FRACTAL FORMAT FOR BITMAP IMAGES

Agnieszka Siwocha[1], Andrzej Cader[1],
Marcin Kolibabka[1], Marcin Krupski[2]

[1] IT Institute, University of Management, Lodz, Poland
(mkolibabka, acader, asiwocha)@spoleczna.pl

[2] Department of Computer Science in Economics
Institute of Applied Economics and Informatics
Faculty of Economics and Sociology
University of Lodz, Poland
mkrupski@uni.lodz.pl

Abstract

The conception of proposed recording format is the example of the theoretical and practical application of the FBS method, which was precisely described in thesis [18,19,20]. The foundation of the presented recording format is the use of a new method of fractal basis splines (FB-splines), which allows the reconstruction of complex geometric structures with the properties of fractals. Fractral basis splines method is based on the use of non-local characteristics to describe the interpolation nodes. With that the one-parameter family of fractal curves is used as the basic approximating elements.

Keywords: fractal interpolation method, image recording formats, fractal dimension, multifractal analysis, non-local characteristics.

1 Introduction

The mode of saving the image in the computer's memory essentially distinguishes between bitmapped graphic and vector graphics. Taking into account the criterion of recording the image still fractal graphic needs to be included in the above distribution, due to the fact that fractals can be recorded as system of iterated mapping IFS (Iterated Function System).

In vector graphics primitive components (objects) are stored using mathematical equations, which causes that they can be freely scaled without loosing the quality. This vector graphics formats are software formats, in which vector graphics is generated: AI (Illustrator) and SWF (Flash), CDR (CorelDraw program), SVG (Scalable Vector Graphics) – format created by the consortium World Wide Web Consortium (W3C) - the format based on XML language, promoted as the standard for vector graphics. Vector graphics is ideal for creating graphic designs and technical documentation. Files that store the content of the drawing have small size.

Coding bitmapped graphics formats can be divided into:

- using lossless compression - moderately reducing the file size without losing any information: RLE (run-length encoding) - sequence length encoding and LZW (Lemple-Ziv-Welch);
- using lossy compression significantly reducing the file size (up to 1/100) at the expense of loss of information, with strong compression a significant deterioration in the quality of the image can be seen (jpeg);
- not using compression.

The most popular bitmapped graphics formats: GIF (Graphics Interchange Format), PNG (Portable Network Graphics), TIFF (Tagged Image File Format), JPEG (Joint Photographic Expert Group), JPEG (JPEG Interchange Format), BMP (Bitmap Format).

The format, which describes the fractal image is the FIF (Fractal Interchange Format)format, which is based on the local image self-similarity and using fractal compression. It was introduced and patented by Interated Systems Inc. in 1987. FIF is something in between bitmapped and vector graphic, has many raster features , but it can also be scaled without loss of quality - such as in vector graphics. The file format created by Iterated Systems, Inc. is based on the mathematical thesis on fractal geometry from 1984. According to the princeples the images can be enlarged indefinitely without loss of detail, in practice, selecting the appropriate system of functions is difficult and can deliver extreme results from very good to quite average.

As an alternative to the FIF format, the authors suggest a different way of encoding a bitmap into a fractal form. Presented later in this article the algorithm encodes a bitmap into a fractal form, exactly using the FSB method (fractal splines Base) [18,19,20] wherein the chrominance chart of subsequent pixels of the raster image serves as the interpolated curve. The fractal record of the interpolation curve makes the encoded image form. The image is saved in the form of fractal interpolation curve decsribed in the interpolation nodes by fractal splines base scaling multipliers and non-local characteristics including the fractal dimension.

2 Theoretical basics of the method

The suggested method assumes that brightness values of consecutive pixels in the image line constitute a sequence which is a result of sampling of a sophisticated structure acting as a multi-fractal and being a reflection of the real image. Such a sequence may be analyzed by means of well-known methods based on multi-fractal spectrum analysis [18,19]. The sequence is a set of numerical values, each out of which automobile can be treated as a result of a Certain stochastic Process realization $P(x)$. The values in our expression (T_1, T_2,..., T_N) are established for the equal ranges Δx, where:

$$T_i = P(x = i\Delta x) \tag{1}$$

To make things easier, the sequence has been standardized:

$$\frac{1}{N}\sum_{k=1}^{N} T_k = 1 \tag{2}$$

A coarse function is defined:

$$T(x) = \sum_{k=1}^{N} T_k I_{[(k-1)\Delta x, k\Delta x]}(x) \tag{3}$$

where $I_A(x)$ is a specific function of set A in the form of:

$$I_A(x) = \begin{cases} 1, & for \quad x \in A \\ 0, & for \quad x \notin A \end{cases} \tag{4}$$

where the ensemble A in the presented method is the sum of intervals Δx.

The multi-fractal analysis is aimed at finding a new sequence $S(x)$ composed of whole numbers σ_i and chosen from the range $1, 2, \ldots, Q << N$, where each of these numbers corresponds to the value of the coarse Holder exponent from the range $[(k\text{-}1)\Delta x, k\Delta x]$ — a form of the Holder exponent α digitization:

$$S(x) = \sum_{k=1}^{N} \sigma_k I_{[(k-1)\Delta x, k\Delta x]}(x) \tag{5}$$

The purpose of these proceedings is to select subsets composed of ranges with the same σ_i value (discreet value of the Holder exponent).

In practice, we deal with discreet structures presented with a given resolution, for example in computer algorithms conditioned by the pixel size. In such a case, the coarse Holder exponent is used. Using a dependency, $\alpha(x,\delta x)$ is defined for the rane δx around the point x:

$$G_{\alpha(x,\delta x)}(x) = \mu\left(B_{\delta x}(x)\right)\left(\delta x\right)^{-\alpha(x,\delta x)} \tag{6}$$

where μ(Bδx (x)) is a certain measure indicating the probability that a randomly selected point is in the area of $B_{\delta x}(x)$ defined by the interval δx around the point x. In generality, the $G_{\alpha(x,\delta x)}(x)$ for any value of $\alpha(x,\delta x)$ takes the value of zero or infinity, there is only one value, for which the above measure is finite. This value determines the Holder exponent at the point x. $\alpha(x,\delta x)$ may be calculated by logarithming the above dependency:

$$\ln\mu\left(B_{\delta x}\left(x\right)\right)=\alpha\left(x,\delta x\right)\ln\delta x+\ln G_{\alpha\left(x,\delta x\right)}\left(x\right) \tag{7}$$

and by using a well-known procedure designates points in the coordinates $(ln\mu(B_{\delta x}(x)),\ ln(\delta x))$ for the ranges of various sizes. The coarse Holder exponent determines the inclination of a straight line adjusted with the least squares method. The procedure should be used for each point (pixel).

The notion of an interpolation node plays an important role in fractal interpolation methods. In case of reconstructing complex fractal structures, however, the classic notion of an interpolation node is no longer sufficient as it can not be attributed to one, clear-cut value. What follows is a necessity to introduce a new quantity defining a place of data gathering and being able to reconstruct the structure in the future. The Local Interpolation Window (LIW) [18.21] can be treated as such a new term. LIW is absolutely crucial with regards to gathering data needed for a local reconstruction of the interpolated shape. Numerical data from the LIW area, for example a local fractal dimension, create a data set on the basis of which a curve, representing a dependency of brightness on the pixel position, is reconstructed.

Using a similar approach, the coarse Holder exponent $\alpha(x,\delta x)$ can be averaged with the ranges representing the LIW width:

$$\alpha\left(x\right)=\sum_{i=1}^{N}\left\langle\alpha\left(x,\delta x\right)\right\rangle_{\Delta x_i} I_{\Delta x_i}\left(x\right) \tag{8}$$

Where $I_{\Delta x_i}\left(x\right)$ is a characteristic function (4) $\left\langle\alpha\left(x,\delta x\right)\right\rangle_{\Delta x_i}$ — medium value $\alpha(x,\delta x)$ in the range of Δx_i.

As a result, the multi-fractal curve breaks up, with a certain approximation dependent on the LIW width, into local (within the ranges of Δx_i) monofractal sets characterized by one, unique value of the fractal dimension $D(\Delta x_i)=D_i$ and in each range. The original curve may then be locally reconstructed by means of set fractal curves with the right dimensions. Practically, this is a case where a dimension may be calculated for every window as a box dimension, which greatly facilitates the calculations.

For the interpolated structure to be reconstructed, it is necessary to have a set of base curves fractals defined in the standard Δx range, for example: Δx

=[0,1]. For our practical proceedings, we have chosen a family of curves based on the well-known Koch's curve, and we shall call them Generalized von Koch Curves (GKC) [18]. One advantage of our choice is a simple dependency defining the GKC fractal dimension:

$$D = \frac{\ln 4}{\ln\left[2\left(1+\cos\varphi\right)\right]} \tag{9}$$

in the above parameter $\varphi \in [0, \frac{\pi}{2})$, is the opening angle of breaking in the curve.

Interpolated curve may be marked by dependency means:

$$L \approx \sum_{i=0}^{N} s_i K_i (x - i\Delta x, y, D_i) \tag{10}$$

where: s_i – interpolation coefficients, and fractal curves $K_i(x,y)$ are placed centrally (with the highest possible value) he interpolation points.

Let us assume that we know the values of the curves y_1, y_2, ..., y_n in the set interpolation nodes x_1, x_2,..., x_n (they represent the brightness of the marked pixels) as well as the fractal dimensions D_1, D_2,..., D_n of the interpolated curve L defined in the respective LIW interpolation windows corresponding to the given nodes The s_i coefficients are calculated iteratively on the basis of the expression:

$$y_i = \sum_{j=0}^{N} s_j^{n+1} y_j \left(\frac{(i-j)\Delta x}{s_j^n}, D_j \right)^* \tag{11}$$

where: s^0_j=1, y_i - set values of interpolated nodes, $y_j(x,D_j)$ is a set of y values of the curve $K_j(x,y,D_j)$ numbered j for the set value of x, and $K_j(x,y,D_j)$ - starting FB-splines are centrally located on each interpolation point. The basic curves' span equal to $6\Delta x$ has been assumed for our calculations. In expression (11), a possibility of occurrence of many FB-splines values with a set x has been taken into account. Therefore:

$$y_j^n(x, D_j)^* = \frac{1}{2}\left[\sup y_j^n(x, D_j) + \inf y_j^n(x, D_j)\right] \tag{12}$$

The mean square error of iteration is an accuracy measure of the calculations:

$$\delta^n = \sum_{i=0}^{N} \left(y_i - \sum_{j=0}^{N} y_j \left((i-j)\Delta x, D_j \right)^* \right)^2 \tag{13}$$

While conducting the interpolation, we have to remember about the notorious problem of lack of data at the ending points of the interpolated curve. These values have to be defined separately.

3 The description of the raster image processing into a fractal form with record algorithm

The concept of processing the raster image to the record in the fractal form is based on the method of fractal splines basis to store the level of chrominance or exertion of individual components RGB following image pixels. Therefore the chrominance pixel of the bitmap graph can be restored by using the method of fractal splines basis. We encode the image data into a data form needed to calculate the interpolation function by using the FSB (fractal splines base) method, instead of direct record of chrominance level values or intensity of the individual components of the following image pixels. The whole image is treated as a single sequence of pixels, without considering the division into the lines. In order to increase the probability of implying the close pixels position on the interpolated curve by their close position on the image, the encoding of the odd-numbered lines of the image in order from left to right was used, while with the even-numbered lines - from right to left. The existence of the above relation facilitates the application of fractal dimension read from a local interpolation window as an additional parameter describing the image, since it is closely related then to the specific consistent area.

Image can be encoded in the mode of three additive components - then it is treated as three separate monochromatic images representing each component. In order to encode the image based on all its pixels a chrominance chart is being created. In the case of using the RGB model the component G has the highest bit depth of the parameters, the most severe error criterion and the largest width of the local interpolating window , setting out the analysis range of non-local parameters of the curve. Due to the fact that the final outcome it is difficult to be predicted, that is the number of iterations and the total error of the using the fractal splines base algorithm, it is necessary to submit the image data to many attempts to recover.

The aim of the proposed method presupposes the attempt to recover the initially given segment of the generated graph so that in the case of failure of its reconstruction by using the method of fractal splines basis with the result, which easy to describe with the smaller amount of data, be similarly examined, but in truncated form.

Therefore, the number of blocks into which the entire chrominance graph is going to be divided as well as the length of each block can not be determined in advance. In relation to that, the authors propose to base the image recording process on parameters, such as: the number interpolation nodes; permissible number of iterations the fractal splines basis method; permissible error of the curve projection using fractal spline basis method; the initial length of encoded in the block graph segment; shortening multiplier of the graph block.

The first parameter, called the number of interpolation nodes is the value required to start, here treated as the service, fractal splines basis method. The number of interpolation nodes used to encode of each of the analyzed segments of the graph is imposed on by the user prior to the encoding process. It has a significant impact on the reconstruction of the image - the greater number of interpolation nodes helps to obtain a better mapping, however it may turn out that achieving such conditions entails the consequence of carrying out many more operations.

The process of converting an image into a curve treated later in the process as the fractal shape has a remarkable influence on the further course of the algorithm. The conducted tests showed, that the result obtained, in particular the file size and accuracy of mapping is extremely sensitive to the values such as: length of initially being tested to be reconstructed segment of the particular graph in the *initial length of the interpolated curve* parameter; the expected maximum value of the deviation of interpolated curve which is the medium encoding the part of the graph from the relevant graph, acting as the interpolated function here; permissible number of iterative repetitions of the fractal splines basis algorithm

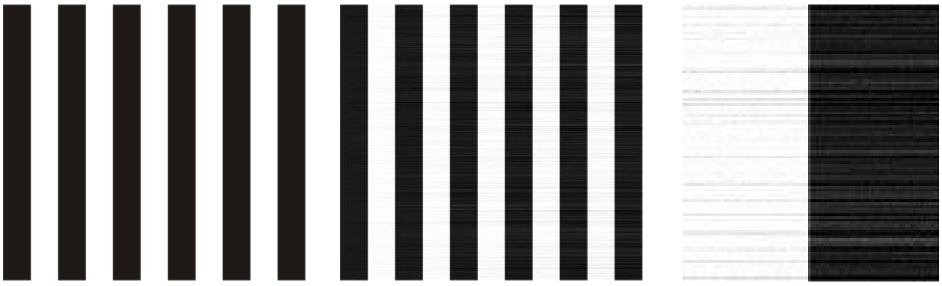
On the serialization speed, apart from *maximum number of iterations in the interpolation* impacts primarily reduction mulitiplier of the encoded segment of the graph *(interpolated curve reducing).* Reducing occurs when after performing a specified number of repetitions during any of the iterations it failed to get the error below the value specified in the *the maximum permissible error on the pixel* parameter. The authors aim is such a match of detailed algorithm solutions and also other parameters values that the remaining values, that is *the ratio of the number of pixels to the length of the curve,* and *the value for the white color,* which are the multipliers of the horizontal and vertical fractal curve graduation did not affect the time and the result of image serialization.

In the process of image encoding the fractal dimension plays a significant role. However, its use is possible only in the case of a large size images. It results from the fact that used in the Fractal Splines Basis method method of researching the box-counting dimension based on the Kolmogorov algorithm requires high resolution. For the treated as a fractal curve chrominance graph to meet this requirement, the image should consist at least of one-fourth, and preferably of half of other megapixel. The authors suggest the use of the wide, approximately 2048 pixels, Local Interpolation Windows, since only similar values provide the opportunity to detect the fractal structure of the image. Due to construction of the FBS method, elementary approximating shapes will be in the selected nodes having a fractal structure similar to the value read from the image. It should be taken into consideration that both the impact of the dimension value on the image and the method of its reading do not apply to a single pixel, or a designated spot in the image. They are connected with the widely understood environment. With this use of fractal dimension as a non-local characteristics it will be able to get a chance of a single isolation of part of common image fragment features with simultaneus, taking into account differences saved by graduation basic splines multipliers of automatically placed interpolation nodes.

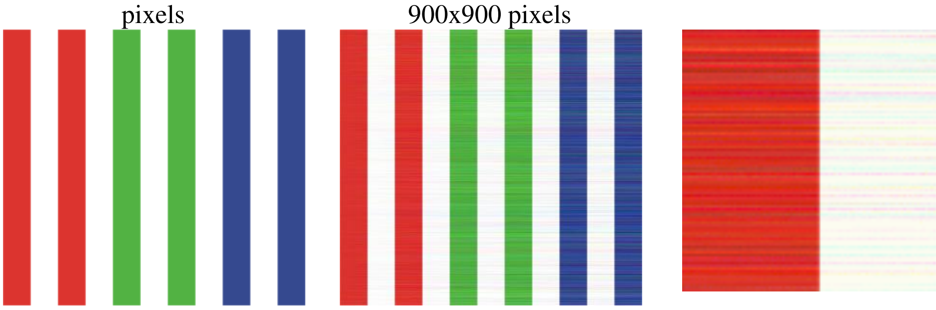
5 The results of research on the sample images

Tests were carried out on a wide range of images, gained both from imaging devices and generated algorithmically. Among many tests on the images examples were selected, which in the best way are showing issues that have yet to be solved, and the possibilities of the proposed method. The main weakness of the method is most often showed as a deviation of color saturation or brightness value occurring for all the horizontal lines of the image. Is most common where there are step changes in the values of these parameters.

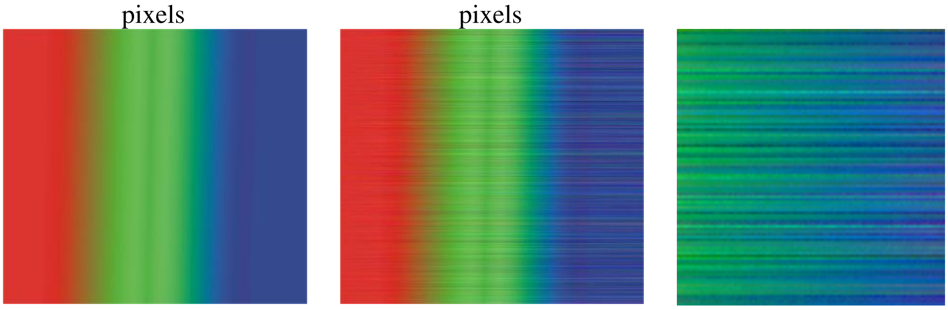
In the case of algorithmically generated image in shades of gray, the horizontal lines in the color that differs from the color of their equivalents in the source image are existing in the areas of the color similar to black. This situation reflects serious problems in obtaining, with the fractal basic splines method, unified smooth curve with the value close to zero. As part of further work on improving the method, the authors plan inter alia to expand it with a mechanism enabling the elimination of smooth or linear graph fragments from the curve undergoing fractal reconstruction. The fragments of the classic curves will be saved in a way that better reviewers their character.



a)BMP 2,54 MB (2 674 194 bytes) 900x900 pixels | b) BMP (z FZO) 640 KB (655 360 bytes) 900x900 pixels | c) FZO 1,27 MB (1 339 392 bytes) (1:1)



a)BMP 2,54 MB (2 670 592 bytes) 900x900 pixels | b) BMP (z FZO) 884 KB (905 216 bytes) 900x900 pixels | c) FZO(1:1) 1,69 MB (1 781 760 bytes) (1:1)



a)BMP 2,55 MB (2 678 784 bytes) 900x900 pixels | b) BMP (z FZO) 1,11 MB (1 171 456 bytes) 900x900 pixels | c) FZO 1,70 MB (1 785 856 bytes) (1:1)

Figure 1. a) original image – BMP format - a computer-generated image; b) fractal image encoded using fractal basic splines method (FB-splines) - save in the bmp format ; c) Image encoded using fractal base splines method (FB-splines) saved in the proposed FZO format, decoded in the file browser FZO

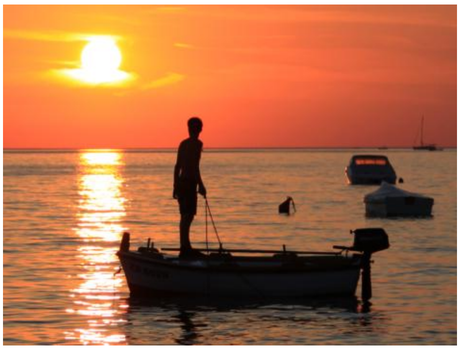
The applied above example proves that the method can handle sharp edges very well. In addition, good results are guaranted iteratively invoking the fractal splines basis method for graph fragments of different length. The above result was clearly much improved by implemented mechanism of searching and a separately encoding, different from the surrounding points of the graph, so-called high and low peaks. Because these elements were present in the above examples exceptionally often, resulting in the repetition of a particular sequence of bytes in the file that stores the encrypted form of the image, the algorithm using compression zip, could additionally reduce the size of that file.

In case of application work on the image in gray scales the fractal image recording algorithm does not encode three separate components RGB, only one chrominance graph. Therefore there is no danger of values being inconsistent for any part of the image, where one or two from the components were extremely overestimated, and the other - in this particular part underestimated. Such situations very strongly spoil visually the end result. The described example shows how exactly the fractal image recording method is able to extract from the image the fractal structures and with how high accuracy will it later be able to restore them. At this point, the authors are convinced, that from every image it can be extracted smaller or bigger group of data, having such chaotic, irregular or jagged runs, just in order to subject them to the encoding with the suggested method.

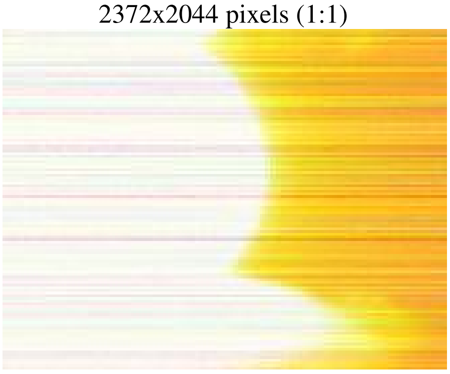
Analogical used of the suggested method for the color image creates a threat of the previously mentioned components being inconsistent. Situations, in which there unexpectedly appear significant differences between pixels of the subsequent lines of image on the smooth areas is mainly caused by the fact that the use of the fractal methods do not work for processing of smooth shapes. The above problem can be fixed by averaging the subsequent pixels with neighbors lying below and above. Unfortunately, the unavoidable consequence of such operation must be the loss of data, and consequently a blured image. Homogeneous smooth large areas detected by the algorithm results in greater reproducibility of data in the binary file containing the encoded image, which was visible in the stronger compression of the file using zip compression.

a) JPG 1,81 MB (1 905 711 bytes)
2372x2044 pixels

b) FZO 9,24 MB (9 696 742 bytes),
ZIP 6,63 MB (6 955 008 bytes)
2372x2044 pixels (1:1)

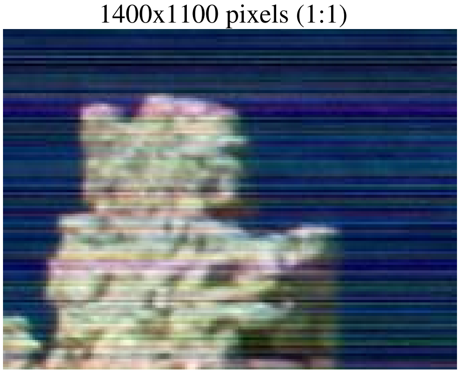


a) JPG 850 KB (870 977 bytes)
1400x1100 pixels



b) FZO 3,09 MB (3 250 182 bytes),
ZIP 1,83 MB (1 925 120 bytes)
1400x1100 pixels (1:1)

a) JPG 980 KB (1 003 520 bytes)
1452x984



b) FZO 2,72 MB (2 857 542 bytes)
ZIP 2,04 MB (2 142 184 bytes)
1400x1100 pixels (1:1)

Figure 1. a) original image – JPG format; b) Image encoded using fractal base splines method (FB-splines) saved in the proposed FZO format, decoded in the file browser FZO

The above example specifically illustrates the restrictions of the use of the suggested method to images with contain information of visibly low frequency. In large parts of the image large step changes appear only in the dominant red component. Also a small green element can bo spotted. In addition, the image looks exactly like the result of the applying of a strong low-pass filter. According to the authors realizing described earlier in this thesis mechanism of separately encoding fragments of classical curves will significantly improve the algorithm performance of the algorithm in such applications as described above.

5 Summary and conclusions

The presented above juxtaposition of bitmap is selected in such a way, to present the perceived advantages of the proposed recording method, but also in order to illustrate the particularly troubling drawbacks and deficiencies, with particular emphasis on those, with which the authors are planning to face in the near future. To the already mentioned advantages of the suggested method belongs, first of all the ability to match the encoding media into the irregular structures having chaotic and rough course. Much better result were achieved trying to save images with high frequency, than images partially averaged with a low-pass filter. As shown by the above examples, the majority of the work awaits the authors in areas for service in such situations, where the graph of color intensity or chrominancy takes the form of the classical curve, in particular linear function. Selecting such areas and their encoding using different algorithms is the major authors hope to improve the quality of decoded data achieved and the size of the files that contain their encoded form.

There are also no doubt that a lot of focus in the future will need to be given to the selection of the points separating the chrominancy graph on individual fragments treated as fractal curves undergoing reconstruction. The proper dividing of the graph to the relevant parts will provide a better fit of the interpolating curve to the interpolated one due to a better separation of certain different local characters of the examined runs. As a result, it will be possible to obtain improved image quality and reduced size of the file with the encoded data. It also seems necessary to return to the subject of the sequence of processing the source image into chrominancy curve or intensity of the component color. An attempt was made to apply to this purpose the Peano curve, however it did not bring the expected results. The revised order of reading the pixels, increased the probability of grouping similar pixels close to each other, but unfortunately it did not provide continuity of the most important characteristics of fractal structures.

It will also be necessary focus future research on the linking the image transformation algorithm into a graph with the detection of edges, surface, or other initial image analysis. The ideal solution seems to be the concept that is being able to connect the two recently discussed issues, that is, the enrichement of the algorithm with the ability to initialy divide the image into areas with similar fractal structure and connecting with them relevant, consistent in terms of the many features fragment of the analyzed later graph.

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