Sub-phase power electronics inverters

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Dynamic changes of parameters of energy sources and receivers are reasons that decrease exactitude of output signals towards reference signals. To improve these parameters more advanced solutions of power electronics systems are necessary. Ones of possible solutions are a sub-phase inverters. The output quantity regulator there is based on Generalized Sampling Expansion, being crucial extension of Whittaker-Kotelnikov-Shannon sampling theorem. Wide-band power electronics current source with regulator based on that extension is analyzed further.

1. Introduction

Dynamic changes of parameters of electrical energy receivers as well as non-linearity of power inverters are reasons that decrease exactitude of static and dynamic output signals towards reference signals. To improve these parameters more advanced solutions of power electronics systems are necessary. One of these is power electronics wide-band voltage controlled current source [3, 4, 6, 7]. This special inverter may found many applications as: execution block of compensators of currents deformation in power grids [3, 6], systems with unified power flow controllers (UPFC), power electronics reference generators [7], modern electrical drives [16] and equipment for medicine [4, 17].

Proposed power electronics sub-phase inverters works with negative feedback and contains output current regulator based on Generalized Sampling Expansion (GSE) [14]. On base of such extension of Whittaker-Kotelnikov-Shannon (WKS) sampling theorem, small-signal and simulation model of sub-phase current source is proposed and analyzed further.

2. Generalized sampling expansion

Assume signal $x(t) \in \mathrm{L}^2(\Re)$, the space of square-integrable functions, where $\Re$ - real numbers domain, and its Fourier transform $X(\mathrm{j}\omega)$ exists. Since the function $x(t)$ is band-limited, the support of $X(\mathrm{j}\omega)$ is within the interval $[-\omega_{\mathrm{MAX}}, \omega_{\mathrm{MAX}}]$. The function $X(\mathrm{j}\omega)$ lies within the space $\mathrm{L}^2[-\omega_{\mathrm{MAX}}, \omega_{\mathrm{MAX}}]$ and can, therefore, be written in terms of a Fourier series.

Taking into consideration WKS sampling theorem while sampling of $x(t)$ is uniform and ideal, it means with utilization of Dirac series $\sum_n \delta(t - nT_S)$, where $T_S \leq \frac{\pi}{\omega_{MAX}}$ is the sampling period, the $x(t)$ can be reconstructed using inverse Fourier transform formula:

$$x(t) = \frac{1}{2\omega_{MAX}} \sum_{n=-\infty}^{\infty} x(nT_S) \int_{-\omega_{MAX}}^{\omega_{MAX}} e^{j\omega(t-nT_S)} d\omega. \tag{1}$$

A one of major extension of Whittaker-Kotelnikov-Shannon (WKS) sampling theorem was formulated by Papoulis [14] which unifies a broad class of extensions. The general setting is that a signal $x(t)$ is processed by a linear Multi-Dimensional Sampling System (MSS) – Fig. 1.

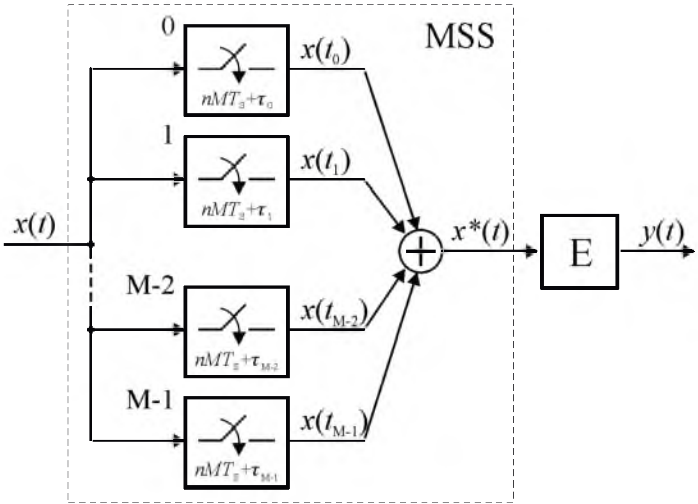

Fig. 1. Multi-Dimensional sampling system (MSS)

Suppose now that $x(t)$ is a common input to M sampling systems, being sub-systems of MSS. Each sub-system samples at a rate of $\frac{1}{M}$ times of the Nyquist rate. Assuming individual delays $\tau_i = iT_S$, the $y(t)$ can be reconstructed using inverse Fourier transform formula:

$$y(t)=\frac{1}{2\omega_{\text{MAX}}}\sum_{n=-\infty}^{\infty}\sum_{i=0}^{M-1}\left\{x\left[\left(n+\frac{i}{M}\right)MT_{\text{S}}\right]\int_{-\omega_{\text{MAX}}}^{\omega_{\text{MAX}}}e^{j\omega\left(t-(n+\frac{i}{M})MT_{\text{S}}\right)}\text{d}\omega\right\}. \quad (2)$$

The overall sampling rate still satisfies the Nyquist criterion. Finally, signal $y(t)=x(t)$ can be retrieved from extrapolator E on base a sum of $x(t_i)$ samples. The multi-dimensional sampling system, being a consequence of GSE, lets to reduce the required sampling frequency to $\frac{1}{M}$, comparing to 1-dimensional sampling system (WKS sampling theorem).

3. Idea of sub-phase power electronics inverters

A general scheme of sub-phase inverter, for instance controlled current source, is shown on Fig. 2. The sub-phase inverters are extension of idea of interleaved inverters [10, 15] – toward more wide-band while final limitation of frequency is a pulse modulation carrier frequency.

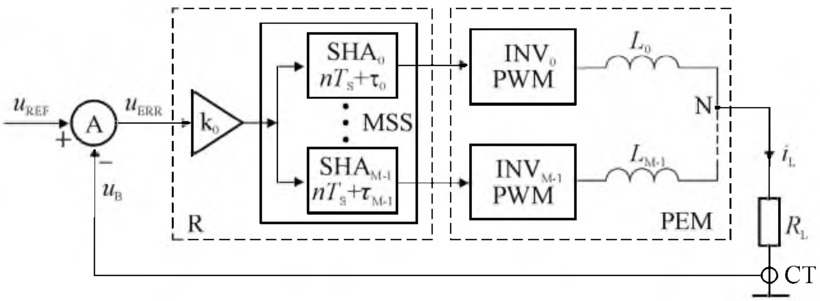

Fig. 2. General block scheme of power electronics wide-band controlled current source with GSE based current regulator

The current source works with negative feedback and consists of two main modules: R – regulator, PEM – power electronics module. CT is a current transducer.

The regulator consists of M^{th} order MSS and gain block (k_0) while the power electronics module consists of M PWM based inverters and M output inductors with common current summing node (N). The adder A produces error signal: $u_{ERR}(t)=u_{REF}(t)-u_B(t)$. The output current $i_L(t)$ is proportional to input (reference) voltage $u_{REF}(t)$.

Since the sampled function does not carry any energy, additional module converting sampled function into continuous-time domain, is necessary. Most wide utilized in theory and practice converting module is 0-order extrapolator in form of Sample-And-Hold Amplifier (SHA) [1]. Thus, for implementation of GSE in regulator (R) crucial modification of MSS, according to Fig. 1, is necessary. In each of sampling sub-system 0-order extrapolator (SHA) is placed now. All SHAs sample at T_S rate with individual $\tau_i = i\frac{T_S}{M}$ time shift. Order of MSS is M .

Sub-phase inverters carry a lot of advantageous. Most common are: crucial increasing the regulator gain k_0 while system stability is behaved and significant minimization of ripples in output signal caused by pulse (e.g. PWM) modulation.

4. Stability analysis of sub-phase power electronics current source

Assuming now $h_{SHA}(t)$ is a pulse response and taking into consideration scale factor $\frac{1}{T_S}$ [9], a transfer function of SHA is given by following equation:

$$H_{SHA}(s) = \Im[h_{SHA}(t)] = \frac{1}{T_S}\lim_{\varepsilon\to 0}\int_0^{T_S-\varepsilon} e^{-st}dt = \frac{1}{T_S}\frac{1-e^{-sT_S}}{s}\Big|_{s=j\omega} = e^{-j\omega\frac{T_S}{2}}\,\mathrm{Sa}\left(\omega\frac{T_S}{2}\right). \quad (3)$$

The integral in formula (3) is improper because of T_S is excluded from the sampling interval $\langle 0, T_S)$.

In fact, relationship between $X(j\omega)$ and $Y(j\omega)$ apart *static* (time invariant) component (3), contains *dynamic* (time variant) component $\sum_{n=-\infty}^{\infty} x(nT_S)e^{j\omega nT_S}$ yet, so formal transfer function of SHA doesn't exist [2, 11, 12]. The *dynamic* component is related to an aliasing effects also, by which the high frequency poles are folded back into lower frequencies. Although respecting the *static* part only of equivalent transfer function (ETF) of SHA gives, in most cases of system stability analysis [3, 4, 7], satisfying results the crucial knowledge is that the aliasing mechanism can cause loss of stabilization at critical frequencies [2, 5, 8, 11].

The equation (3) is valid assuming the SHA works in open loop, that is, feedback has not an effect. However while feedback is present equation (3) has to be modified toward ETF system with feedback. The modification follows a fact that sampling system is a modulator producing an infinite number of aliases of modulator input signal spectrum in basic band. A modulator input signal is a difference of input (reference) and output signals [2, 5, 8, 11].

General case of sampling functions, in form of Dirac series while M =2, is shown on Fig. 3. Sampling is uniform one and both of series are shifted each other by $2aT_S$, where shift factor $a \in \left\langle -\frac{1}{4}, \frac{1}{4} \right\rangle$. In particular cases: a =0 sampling is 1st order, $a = \pm 1/4$ sampling is 2nd order and orthogonal. Formally, sampling functions are given by following equations:

$$s_i(t) = \sum_{n=-\infty}^{\infty} \delta\left[t - \left(n - (-1)^i a\right) T_S\right] \tag{4}$$

where: $i = 0$ or $i = 1$.

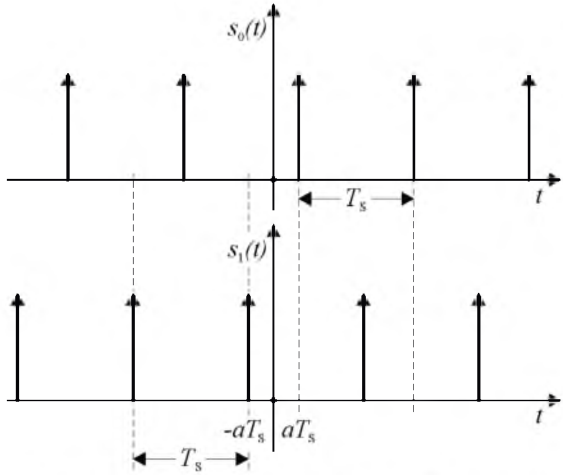

Fig. 3. Sampling functions for achieving 2nd order MSS

In order of stability analysis system should be more specified, that is, choice of output circuit configuration is necessary. Thus, further consideration concerns the small-signal (linear) model of controlled current source in form shown on Fig. 4. The general assumption is that one works with negative feedback. Regulator (R) of output current $i_L(t)$ is 2nd order MSS (M =2) and consists of two SHAs and gain block k_0.

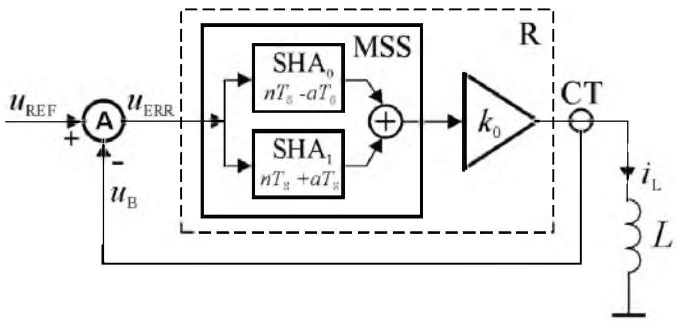

Fig. 4. Linear model of controlled current source with 2nd order MSS based regulator

Proposed small-signal model of such input-output system is one-dimensional, time-invariant and can be completely described in terms of continuous-time system with transfer function: $G(\mathrm{j}\omega)=\dfrac{I_{\mathrm{L}}(\mathrm{j}\omega)}{U_{\mathrm{REF}}(\mathrm{j}\omega)}=\dfrac{K(\mathrm{j}\omega)}{1+K(\mathrm{j}\omega)}$. Stability analysis takes advantage of Nyquist criterion [2,12,13]. Hence, characteristic equation of model: $R(\mathrm{j}\omega)=1+K(\mathrm{j}\omega)=0$, while model transfer function $K(\mathrm{j}\omega)$:

$$K(\mathrm{j}\omega)=2r_{\mathrm{CT}}k_0\sum_{n=-\infty}^{\infty}\left\{\mathrm{e}^{-\mathrm{j}(\omega-n\omega_{\mathrm{S}})\frac{T_{\mathrm{S}}}{2}}\cos[(\omega-n\omega_{\mathrm{S}})aT_{\mathrm{S}}]\mathrm{Sa}\left[(\omega-n\omega_{\mathrm{S}})\frac{T_{\mathrm{S}}}{2}\right]\frac{1}{\mathrm{j}(\omega-n\omega_{\mathrm{S}})L}\right\}. \quad (5)$$

Effective regulator gain $k_{\mathrm{e}}=2k_0$ is increased by 2, comparing to gain of 1st order MSS [9]. The function (5) possesses infinity number of singular points $\omega=n\omega_{\mathrm{S}}$.

The Nyquist diagram, on base of equation (5), shown on Fig. 5, concerns following hypothetical conditions: r_{CT}=1 V/A, L=5 mH, T_{S}=0,1 ms and k_0=60 V/V (effective gain: k_{e}=120 V/V). Since (5) is periodic function in ω with period T_{S}, the (P,Q) diagram is also periodic in ω intervals: $\left\langle n\dfrac{2\pi}{T_{\mathrm{S}}},(n+1)\dfrac{2\pi}{T_{\mathrm{S}}}\right\rangle$.

For hypothetical particular gain k_0=60 V/V (k_{e}=120 V/V) and a=0,10; 0,15; 0,20; 0,25 the (P,Q) phase curve encircles the Nyquist point ($-1+\mathrm{j}0$) on the right side what means system is stable. Characteristic effect of 2nd order sampling is decreasing gain of MSS while frequencies are closed to characteristic point (Nyquist frequency) $\omega=\dfrac{\pi}{T_{\mathrm{S}}}$. For other ones the gain is relatively high. Gain minimizing effect is maximal while sampling is orthogonal.

In terms of Nyquist criterion maximal regulator gain up to that system is stable defines equation (6) while $\omega=-\pi$:

$$\left|K(\mathrm{j}\omega)\right|\Big|_{\omega=\omega_\pi}=2\frac{r_{\mathrm{CT}}k_0}{L}\frac{2}{\pi^2}\left|\sum_{n=-\infty}^{\infty}\cos\left[\frac{\pi}{4}(1-2n)\right]\frac{1}{(1-2n)^2}\right|<1. \quad (6)$$

By solving the equation (6), the maximal value of regulator gain $k_{0,\mathrm{MAX}}$ can be determined as follows:

$$k_{0,\mathrm{MAX}}<2\frac{L}{r_{\mathrm{CT}}T_{\mathrm{S}}}, \quad (7)$$

$$k_{\mathrm{e,MAX}} = 2k_{0,\mathrm{MAX}}\,. \tag{8}$$

For given hypothetical system parameters (r_{CT}=1 V/A, L=5 mH, T_{S}=0,1 ms): $k_{0,\mathrm{MAX}} < 100$, thereby: $k_{\mathrm{e,MAX}} < 200$.

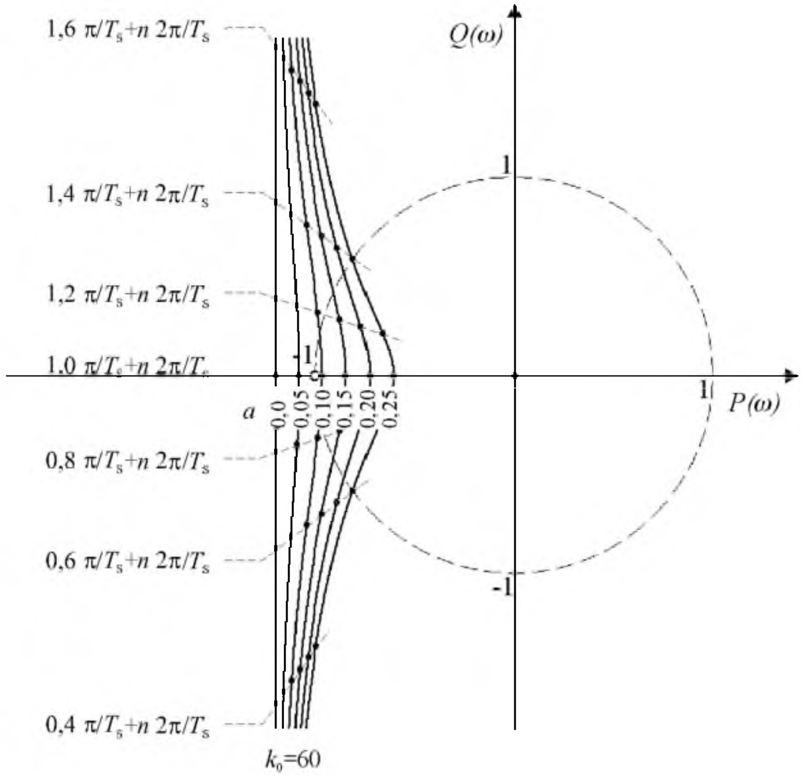

Fig. 5. Nyquist diagram for characteristic equation $R(\mathrm{j}\omega)$ of small-signal model of current source with MSS and M =2; k_0 =60 V/V

5. Simulation model of current source

For acknowledgment of mathematical and small-signal model considerations, simulation models of controlled current source with 2nd order MSS in ORCAD environment has been proposed – Fig. 6. The PEM block there consists of two pairs of inverters. Thanks to that, implementation of one-pole PWM strategy has been possible.

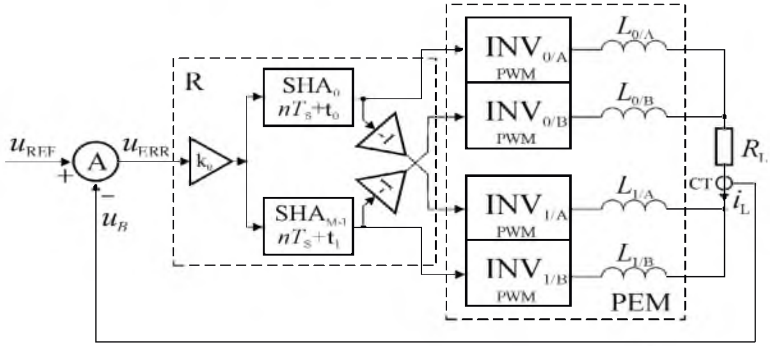

Fig. 6. Block scheme of power electronics current source with 2nd order MSS and one-pole PWM implemented

Selected investigation results of simulation model are shown on Fig. 7. Values of system quantities are as follows: k_0=50 V/V, r=1 V/A, L_X=5 mH, $R_L=0\,\Omega$, T_S=0,1 ms. The PWM is 2-sided and carrier frequency $f_{PWM,C}$=5 kHz.

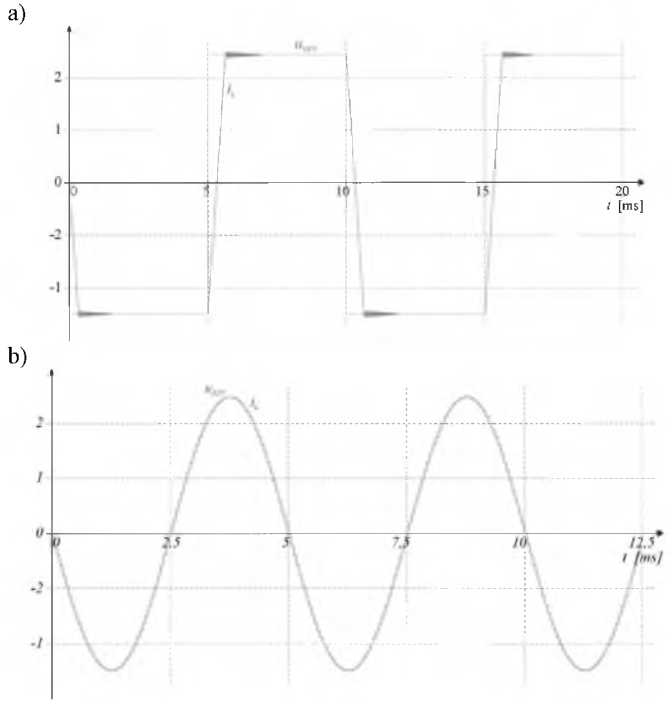

Fig. 7. Selected waveforms in simulation model of current source with 2nd order MSS and one-pole PWM implemented: a) $u_{REF}(t)$ is rectangular and f_{REF} =50 Hz, b) $u_{REF}(t)$ is sinusoidal and f_{REF} =200 Hz

Simulation model investigation results point at fact that 2nd order MSS lets crucially increase effective system gain, comparing to regulator on base of WKS (or MSS and $M=1$). Another advantage of MSS based regulator is better stop-band properties of this at frequencies closed to Nyquist frequency, what lets more effective minimize aliasing effects.

Investigation results show that output current parameters are much improved. Commonly, referencing the output current waveform within reference signal is much better, comparing to conventional solution of such inverters. Output quantity referencing error is understood as $u_{ERR}(t)$ (Fig. 6) and can be easily measured.

6. Conclusions

Presented idea of sub-phase inverters with regulator on base of GSE lets maximize significantly regulator gain and achieve more advantageous system transfer function. Thanks of that affecting of aliasing effects on system work is reduced. The ripples in output signal, caused by pulse (e.g. PWM) modulation, are significantly minimized also. As a result, referencing the output quantity waveform within input (reference) signal is much better, comparing to classical solution of inverters. It seems necessary to continue investigations of such type of power electronics equipments. The reason is their advantages and possibilities of direct implementation in modern power electronics systems with acceptable today complication of these.

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