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Dynamics models of drive systems with DC and AC motors

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Abstract. Mathematical models of two electromechanical systems consisting of a DC motor and an AC motor were built. To determine the moment of active (driving) torque as a function of angular velocity, Kirchhoff voltage equations were used in the system with the DC motor. In the system with the alternating current, equations determining the components of the stator and rotor vectors and the voltage equation were used. Both models include susceptibility and suppression of motion transmission systems. It was assumed that the speed control is carried out according to the preset traffic pattern in the feedback system in relation to position, speed and acceleration. The Runge-Kutta method was used to solve the equations. The author's own simulation program was written, which allowed to determine the time of displacement, speed and acceleration of the output shaft depending on the resistance of motion and the adopted method of rotational speed control. Calculation examples have been provided. Conclusions and suggestions resulting from the simulation have been formulated.

Key words: DC motor, AC motor, dynamics, drive system, theoretical model, motion simulation.

INTRODUCTION

Nowadays, it is difficult to imagine life without electrical devices. Micromotors are produced with the power of several dozen milliwatts and motors with the capacity of several gigawatts. Low power electric drive is used in everyday and household appliances, medium and high power in industrial equipment (e.g. in cement plants) and transport equipment (e.g. in locomotives, trams, trolleybuses). Thanks to the development of new technologies for the production of electric batteries, electric motors are also used to drive vehicles without access to the power grid [5, 7, 10, 19, 24]. It is, among others, a way to reduce the consumption of liquid fuels and carbon dioxide emissions [20, 22].

DC motors are used to drive low and medium power devices. A distinction is made between series motors in which the stator and stator windings are connected in series. They are often called universal motors because they work on DC as well as on alternating current. Motors in which windings are connected in parallel are called bypass motors. Due to the "rigid" torque characteristics, they are used in devices that require precise speed control (e.g. in manipulators). To drive small devices, permanent magnet motors are used, often equipped with an electronic commutator, which does not cause generating electromagnetic interference. Speed control in DC motors is only possible by changing the value of the supply voltage.

A separate chapter for the development of electric drives are AC motors. In 1889, Pole Michał Doliwo-Dobrowolski, electrical engineer and inventor, pioneer of three-phase current technology, constructed a three-phase induction motor (asynchronous) with a 80W squared impeller [9]. The construction of the invented engine was improved in the following years. The shape of the rotor cage was modified. This resulted in the two-well cage and deep groove rotors.

So-called ring motors with a three-phase armature winding led out through three slip rings by means of brackets adhering to the rings. When the three pins were shorted, the rotor obtained the maximum speed. The reduction of the current and speed consumed was obtained by increasing the resistance of the armature windings by inserting resistors into its circuit or by using a chopper (current chopper). In addition to the regulation of the additional resistance value, a cascade control system based on supplying the armature from an external power supply was used [8]. Ring motors are less and less used due to the wear of brushes.

Asynchronous cage motor has a simpler structure than a DC motor and less than about 30% mass at the same power, it is reliable, resistant to overload and works quietly. Thanks to its advantages, the cage engine is also commonly used to drive machines and devices for everyday use.

The use of DC caged asynchronous motors from accumulators or traction requires the use of so-called inverters - advanced converters (converters) of direct current into three-phase alternating current. Three-phase asynchronous cage motors powered by the inverter are currently used in traction vehicles (locomotives, trams, trolleybuses) and vehicles equipped with lithium-ion batteries (passenger cars, trucks, buses and even motorcycles) [3, 21, 23]. Changing the cage motor speed is only possible by changing the frequency of the stator supply voltage. In speed control systems, power transistors, triacs and microprocessors are used. In order to avoid mechanical and electrical overloads, starting and braking of the motor is often done according to the set "motion pattern" defining the time courses of displacement, velocity and acceleration.

For modeling and computer simulation of electric drives and power systems, mathematical models [1, 16, 18] and commercial programs are used, among others MatLab Simulink [6, 12, 13, 15] and others [2]. There are also used methods of artificial intelligence and fuzzy models [11, 14, 25] and even statistical programs [4].

The work presents calculation algorithms for drive systems with a DC bypass motor and an AC motor. In the mathematical models of dynamics of drive systems, dynamic characteristics of engines with control were taken into account. The presented models, despite the existing differences, are similar modules that can be used interchangeably depending on the type of drive selected. The author's simulation program was created that links geometrical parameters (gear ratio), kinematic (displacement, speed, acceleration), dynamic (the course of active and passive forces in the drive system) and electrical (voltage, current, frequency of current). With the use of the software, times mileages of the kinematic courses of output shaft drive systems for a sinusoidal "motion pattern" were determined.

DEFINING THE TASK OF A DRIVING SYSTEM MODELING

Exemplary drive systems are shown in Figure 1. Drive sets consist of motors (2) – DC, AC. The motors are supplied via the regulator (1) – of the controlled rectifier and the inverter. Rotor speed is controlled by changing the supply voltage, the frequency of the supply voltage u_z. The displacement Θ_1 is measured by the sensor (6), while the speed is measured by the tacho electric generator (7). The angular acceleration signal is obtained from the angular velocity measurement in subsequent time intervals. Signals of measured values are compared with setpoint signals from the "motion pattern" generator (4). In the controller (5), the error signals are amplified and converted into control signals of the regulator. The driving torque M_n is transferred by the gearbox (3) with the i_p gear ratio and then via a flexible shaft with torsional elastic coefficients k and damping l to the driven device at the moment of load M_o.

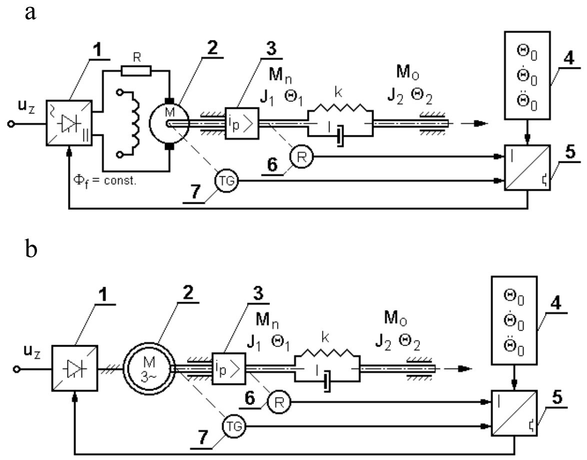

Fig. 1. Driving systems, a – of DC motor, b – of AC motor

The mathematical model of the DC motor assembly was constructed using Kirchhoff voltage equations. In the model of the AC motor, the equations of the stator and armature streams were used. The susceptibility and damping of the motion transmission system's vibrations, feedback control in relation to position, speed and acceleration were taken into account. Mechanical and magnetic hysteresis, influence of eddy currents and magnetic saturation were omitted. It was assumed that the three-phase machine is symmetrical and the voltage waveforms are sinusoidal.

MATHEMATICAL MODEL OF DRIVING SYSTEMS

For a DC motor, the Kirchhoff equation for the armature circuit has the form [17]:

$$u_a(t) = ci_p \frac{d\Theta_1(t)}{dt} + R_a i_a(t) + L_a \frac{di_a(t)}{dt}, \quad (1)$$

where: c – motor constant [V·s], $i_a(t)$ – armature current [A], i_p – reduction gear ratio, L_a – armature inductance [H], R_a – armature circuit resistance [Ω], $u_a(t)$ – armature voltage [V], $\Theta_1(t)$ – angular displacement of the transmission output shaft [rad].

The electric torque of the engine:

$$M_s(t) = c\, i_a(t)\,. \quad (2)$$

The equation of mass balance with reduced inertia J_1 has the form:

$$J_1 \frac{d^2\Theta_1(t)}{dt^2} = M_n(t) - M_{kl}(t)\,, \quad (3)$$

where:

$$M_n(t) = M_s(t)\, i_p\,, \quad (4)$$

$$M_{kl} = k[\Theta_1(t) - \Theta_2(t)] + l\left[\frac{d\Theta_1(t)}{dt} - \frac{d\Theta_2(t)}{dt}\right], \quad (5)$$

$\Theta_2(t)$ – angular displacement of the output shaft [rad].

Assuming the load of the output shaft of the system $M_o(t)$, after the system is reduced to its axis, the equation of the balance of the second mass of inertia J_2 for a system with two degrees of freedom has the form:

$$J_2 \frac{d^2\Theta_2(t)}{dt^2} = M_{kl}(t) - M_o(t)\,. \quad (6)$$

The above equation is valid when the value of the moment of inertia J_2 is unchanged over time.

After making minor transformations and entering the coordinates of the state:

$$Y_1 = \dot{\Theta}_1, Y_2 = \dot{\Theta}_2, Y_3 = \Theta_1, Y_4 = \Theta_2, Y_5 = i_a, \quad (7)$$

we get the equations of motion in the coordinates of the state:

$$\frac{dY_1}{dt} = \frac{1}{J_1}(M_n - M_{kl}), \quad \frac{dY_2}{dt} = \frac{1}{J_2}(M_{kl} - M_o),$$

$$\frac{dY_3}{dt} = Y_1, \quad \frac{dY_4}{dt} = Y_2,$$

$$\frac{dY_5}{dt} = \frac{1}{L_a}\left(u_a(t) - R_a Y_5 - ci_p Y_1\right). \quad (8)$$

The initial conditions were accepted:

$$Y_1(0) = 0, Y_2(0) = 0, Y_3(0) = \Theta_0(0),$$

$$Y_4(0) = \Theta_0(0) - M_{st}(0)/k, Y_5(0) = 0,$$

$M_{st}(t)$ – load torque at start-up [N·m], $\Theta_0(t)$ – initial angular displacement [rad].

For an AC motor, the equations determining the constituent values of stator and armature vectors are [17]:

$$\frac{di_{s\alpha}}{dt} = \sigma^{-1}\left(-m_s i_{s\alpha} + \delta_s i_{s\beta} + k_s m_r i_{r\alpha} + \dot{\Theta}_1 i_p k_s i_{r\beta} + g_s U_m\right),$$

$$\frac{di_{s\beta}}{dt} = \sigma^{-1}\left(-\delta_s i_{s\alpha} - m_s i_{s\beta} - \dot{\Theta}_1 i_p k_s i_{r\alpha} + m_r k_s i_{r\beta}\right), \quad (9)$$

$$\frac{di_{r\alpha}}{dt} = \sigma^{-1}\left(m_s k_r i_{s\alpha} - \dot{\Theta}_1 i_p k_r i_{s\beta} - m_r i_{r\alpha} + \delta_r i_{r\beta} - g_s k_r U_m\right),$$

$$\frac{di_{r\beta}}{dt} = \sigma^{-1}\left(k_r \dot{\Theta}_1 i_p i_{s\alpha} + m_s k_r i_{s\beta} - \delta_r i_{r\alpha} - m_r i_{r\beta}\right).$$

The electric torque of the engine is expressed by the formula:

$$M_s(t) = \frac{3}{2} p L_m \left(i_{s\beta} i_{r\alpha} - i_{s\alpha} i_{r\beta}\right), \quad (10)$$

where:

$$g_s = \frac{1}{L_s},\ k_s = \frac{L_m}{L_{si}},\ k_r = \frac{L_m}{L_r},\ m_s = \frac{R_s}{L_s},$$

$$m_r = \frac{R_r}{L_r},\ \sigma = 1 - k_s k_r,$$

$$\delta_s = \omega_{sn}\sigma + \dot{\Theta}_1 i_p k_s k_r,\ \delta_r = \omega_{sn}\sigma - \dot{\Theta}_1 i_p,,$$

$$\omega_{sn} = 2\pi f,$$

f – frequency of the stator power supply [Hz], $i_{r\alpha}$, $i_{r\beta}$ – components of the armature current vector in the directions α and β [A], $i_{s\alpha}$, $i_{s\beta}$ – components of the stator current vector in the directions α and β [A], k_s, k_r – stator and armature coupling factors, L_m – stator and armature inductance [H], L_s, L_r – stator circuits inductance, armature [H], m_s, m_r – stator and armature damping coefficients [$\Omega\cdot H^{-1}$], p – number of pole pairs, R_s, R_r – resistance of stator and armature circuits [Ω], U_m – amplitude of stator voltage supply [V], σ – resultant dispersion factor (for asynchronous machines is about 1/15), ω_{sn} – pulsation stator power supply [s^{-1}].

The mechanical part of the drive unit is identical to that of the DC motor drive and is described by equations (3) ÷ (6).

After making minor transformations and entering the coordinates of the state:

$$Y_1 = \dot{\Theta}_1,\ Y_2 = \dot{\Theta}_2,\ Y_3 = \Theta_1,\ Y_4 = \Theta_2, \qquad (11)$$

$$Y_5 = i_{s\alpha},\ Y_6 = i_{s\beta},\ Y_7 = i_{r\alpha},\ Y_8 = i_{r\beta},$$

we get the equations of motion in the coordinates of the state:

$$\frac{dY_1}{dt} = \frac{1}{J_1}\left(M_n - M_{kl}\right),\ \frac{dY_2}{dt} = \frac{1}{J_2}\left(M_{kl} - M_o\right),$$

$$\frac{dY_3}{dt} = Y_1,\ \frac{dY_4}{dt} = Y_2,$$

$$\frac{dY_5}{dt} = \sigma^{-1}\left(-m_s Y_5 + \delta_s Y_6 + k_s m_r Y_7 + Y_1 i_p k_s Y_8 + g_s U_m\right),$$

$$\frac{dY_6}{dt} = \sigma^{-1}\left(-\delta_s Y_5 - m_s Y_6 - Y_1 i_p k_s Y_7 + m_r k_s Y_8\right), \qquad (12)$$

$$\frac{dY_7}{dt} = \sigma^{-1}\left(m_s k_r Y_5 - Y_1 i_p k_r Y_6 - m_r Y_7 + \delta_r Y_8 - g_s k_r U_m\right),$$

$$\frac{dY_8}{dt} = \sigma^{-1}\left(k_r Y_1 i_p Y_5 + m_s k_r Y_6 - \delta_r Y_7 - m_r Y_8\right).$$

The initial conditions were accepted:

$$Y_1(0) = 0, Y_2(0) = 0,\ Y_3(0) = \Theta_0(0),$$

$$Y_4(0) = \Theta_0(0) - M_{st}(0)/k,$$

$$Y_5(0) = 0, Y_6(0) = 0, Y_7(0) = 0, Y_8(0) = 0.$$

Any numerical integration method with constant or variable integration step can be used to solve equations.

MATHEMATICAL MODEL OF THE REGULATION SYSTEM

The development of a universal mathematical model of the control system is not possible due to the variety of solutions. Very simple mathematical models of control systems for DC and AC motors will be presented. Taking into account feedback control with respect to position, speed and accelerations, setting quantities are defined by the following relations:

$$\kappa_\varphi = K_\varphi\left[\Theta_1(t_n) - \Theta_0(t_n) - \frac{M_{st}(t_n)}{k}\right], \qquad (13)$$

$$\kappa_\omega = K_\omega\left[\dot{\Theta}_1(t_n) - \dot{\Theta}_0(t_n)\right], \qquad (14)$$

$$\kappa_\varepsilon = K_\varepsilon\left[\frac{\dot{\Theta}_1(t_n) - \dot{\Theta}_1(t_{n-1})}{t_n - t_{n-1}} - \ddot{\Theta}_0(t_n)\right], \qquad (15)$$

where: $K\varphi$, $K\omega$, $K\varepsilon$ – coefficients of amplification of position, velocity, acceleration, $K\varphi$, $K\omega$, $K\varepsilon$ – setting variables, $\Theta_0(t_n), \dot{\Theta}_0(t_n), \ddot{\Theta}_0(t_n)$ – set, reference values of kinematic parameters in the time step n.

In the current sampling time step, the value of the control parameter τ_{rn} is calculated as the difference between the parameter value from the previous step and the sum of the control values:

$$\tau_{rn} = \tau_{rn-1} - (\kappa_{\varphi} + \kappa_{\omega} + \kappa_{\varepsilon}) . \tag{16}$$

The parameter of the drive regulation with the DC bypass motor, in the time step n will be the armature voltage u_a (then τ_{rn} = u_{an}) or indirectly the angle of thyristor control ϑ_g (then τ_{rn} = ϑ_{gn}) and then the supply voltage [8]

$$u_a(t) = \frac{u_z m\sqrt{2}}{2\pi}\left[1 - \cos\vartheta_g(t)\right]\operatorname{sgn}\vartheta_g(t) , \tag{17}$$

where: m – number of phases, u_z – effective value of the alternating voltage supplying the armature [V], ϑ_g – thyristor extinguishing angle [rad].

The change of the kinematic parameters of the drive with the AC motor is obtained by changing the frequency f of the stator supply voltage (then $\tau_{rn} = f_n$).

All the dependences discussed in this chapter, after passing the description to the coordinates of the state, constitute a complete mathematical model of drive systems together with control systems, adapted to numerical integration.

NUMBERS EXAMPLES

The motion of drive units was simulated. A sinusoidal speed standard, a constant value of the mass moment of inertia J_2 and a time-varying load torque were assumed. The equations of state were solved by the Runge–Kutta method with a time step of 0.001 s. The following parameter values were adopted: – for a sinusoidal speed standard:
Θ_{0p} = 1.40 rad, Θ_{0k} = 2.49 rad, $(d\Theta_0/dt)_{\max}$ = 1.2 rad·s^{-1}, $(d^2\Theta_0/dt^2)_{\max}$ = 4 rad·s^{-2}, – load moment: $M_o(t)$ = 300sinΘ_2 N·m, – for the mechanical system: J_1 = 48 kg·m^2, J_2 = 2 kg·m^2, k = 90000 N·m·rad^{-1}, l = 500 kg·m·s^{-1}, i_p = 150, – for DC motor: c = 0.442 V·s, n = 3000 rpm, L_a = 0.0012 H, R_a = 15.2 Ω, m = 3, u_z =120 V~, N = 1000 W, – for AC motor: N = 1000 W, n = 1390 rpm, u_z = 220 V, f = 50 Hz, p = 2, L_m = 0.1695 H, m_s = 19 Ω·H^{-1}, m_r = 22 Ω·H^{-1}, k_s = 0.957, k_r = 0.906.

Figure 2 presents the motions of displacements, velocities and accelerations of the output shaft given by the "motion pattern" and obtained from the simulation.

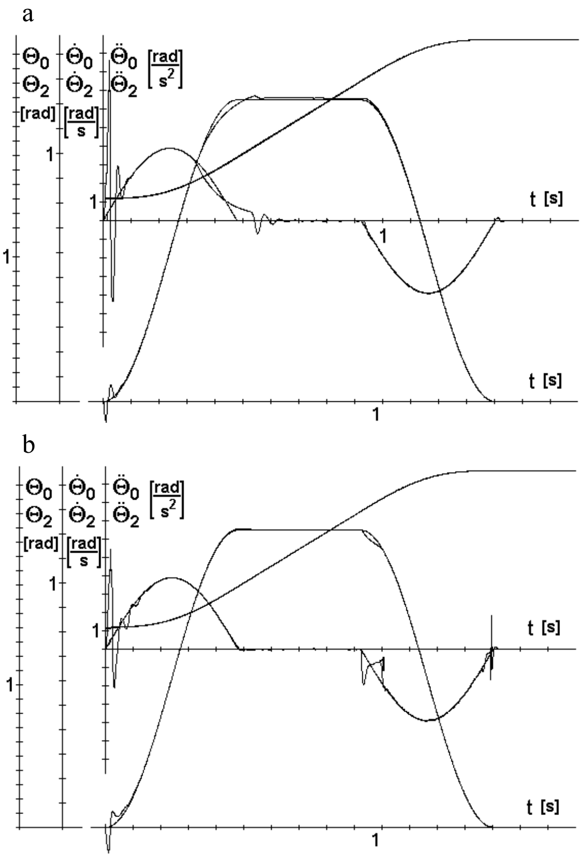

Fig. 2. Time related courses of kinematic parameters. a – of DC motor, b – of AC motor

CONCLUSIONS

The proposed mathematical models allow the analysis and selection of drive parameters, taking into account the susceptibility and damping of drive system components. It is possible to analyze vibration processes depending on kinematic parameters and loads. Models are ready-made "modules" to determine kinematic motion parameters.

Based on the simulations carried out, the following conclusions can be drawn:

1. The feedback control with respect to position, velocity and acceleration allows for a satisfactory mapping of given kinematic parameters, the one which strongly depends on the values of the amplification coefficients of the errors and the sampling frequency.
2. For the assumed numerical values, the greatest oscillations of accelerations occur in the start-up phase, which for the tested models with a DC motor are 8.7 rad/s^2 and the variable one is 5.5 rad/s^2.
3. Better control properties and less sensitivity to changes in parameters have been observed in the drive system with the DC motor, it is more difficult to adjust the drive system with the AC motor.

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