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A study on the identification of the second-order linear Nomoto model from the zigzag test

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Abstract

In this paper a simple four-point, in terms of time, but eight-value in total, identification method has been developed for the second-order linear Nomoto steering model. The algorithm intrinsically uses the zigzag test data in that it inherited some principles of the well-known procedure for the first-order model, from which it is essentially derived. The performance evaluation was then conducted with both simulated and real data. However, the results of these early, unprecedented efforts are far from satisfactory. Some potential sources of difficulties have been discussed. This calls for further research and improvement in order to provide a practical application of the method.

Introduction

The identification of ship steering dynamic models has long been of interest to naval architects (of hydrodynamic expertise) and ship control engineers since the early 20th century, always aiming to improve the performance of a ship. After introducing ship simulators to support nautical studies, waterway and harbour design, and operator training, a new challenging field of application has emerged. This field, that also provides some feedback towards ship design itself, covers all the matters related to developing mathematical models of ship manoeuvring for full-mission simulators. These ship models, with multiple degrees-of-freedom, and of strongly nonlinear, four-quadrant, and lookup-table type, are imposing new demands in terms of their input data quality and range.

The linear steering models are by far mostly associated with pure yaw motion. Traditionally or colloquially, they have been referred to as the Nomoto models. They were primarily developed to analyse ships course-keeping ability during ship design and to apply some control laws to the ship's autopilots. They are a simple and analytical solution, with a sound interpretation and some identification procedures. The linear models also seem to function well in determining (identifying or calibrating) some regions of the more sophisticated, aforementioned full-mission manoeuvring models.

The existing linear steering models almost entirely fall into two categories: first- and second-order models. They consist of two- and four-parameter structures, accordingly. The first-order model relatively dominates in scientific, engineering, and practical interests. However, only the second-order model, as it possesses the much better or even stricter hydrodynamic background (derivation), can be used for the purpose of tuning the full-mission models.

In spite of its long history, the additional motion 'effects' of the second-order linear model, as compared to the first-order one, are still to be discovered. One of the most valuable and sufficiently in depth contributions to the second-order linear theory belongs to Norrbin (Norrbin, 1996), which contains a lot of other useful references. In the literature (Artyszuk, 2016) it was realised, against the old original validation results (from the nineteen fifties), that the traditional reduction of second- to first-order model often leads to an incorrect performance of the zigzag test simulation. Both overshoot angles and zigzag period are therein seriously affected, which are very sensitive to the ratio of the so-called T_3 to T_2 time constants. A quantitative insight to this phenomenon was undertaken in the literature (Artyszuk, 2017). Of course, as stated in the literature (Artyszuk, 2016), one might attempt to directly identify the first-order model from the zigzag test data using the well-established procedure (Nomoto, 1960), but then a quite different meaning of the model's two parameters (K and T) would be received and the consequences associated therewith.

This paper was aimed at improving and adapting the Nomoto approach (Nomoto, 1960) to the second-order model for the purpose of directly getting its four parameters. The effectiveness of the new procedure has been thoroughly examined and reported.

In the first section, the fundamentals of the first-order model identification from the zigzag test have been considered, in which the latter test type played a vital role. Some features of this algorithm have also been discussed therein. Subsequently, the procedure for the second-order model has been developed. In the next two sections, the performance of is the model was tested (on simulated, as well as on full scale trial, data) and a comprehensive but simple sensitivity analysis of the identification of potential errors in the input data was carried out.

Classical approach to the first-order Nomoto model identification from the zigzag test

The first-order linear model, in fully dimensionless form (thus directly applicable to any ship's length and speed with the same parameters, if the geometric similitude is valid,), reads:

$$T\frac{\mathrm{d}\omega_z'}{\mathrm{d}s'}+\omega_z'=K\delta \tag{1}$$

$$\mathrm{d}s'=\frac{\mathrm{d}s}{L}=\frac{v\,\mathrm{d}t}{L}=\frac{\mathrm{d}t}{t_L} \tag{2}$$

$$\omega_z'=\omega_z\frac{L}{v},\ \omega_z=\frac{\mathrm{d}\psi}{\mathrm{d}t}\Rightarrow\omega_z'=\frac{\mathrm{d}\psi}{\mathrm{d}s'} \tag{3}$$

$$\varepsilon_z'=\frac{\mathrm{d}\omega_z'}{\mathrm{d}s'} \tag{4}$$

where:

- T – dimensionless distance constant (representing inertia);
- K – dimensionless gain constant;
- s' – dimensionless (instantaneous) distance;
- ω'_z – dimensionless yaw velocity;
- δ – helm (rudder) angle, as control variable;
- s – absolute distance travelled;
- t – absolute time elapsed;
- v – ship's speed;
- L – ship's length;
- t_L – time taken to travel 1L;
- ω_z – yaw velocity;
- ψ – heading;
- ε'_z – dimensionless yaw acceleration (mentioned here as to be further used).

The dimensionless distance constant T and dimensionless distance s' above can also be alternatively referred to as the dimensionless time constant (also the often used term and the origin for the symbol 'T') and dimensionless time, accordingly. Although they have no units, they really express the distance (of either aspect) in units of ship length (L), or the time in units of the time taken to travel one ship's length.

Assuming the zero initial conditions of the yaw velocity and heading, as standard for the zigzag test, i.e. $\omega'_z(0) = \psi(0) = 0$, after the both-sided integration from 0 to arbitrary s' equation (1) yields:

$$T\omega_z'(s')+\psi(s')=K\int_0^{s'}\delta\,\mathrm{d}s' \tag{5}$$

Marking the right-hand side integral in (5), which is easily analytically computable for the usual trapezoidal rudder steering (of finite, constant-speed deflection), as:

$$\Delta(s')=\int_0^{s'}\delta\,\mathrm{d}s' \tag{6}$$

one gets:

$$T\omega_z'(s')+\psi(s')=K\cdot\Delta(s') \tag{7}$$

The classical method of Nomoto (Nomoto, 1960) uses two special points from the zigzag record to also arrive at the two model parameters T and K in (7). These correspond to the abscissas of the first overshoot angle and the first zero-crossing heading (see Figure 1 – for an example of 10°/10° zigzag, points A and B) as follows:

$$s'=s'_{OS1},\quad \underline{\psi(s'_{OS1})=\psi_{OS1}},\quad \omega_z'(s'_{OS1})=0 \tag{8}$$

$$s'=s'_{Z1},\quad \psi(s'_{Z1})=0,\quad \underline{\omega_z'(s'_{Z1})=\omega_z'\big|_{Z1}} \tag{9}$$

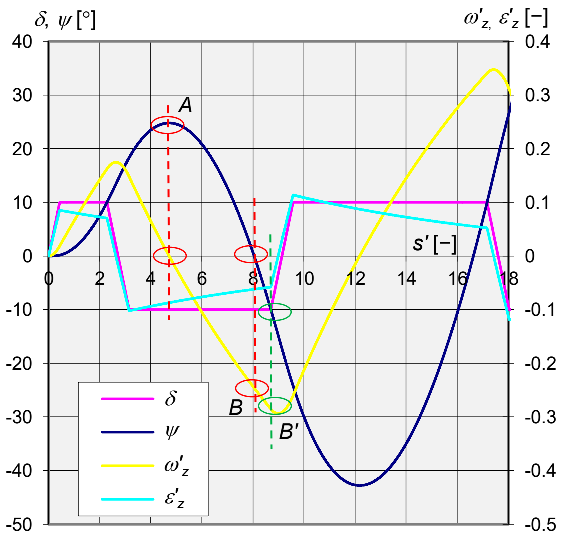

Figure 1. Definition of the basic zigzag points for 1st-order identification

The model parameters are then simply calculated stepwise according to:

$$K = \frac{\psi_{OS1}}{\Delta(s'_{OS1})} \tag{10}$$

$$T = \frac{K \cdot \Delta(s'_{Z1})}{\omega'_z|_{Z1}} \tag{11}$$

which basically constitute **the solution of a set of two linear equations** based on (7). Note that it is really necessary to substitute herein two points of both the heading and the yaw velocity curve, thus leading in total to **four** points. However, they originate from only two distance instants (cross-sections of the zigzag full record, see Figure 1).

Of course, arbitrary pairs of these 'dual-type' points, not only the adjoining ones as commonly used, can be applied in the same way, without complicating the computation procedure at all. The integration mentioned just extends them to multiple half-cycles. Since the half-cycles of the zigzag are often not 'comparable' to or 'deducible' from each other in the light of the information carried, they may lead to essentially different identification results. A reason for this may lie in various nonlinear or higher-order linear effects, and in some inherent errors. This is why an average of the identification results originating from the different zigzag intervals is usually undertaken (usually up to a maximum of the first three half-cycles). However, strictly speaking, such an 'identification', even for a single half-cycle, looks more like a very specific first-order linear approximation (linearization) of the ship's real behaviour, which more or less adequately fits this simple model (1).

It shall be emphasised here that the yaw angular velocity ω'_z, as shown in Figure 1 (yellow-marked) and used in (7) or (11), is often neither recorded in shipyard zigzag trials, nor reported. Far more significant, the same lack of measurement data exists with regard to the derivative of the yaw velocity, ε'_z, that is also presented in Figure 1 (cyan-coloured). In such a situation, obtaining the yaw velocity for the model identification, expressed by (10) and (11), requires projecting a tangent line to the heading curve at the point of concern. The tangent of its slope angle indicates the desired value of the yaw velocity.

For a zero-crossing heading, the yaw velocity is close in magnitude, but not equal, to the maximum value at each zigzag cycle, refer to Figure 1. Such maximum values are 'practically' linked to the counter-rudders, see e.g. the point B' in Figure 1, and are much easier to visually assess based on the heading itself. As mentioned before, the counter-rudder abscissas are also required to compute the integral (6). Therefore, rather than using zero-crossing points, the author by virtue of economy has suggested taking a wide advantage of the counter-rudder points. At the counter-rudder, the heading deviation assumes its nominal, known value. In the case of the 10°/10° zigzag, this equals 10° (=π/18 rad). The formula (11) can then be replaced by:

$$T = \frac{K \cdot \Delta(s'_{CR2}) + \frac{\pi}{18}}{\omega'_z|_{CR2}} \tag{12}$$

where s'_{CR2} is the distance from the second counter-rudder (point B').

The identification procedure discussed in this section, let us say for a dynamic model of a certain differential structure, takes two discrete, but special, points of a ship's response in the zigzag test. This two-point identification may often be considered advantageous. However, one may attempt to engage more points, of various types, in order to someway obtain a more representative or average model, thus transforming the identification into a somewhat 'continuous' type. It is worthwhile to note that the two basic points refer to 'partially' independent dynamic states, consisting of a basic response variable (heading) and its first derivative, which improve the structural adequacy of the identified dynamic model. The structural adequacy might be missing if, in contrast, someone would attempt to identify the model, e.g. treating it as a black box, with the heading alone.

Proposal for the second-order model identification

In the dimensionless form, the aforementioned second-order linear Nomoto model appears as:

$$T_1 T_2 \frac{\mathrm{d}^2 \omega'_z}{\mathrm{d}s'^2} + (T_1 + T_2)\frac{\mathrm{d}\omega'_z}{\mathrm{d}s'} + \omega'_z = K\left(\delta + T_3 \frac{\mathrm{d}\delta}{\mathrm{d}s'}\right) \tag{13}$$

where:

T_1, T_2 – dimensionless distance constants (due to symmetry $T_1 > T_2$ is commonly assumed);

T_3 – dimensionless distance constant (as appears in the exciting term on the right-hand side).

Integrating (13), analogically as done before with the first-order model, one obtains:

$$T_1 T_2 \varepsilon'_z(s') + (T_1 + T_2)\omega'_z(s') + \psi(s') = = K \cdot \Delta(s') + K T_3 \delta(s') \tag{14}$$

or:

$$T_1 T_2 \varepsilon'_z(s') + (T_1 + T_2)\omega'_z(s') - K \cdot \Delta(s') + - K T_3 \delta(s') = -\psi(s') \tag{15}$$

which essentially is a linear equation with four unknowns (in terms of the model parameters).

Compared to the first-order case, equation (15) now requires the provision of the three state variables lying at four different distances to define a set of four linear equations. Obviously, the distance cross-sections of the zigzag record may not be arbitrary, otherwise the motion data could be correlated with each other and prevent the solution. It seems that this condition is met by the natural cross-sections of interest as drawn in Figure 2 with '#'. However, this is not considered to be the best choice, but is believed to be sufficient in view of the present preliminary investigation.

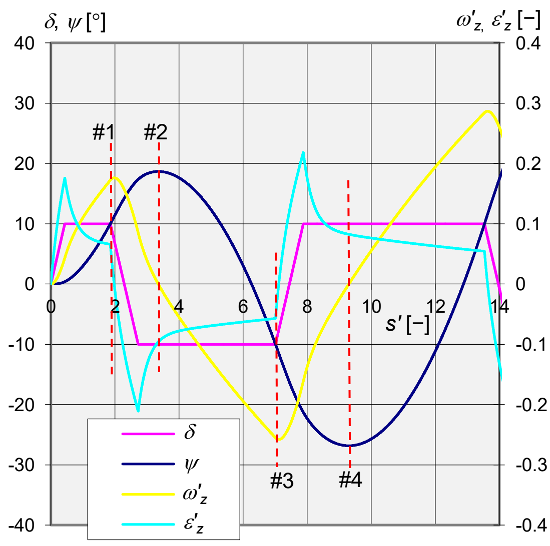

Figure 2. Definition of reference lines for 2nd-order identification

Let us redefine the unknowns:

$$x_1 = T_1 T_2, \quad x_2 = T_1 + T_2, \quad x_3 = -K, \quad x_4 = -K T_3 \tag{16}$$

Then, in matrix form, this finally reads:

$$\begin{bmatrix} \varepsilon'_{\#1} & \omega'_{\#1} & \Delta_{\#1} & \delta_{\#1} \\ \varepsilon'_{\#2} & \omega'_{\#2} & \Delta_{\#2} & \delta_{\#2} \\ \varepsilon'_{\#3} & \omega'_{\#3} & \Delta_{\#3} & \delta_{\#3} \\ \varepsilon'_{\#4} & \omega'_{\#4} & \Delta_{\#4} & \delta_{\#4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\psi_{\#1} \\ -\psi_{\#2} \\ -\psi_{\#3} \\ -\psi_{\#4} \end{bmatrix} \tag{17}$$

where the subscripts of the left- and right-hand side coefficients indicate the values at four arbitrarily selected distance abscissas, and for the reference lines per Figure 2, these indices can be rewritten as:

'#1' – '$CR1$', '#2' – '$OS1$',
'#3' – '$CR2$', '#4' – '$OS2$'

to directly show their correspondence with the successively occurring counter-rudders ('CR') and overshoot angles ('OS'). In this special case, for the 10°/10° zigzag test, one gets:

$$\begin{bmatrix} \underline{\varepsilon'_{CR1}} & \underline{\omega'_{CR1}} & \Delta_{CR1} & +\pi/18 \\ \underline{\varepsilon'_{OS1}} & 0 & \Delta_{OS1} & -\pi/18 \\ \underline{\varepsilon'_{CR2}} & \underline{\omega'_{CR2}} & \Delta_{CR2} & -\pi/18 \\ \underline{\varepsilon'_{OS2}} & 0 & \Delta_{OS2} & +\pi/18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\pi/18 \\ -\underline{\psi_{OS1}} \\ +\pi/18 \\ -\underline{\psi_{OS2}} \end{bmatrix} \tag{18}$$

The moment of the first counter-rudder ($CR1$) in the 10°/10° zigzag is very important since it defines the IMO initial turning ability.

Let us introduce the rudder steering distance s'_R (for the rudder trapezoidal steering) as the distance covered by the ship if the rudder changes from amidships to its nominal angle δ_0. This is expressed by:

$$s'_R = \frac{\delta_0}{\dfrac{\mathrm{d}\delta}{\mathrm{d}s'}} \tag{19}$$

In the present work there has been assumed, unless explicitly stated, the reference case of the rudder speed being equal to 23°/L (i.e. a 23° rudder change at a ship's distance equal to 1 L), (Artyszuk, 2016). For the rudder nominal angle 10° in the zigzag of 10°/10° type this resulted in $s'_R = 0.4348$.

Within the periods of constant rudder angle ($\mathrm{d}\delta/\mathrm{d}s' = 0$), delineated by particular counter-rudder phases, see Figure 2, the following analytics were valid for ease of computation in (18):

$$\Delta\left(s'_R < s' \le s'_{CR1}\right) = \frac{\pi}{18}\left(s' - 0.5s'_R\right) \tag{20}$$

$$\Delta\left(s'_{CR1} + 2s'_R < s' \le s'_{CR2}\right) = = \frac{\pi}{18}\left(-s' + 2s'_{CR1} + 1.5s'_R\right) \tag{21}$$

$$\Delta\left(s'_{CR2} + 2s'_R < s' \le s'_{CR3}\right) = = \frac{\pi}{18}\left(s' - 2s'_{CR2} + 2s'_{CR1} - 0.5s'_R\right) \tag{22}$$

The matrix equation (18) is solvable without difficulty using common linear algebra, including the determinant-based (Cramer's) method that could be implemented even in a spreadsheet.

The inverse formulas for the model parameters, see (16), yield:

$$T_1 = 0.5\left(x_2 + \sqrt{x_2^2 - 4x_1}\right), \quad T_2 = 0.5\left(x_2 - \sqrt{x_2^2 - 4x_1}\right)$$
$$K = -x_3, \quad T_3 = \frac{x_4}{x_3} \tag{23}$$

In which the expression under the square root is positive when both T_1 and T_2 are positive (or both negative, or in some cases of opposite signs) as well.

Numerical assessment of second-order model identification

For verification and performance testing of the present algorithm, some simulated input data were used. First, the run of the zigzag test 10°/10°, according to the full second-order model (13), was prepared with the following parameters:

$$K = 4.89577,\ T_1 = 10.49093,$$
$$T_2 = 0.29813,\ T_3 = 0.98319 \tag{24}$$

The numerical integration scheme of (13) was reported in detail by Artyszuk (Artyszuk, 2017), where the step size 0.002 was taken for the dimensionless distance s'. The resulting motion data, based on (24), have already been plotted in Figure 2. However, to improve the accuracy of such data, let us say, for a kind of 'reverse' identification, they were interpolated for every 0.001 of the independent variable. The discrete data as needed for importation into (18) have been displayed, yet rounded, in Table 1. This produced the results given in Table 2, where '%s' indicate the relative change versus the original values of the parameters (24).

Although serving in the present study as just an example, the values of (24) were not accidental.

Table 1. Simulated input to the algorithm (2nd-order)

Point type	s'	ε'_z	ω'_z	Δ	ψ [rad]	ψ [deg]
$CR1$	1.857	0.06522	0.17240	0.28617	0.17453	10
$OS1$	3.352	−0.09620	0	0.17701	0.32594	18.675
$CR2$	7.015	−0.05652	−0.25500	−0.46231	−0.17453	−10
$OS2$	9.305	0.08246	0	−0.21439	−0.46824	−26.828

Table 2. Output of the algorithm (2nd-order)

Parameter	Value	%	Parameter	Value	%
T_1	10.3747	−1.1	K	4.8537	−0.9
T_2	0.2750	−7.8	T_2/T_1	0.0265	−6.7
T_3	0.9534	−3.0	T_3/T_2	3.4672	5.1

They essentially come from a theoretical and empirical estimation of the hull and rudder hydrodynamic derivatives for a small chemical tanker. As partly stated before, the model and its parameters in (13), as figured here from (24), can be uniquely determined from the underlying set of two coupled first-order linear differential equations for the sway and yaw motions. Such a conversion from the hydrodynamic coefficients (derivatives) of this set of equations to the parameters of (13) has become the classic form of ship steering hydrodynamics.

For the next input, the 10°/10° zigzag curves based on the first-order model (1) simulation were prepared, with the similar numerical integration method. The following parameters were used for the model:

$$K = 4.89577, \quad T = 9.80587 \tag{25}$$

K in the above was equal to that given in (24), while T corresponded to the almost classical approximation of the second- to the first-order model with the data from (24), as also reported in (Nomoto, 1960):

$$T = T_1 + T_2 - T_3 \tag{26}$$

The performance of the algorithm with the supplied first-order linearity data was thus studied. According to Artyszuk (Artyszuk, 2016), this is equivalent to the second-order linearity when $T_2 = T_3$ (entirely independent of T_2). The latter may suggest that the output values for T_2 and T_3 in this specific 'simplified' linearity case could be indeterminate both in magnitude and sign, but equal to each other. The data on the input and the results obtained have been gathered in Tables 3 and 4, respectively.

In the third step of the validation, the algorithm was run with a real, yet digitised, sea trial data of the pure heading for a small chemical tanker

Table 3. Simulated input to the algorithm (1st-order)

Point type	s'	ε'_z	ω'_z	Δ	ψ [rad]	ψ [deg]
*CR*1	2.287	0.07037	0.16247	0.36121	0.17453	10
*OS*1	4.719	−0.08714	0	0.08852	0.43252	24.782
*CR*2	8.704	−0.05804	−0.28537	−0.60700	−0.17453	−10
*OS*2	12.180	0.08714	0	−0.15209	−0.74676	−42.786

Table 4. Output of the algorithm (1st-order)

Parameter	Value	%	Parameter	Value	%
T_1 (=T)	9.9023	1.0	K	4.9447	1.0
T_2	−2.2113	n/a	T_2/T_1	−0.2233	n/a
T_3	−2.2050	n/a	T_3/T_2	0.9971	n/a

(L = 97.4 m) in the 10°/10° zigzag test. After digitising the plots of the heading, as found in the sea trial report, every 5 s, and applying the Bezier smoothing (Artyszuk, 2002), the most demanding derivatives of the heading – yaw velocity and acceleration – were analytically and automatically computed. Such a fitting of the primary curve of the heading to an analytical representation, and the subsequent analytical computation of derivatives, ensured the full consistency (correspondence) of both derivatives to the heading. The data have been presented in Figure 3. The dots therein represent the recorded but digitised heading and, for the purpose of providing some insight, also the numerically calculated derivatives, with all the errors of such a direct approach. However, the input to the algorithm, Table 5, only consists of the analytical data as based on the Bezier fitting procedure described herein.

In validating this trial data, one should suspect that some biases or errors may exist in the trial report

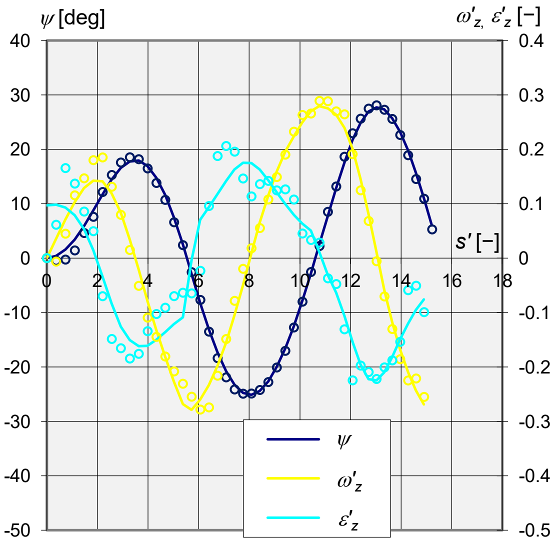

Figure 3. Digitised and smoothed sea trial data of a chemical tanker (dots/lines = raw/smooth data)

Table 5. Smoothed real input to the algorithm (chemical tanker)

Point type	s'	ε'_z	ω'_z	Δ	ψ [rad]	ψ [deg]
*CR*1	2	−0.00674	0.14176	0.32262	0.17453	10
*OS*1	3.6	−0.16082	0.00000	0.14915	0.31254	17.907
*CR*2	6.2	0.07979	−0.24937	−0.30464	−0.17453	−10
*OS*2	8	0.17470	0.00000	−0.09626	−0.43634	−25.000

and in the trial itself. This might include smaller or larger mistakes in ‘manually’ conducting the test, especially with respect to the rudder timing. The rudder angle was not automatically recorded vs. the heading during the trial to allow for the consistency of data assessment. The rudder speed in this real example was about 33°/L (2.4 °/s at v = 7.2 m/s), that affected the Δ values – see (20) to (22). The results of the second-order model identification in this case have been listed in Table 6.

Table 6. Output of the algorithm (chemical tanker)

Parameter	Value	%	Parameter	Value	%
T_1	n/a	1.0	K	−0.194	n/a
T_2	n/a	n/a	T_2/T_1	n/a	n/a
T_3	28.748	n/a	T_3/T_2	n/a	n/a

Some investigators reported, from a numerical perspective, on ill-posed problems in the case of the second-order model identification. For reference, Table 7 has provided the values of the main and the specific determinants involved in (18) for the three cases of input data.

Table 7. Values of determinants for the three cases studied

Determinant symbol	Simulated 2nd-order (Table 1)	Simulated 1st-order (Table 3)	Real (Table 5)
W (main)	2.584E-05	6.233E-07	−2.001E-04
W_{x1}	7.371E-05	−1.365E-05	−1.636E-03
W_{x2}	2.752E-04	4.794E-06	−1.117E-03
W_{x3}	−1.254E-04	−3.082E-06	−3.881E-05
W_{x4}	−1.196E-04	6.796E-06	1.116E-03

Sensitivity study

For the performance testing of the various identification algorithms, it is usual to apply some noise-imposed simulated data. However, such an approach has some limitations, in that even with noise (especially white-noise of zero mean value), the data are ‘too perfect’. In the previous section, only deterministically simulated data were processed to formally verify the algorithm, see Table

Table 8. Identification results for a varied input

Variable (-s) being varied	K	%	T_1	%	T_2	%	T_3	%	T_2/T_1	%	T_3/T_2	%
reference (Table 2)	***4.854***	***−0.9***	***10.375***	***−1.1***	***0.275***	***−7.8***	***0.953***	***−3.0***	***0.027***	***−6.7***	***3.467***	***5.1***
$\varepsilon'_z\|CR1$ + 10%	5.612	14.6	12.479	19.0	0.394	32.2	1.164	18.4	0.032	11.1	2.955	−10.4
−10%	4.544	−7.2	9.531	−9.2	0.211	−29.3	0.847	−13.9	0.022	−22.2	4.018	21.8
$\varepsilon'_z\|OS1$ + 10%	4.846	−1.0	10.346	−1.4	0.161	−46.0	0.837	−14.9	0.016	−45.3	5.203	57.8
−10%	4.902	0.1	10.558	0.6	0.944	216.5	1.641	66.9	0.089	214.5	1.739	−47.3
$\varepsilon'_z\|CR2$ + 10%	4.647	−5.1	9.758	−7.0	0.235	−21.3	0.884	−10.1	0.024	−15.4	3.767	14.2
−10%	5.196	6.1	11.404	8.7	0.332	11.3	1.056	7.4	0.029	2.4	3.183	−3.5
$\varepsilon'_z\|OS2$ + 10%	4.553	−7.0	9.721	−7.3	0.524	75.7	1.220	24.1	0.054	89.6	2.330	−29.4
−10%	4.971	1.5	10.621	1.2	0.187	−37.4	0.858	−12.7	0.018	−38.2	4.600	39.5
$\omega'_z\|CR1$ + 10%	−17.402	−455.5	1.038	−90.1	−55.613	−1.9E+4	2.950	200.1	−53.568	−1.9E+5	−0.053	−101.6
+5%	10.250	109.4	25.783	145.8	0.680	128.2	1.775	80.6	0.026	−7.1	2.609	−20.9
−10%	2.875	−41.3	5.327	−49.2	−0.476	−259.5	−0.121	−112.3	−0.089	−414.2	0.254	−92.3
−5%	3.494	−28.6	6.791	−35.3	−0.125	−141.9	0.346	−64.8	−0.018	−164.7	−2.772	−184.1
$\omega'_z\|CR2$ + 10%	2.961	−39.5	5.092	−51.5	−0.451	−251.4	−0.044	−104.5	−0.089	−412.0	0.098	−97.0
+5%	3.536	−27.8	6.576	−37.3	−0.112	−137.5	0.371	−62.2	−0.017	−159.8	−3.320	−200.7
−10%	−9.862	−301.4	1.003	−90.4	−37.087	−1.3E+4	3.283	233.9	−36.967	−1.3E+5	−0.089	−102.7
−5%	11.015	125.0	29.554	181.7	0.664	122.8	1.827	85.8	0.022	−20.9	2.751	−16.6
$\psi\|OS1$ + 10%	4.870	−0.5	10.434	−0.5	0.505	69.4	1.189	20.9	0.048	70.4	2.354	−28.6
−10%	4.837	−1.2	10.318	−1.6	0.042	−85.9	0.716	−27.2	0.004	−85.6	17.007	415.7
$\psi\|OS2$ + 10%	5.189	6.0	11.069	5.5	0.032	−89.2	0.692	−29.6	0.003	−89.8	21.537	553.1
−10%	4.519	−7.7	9.643	−8.1	0.555	86.1	1.253	27.5	0.058	102.4	2.259	−31.5
all 8 $\{\varepsilon'_z, \omega'_z, \psi\}$+10%	5.205	6.3	10.106	−3.7	0.250	−16.3	0.913	−7.1	0.025	−13.1	3.660	11.0
−10%	4.502	−8.0	10.705	2.0	0.304	2.1	1.000	1.7	0.028	0.0	3.286	−0.4
all 4 $\{\varepsilon'_z\}$ +10%	4.854	−0.9	10.400	−0.9	0.249	−16.4	0.953	−3.0	0.024	−15.6	3.823	15.9
−10%	4.854	−0.9	10.343	−1.4	0.306	2.8	0.953	−3.0	0.030	4.3	3.111	−5.7
all 6 $\{\varepsilon'_z, \omega'_z\}$ +10%	4.854	−0.9	9.406	−10.3	0.276	−7.5	0.953	−3.0	0.029	3.1	3.458	4.8
−10%	4.854	−0.9	11.559	10.2	0.274	−8.0	0.953	−3.0	0.024	−16.5	3.477	5.4
all 2 $\{\omega'_z\}$ +10%	4.854	−0.9	9.377	−10.6	0.304	2.0	0.953	−3.0	0.032	14.2	3.134	−5.0
−10%	4.854	−0.9	11.587	10.4	0.246	−17.4	0.953	−3.0	0.021	−25.2	3.872	17.4
all 4 $\{\omega'_z, \psi\}$ +10%	5.205	6.3	10.080	−3.9	0.275	−7.7	0.913	−7.1	0.027	−3.9	3.319	0.6
−10%	4.502	−8.0	10.736	2.3	0.273	−8.4	1.000	1.7	0.025	−10.5	3.661	11.0

1 and 3. To show the practical and comprehensive performance of the algorithm, exactly the identification of the second-order linear model with biased data, Table 8 has gathered the results of the sensitivity analysis. The input values of Table 1 were different and varied up to ±10%, and have been listed in the left column of Table 8, and their impact on the resulting model parameters was examined, both in absolute and relative terms. Both single and multiple value variations were considered. This was aimed at shedding some light on the inherent structural relationships within the model (13) and/or within the identification scheme (18) itself.

The sensitivity effects of the results in Table 8 can be studied both horizontally, with regard to each model parameter, and vertically, for the significance of each input motion data.

Discussion

The case of the simulated (fully) second-order zigzag input (*Table 2*)*:* The reported differences versus the original values for the four model parameters, in a magnitude of a few percent, can be attributed to a certain numerical inaccuracy (due to rounding, truncation) within the input data provided. This input inaccuracy (say for an 'ideal' input) seems to be unavoidable, even if a lot of care has been exercised. In real measurement conditions the input inaccuracy and the corresponding output inaccuracy of the algorithm, as one might suspect, could be worse. However, the two basic constants, K and T_1, in this simulation experiment were reproduced quite well – with an accuracy of one percent. Table 2 has thus demonstrated a kind of nominal (or maximum achievable) accuracy for the algorithm that has been developed. The fact that all the received values were a little lower than the original ones (negative sign of differences) was not investigated.

The case of the simulated first-order zigzag input (*Table 4*)*:* The achieved accuracy of both first-order model parameters, K and T (= T_1), also tended to be within one percent. However, as compared to the previous case of the second-order input, the identified values here were higher than the original values. Whether a rule or coincidence, this problem was not undertaken in the present research. The sign and magnitude of the additionally determined second-order model specific constants, T_2 and T_3, were insignificant (likely to be accidental) – see the paragraph directly following (26). What is important is that both constants were close to each other as expected, although their negative sign and rather high value might or should call for some future investigation.

The case of the real zigzag input (*Table 6*). It really must be admitted that the identification for the real chemical tanker failed – K is very low (unrealistically so) and negative (like a directionally unstable ship). In addition, no T_1 and T_2 were found due to a negative value under the square roots in (23), and T_3 was exaggerated too much. The problem at least likely lay in the wrong signs of the input yaw accelerations at the counter-rudders, which were changing too rapidly (with sign) at these points – compare qualitatively, for ε'_z, Figure 3 to the 'theoretical' Figure 2, or Table 5 to Table 1. Also, some other inherent biases due to the smoothing procedure adopted from Artyszuk (Artyszuk, 2002) may additionally be further blamed for the failure of the identification algorithm. However, this smoothing method was originally developed for another purpose and thus may need refining in the context of the present research. There obviously could be another problem with this real example, of a hydrodynamic nature, as relating to the ship's nonlinear behaviour. However, this is out of the scope of the present work.

Sensitivity analysis (*Table 8*). Methodically (or mathematically), the sensitivity to one or all variables, at least partly, depends on the chosen reference point, and cannot always be generalized. However, some interesting rough findings can be made from Table 8, a detailed analysis of which has been postponed for the future.

First of all, the influence is essentially not symmetrical against an increase/decrease of particular variable (-s). Such behaviour has often been forgotten or overlooked in other sensitivity studies on different subjects.

With regard to the significance of particular input data, the largest effect was surprisingly observed within the yaw velocity ω'_z variation (corresponding to the potential errors in the real yaw velocity). The model parameters that were then determined were losing a reasonable, physical meaning – being too high in absolute value and/or of unacceptable sign. A very significant effect, but smaller, was also produced by the yaw acceleration (ε'_z) at the first counter-rudder, as associated with the first heading deviation of 10°. The other yaw accelerations, as well as the headings, had similar yet much lesser effects if compared to the previous cases.

The sensitivities of K and T_1 were close to each other and essentially lower than those found for T_2 and T_3 (or for their relative measures in terms of T_2/T_1 and T_3/T_2).

One may of course wonder how the identified model parameters with such 'fouled/varied' input data affect the simulation of the zigzag manoeuvre against the reference run. The issue of such a backward simulation, however, has not been raised in present investigations.

The algorithm developed in this paper is universal. It worked well for both the first- and second-order zigzag performance, at least for a simulated input. The 'type' of zigzag is of course unknown a priori but has been evidenced though the type of output.

It must be strongly emphasised here that the first-order procedure of Nomoto (Nomoto, 1960) is less sensitive to real input data, and seems to be more robust. It only takes heading and its first derivative (yaw velocity) into account. However, this procedure returns, by fitting, the first-order approximation (model) even for an essentially second-order behaviour. It can be said that this is done by forcing $T_2 = T_3$. As such, the resulting K and T values by the Nomoto (Nomoto, 1960) procedure can often and significantly lose physical (hydrodynamic) meaning, as reported by Artyszuk (Artyszuk, 2016) and mentioned in the introduction section of this paper. In other words, such a first-order model mathematically fits the kinematics of the zigzag well, but its parameters are partially useless. Especially if one attempts to somehow reconstruct the steering hydrodynamics as expressed by the set of coupled first-order linear equations.

For the reader's reference, the Nomoto (Nomoto, 1960) procedure applied to the second-order zigzag behaviour with the data of (24), plotted in Figure 2, gave $K = 2.06$ and $T = 3.35$ instead of being close to 4.90 and 10.49 (= T_1), respectively, as of (24). This came from the author's previous paper (Artyszuk, 2016). This is why the first-order procedure was not applied to the real data of Figure 3, which, by definition, always works, but might return an inadequate output, very similar to this example.

Conclusions

In the present study a problem of a simple identification has been formulated with respect to the second-order linear Nomoto model and the zigzag test, and solved analytically. This is a natural extension, which has not been challenged so far and thus is partly new, to the well-known solution for the first-order model. The approach essentially uses the heading and its two derivatives at the four consecutive moments of applying either counter-rudder or reaching overshoot angles. Those points are also well disposed over the zigzag record.

The conducted sensitivity tests revealed a huge vulnerability of the identified four parameters of the second-order model to rather small (of order 10%) excursions from the input motion data. The problem probably lies more within the model than in the identification procedure itself. Specific to the latter are i.e. the number and location of the zigzag moments as selected for the input.

In detail, perhaps the second-order model is either very or too accurate, particularly in terms of simulating the derivatives of the heading and at least in the context of the zigzag, which is hardly noticeable if someone solely and roughly looks at the heading performance. This must impose some special requirements on the accuracy and adequacy of the input data. The adequacy is related to whether the ship being identified (or the input supplied) really exhibits linear behaviour. Otherwise, with nonlinearity, some unexpected or exaggerated results can be produced.

On the other hand, in some aspects, the identification algorithm developed can also be troublesome and contribute to these problems. This algorithm relies on the minimum number of data points, to just simply render the solution, and essentially belongs to the interpolation class, not to the approximation class. In the former, the model (its parameters) is being truly forced to 'hang' on certain discrete data points rather than to approximate them. In contrast, within the multiple- or continuous-point approximation, the model should be less prone to the random input errors or biases and thus be more stable.

Additional future efforts are welcome in both of these aspects (the model and the identification procedure) in order to break down and resolve this problematic behaviour. To some extent, as is well-known, the inconvenience with the second-order model identification has also been reported in the literature. But it is not exactly of the same sort of problems as has been encountered throughout the present investigations.

A validity of linearity in the steering performance based on the zigzag test, of the first- or second-order, is also a good subject for future research. It shall be subject to revision because the published results available, particularly supporting the first-order model superiority, mostly relate to older ship designs. This probably would involve much more advanced measurement instrumentation, especially consisting of accurate inertial sensors for angular velocity and acceleration, and/or data processing techniques. The latter may also cover some sophisticated numerical differentiation (and/or smoothing) methods, when e.g. the pure heading is only being focused on.

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References

1. Artyszuk, J. (2002) Bezier approach to smoothing zigzag data for use in ship maneuvering model identification. *Annual of Navigation* 4, Polish Navigation Forum, pp. 5–12.
2. Artyszuk, J. (2016) Peculiarities of zigzag behaviour in linear models of ship yaw motion. *Annual of Navigation* 26, DOI: 10.1515/aon-2016-0002, pp. 23–38.
3. Artyszuk, J. (2017) Performance of the second-order linear Nomoto model in terms of zigzag curve parameters. In: Weintrit A. (Ed.) *Marine Navigation* (12th Intl. Conf. on Marine Navigation and Safety of Sea Transportation 'TransNav 2017', Gdynia, Jun 21–23). Leiden: CRC Press/Balkema, pp. 403–409.
4. Nomoto, K. (1960) *Analysis of Kempf's standard maneuver test and proposed steering quality indices*. In First Symposium on Ship Maneuverability, May 24–25, DTMB Rep. 1461 (AD 442036). Washington: DTMB, pp. 275–304.
5. Norrbin, N.H. (1996) Further Notes on the Dynamic Stability Parameter and the Prediction of Manoeuvring Characteristics. In: Chislett M.S. (Ed.) *Marine Simulation and Ship Manoeuvrability* (MARSIM '96 Proc., Sep 9–13, DMI, Copenhagen). Rotterdam: Balkema, pp. 527–534.