grained materials characteristics*, in Proceedings of the XIV Balkan Mineral Processing Congress, Tuzla, 1, 57–59.

NIEDOBA T., 2011b, *Zastosowanie krigingu zwyczajnego dla oszacowania zawartości popiołu i siarki w węglu w zależności od gęstości i rozmiaru ziarna*, Górnictwo i Geoinżynieria, 2, 159–166.

NIEDOBA T., TUMIDAJSKI T., 2008, *Multidimensional analysis of coal separation processes*, in Proceedings of XXIV International Mineral Processing Congress, Beijing, 2, 2357–2364.

SARAMAK D., TUMIDAJSKI T., 2006, *Rola i sens aproksymacji krzywych składu ziarnowego surowców mineralnych*, Górnictwo i Geoinżynieria, 3/1, 301–313.

SCARIA J., NAIR N.U., 1999, *On concominants of order statistics from Morgenstern family*, Biometrical Journal, 41, 4, 483–489.

SOBCZYK M., 2001, *Statystyka*, PWN, Warszawa.

TUMIDAJSKI T., 1997, *Stochastyczna analiza własności materiałów uziarnionych i procesów ich rozdziałów*, Wydawnictwo AGH, Kraków.